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CONCEPT AND USE OF MODELS
(PARTS I AND II)

W. H. HOPPMANN II

NOVEMBER 1977

U.S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
Aberdeen Proving Ground, Maryland

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An attempt is made to present a unified view of modeling. Beginning with the earliest concept of model a brief history of the subject is presented for the period beginning from prehistoric times into the twentieth century.

The evolution of the concept of model is discussed in some detail and a taxonomy of the subject is suggested.

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20. ABSTRACT (Continued)

Special attention is given to the relationship between the concept of model and the prevailing ideas of space, time, and matter. The role of dimensional analysis is reviewed and its relationship to modeling examined at length.

Specific types of models which constitute the taxonomy are defined and illustrated with numerous examples. The idea of a Newtonian model is introduced and then extended to cover many physical cases.

The new concept of disclosive model and its role is defined and illustrated. Its historical origin is reviewed. Its value in the evolution of generalized modeling is examined at length.

In Part II the treatment of the concept model has been greatly extended. In Part I we presented a brief history of the subject from the earliest days of man to the latter part of the twentieth century. Also, a tentative classification of models was suggested. It consisted of what we called the iconic, the analogic, the similitudinous, the Newtonian, the extended Newtonian, and the disclosive models. The latter category was suggested for the first time to our knowledge. In Part II we have explored somewhat at length our understanding of the concept of disclosive model and its ramifications. Also, we found it necessary to distinguish between the use of model in mathematics and mathematical modeling, as applied in the various scientific areas.

An important viewpoint is presented in connection with what we call adjectival modifiers. These are designated as static, dynamic, deterministic, and stochastic. Modern mathematical methods used with the latter are treated at length.

The important concepts of system, process, and model are critically reviewed with the objective of firmly establishing the unique position of the concept of model.

All of the universe of things which are of interest to man and which are subject to modeling are considered. For our purpose the universe is divided into mechanistic things-to-be-modeled and humanistic things-to-be-modeled.

Finally our conclusions, which are based on an extensive study of the concept and use of models, are presented. It seems that an axiomatic basis can be developed and we present what we consider to be some important ideas for the purpose.

19. KEY WORDS (Continued)

system, process
mechanistic things-to-be-modeled
humanistic things-to-be-modeled
axiomatic basis for modeling
static model
dynamic model
deterministic model
stochastic model

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PREFACE

Although there is an increasing interest in the theory and practice of models, there is no single source to which the engineer as well as representatives of other scholarly areas can turn for a comprehensive introduction to the broad subject. The present volume is intended to fill the need. It presents a compact yet usable summary of the major types of models written at a level of difficulty which is at once manageable by the general technical reader as well as by the university student or the professional specialist. From the book the reader should be able to obtain a useful overview of modeling theory and at the same time prepare himself to read original sources and apply his knowledge to everyday problems. It is our hope that the book will serve a purpose in the field of generalized modeling which, for example, Langhaar's Dimensional Analysis and Theory of Models does in a more restricted field.

Obviously no single volume can adequately cover a subject which is by nature encyclopedic. However, the authors consider that their book should provide such an overview as to stimulate the reader and supply a cross-fertilization of various disciplines which should be helpful. It would seem that the time to take the broader theoretic and philosophic view has arrived. There is no doubt that considerable technological progress can be made by those who develop a thorough understanding of the basic concept of model and its relationship to the development of many fields of importance to man. The history of the subject shows that up to the present time, every analyst has used models only as if they were unique to his own professional field. There is no doubt in our minds that everyone who is at all aware of the use of models will profit by a comprehensive study of the entire subject without limiting himself in the beginning to a restricted view.

It may seem awkward at first for a student to examine fields as diverse as aeronautics and theology, but the authors are convinced by their study that considerable increase in mastery of any special subject will result as a consequence of the broader program. Furthermore anyone interested in the philosophy of models, must pursue some such course as outlined in our book in order to obtain a sufficiently lucid view to begin critical analysis.

We have attempted to present sufficient material from what may be called the classical engineering approach in order to provide a workable base for the uninitiated to progress and, also, for the better informed to read without reviewing old material. It is our objective to project the development of the subject from the more established foundations of model theory to what may be called the forefront of the subject. Because of the seemingly necessary provincial treatment of the subject by investigators in specific fields, it appears that the universality of the method of models is overlooked. Accordingly, a full

and vigorous pursuit of knowledge, particularly in the recently developed disciplines, is not possible.

At the end of each chapter is a brief list of primary sources which represent some of the most important references concerning the theory. At the end of the book is a fairly long list of general references on the various topics. We have made no attempt to evaluate the various references. Rather, we have attempted to present them in expository terms that will demonstrate their usefulness or what promise they hold for the individual who pursues them. The length of a chapter does not reflect our judgment of its relative importance. Each subject is presented in what seemed to us the smallest number of pages necessary to represent its essential features accurately.

In the preparation of the book we have found ourselves indebted to the authors whose work constitutes a very large literature of the subject. Also, we are grateful to our former students and to our colleagues, both present and past.

The basis for the present study was provided by the research experience obtained by one of the authors in the research laboratories of the U.S. Navy. Also important were his research opportunities provided at the Johns Hopkins University, the Rensselaer Polytechnic Institute, and the College of Engineering at the University of South Carolina. The other author, who is presently Director of the U.S. Army Materiel Systems Analysis Activity, received much of his experience in doctoral research at the John Hopkins University, as well as from a long career in engineering research at the Aberdeen Proving Ground, Maryland. Both authors take great pleasure in acknowledging their debt to these Institutions and Agencies for their lifetime opportunities to develop their knowledge of models.

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CONCEPT AND USE OF MODELS

CHAPTER 1

INTRODUCTION

Like all serious subjects which are of interest to man, that of modeling has a long history, even though the extensive use of the term in the current literature might cause one to think otherwise. It is apparently used for diverse purposes and sometimes even in a trivial manner. In the course of the present book we wish to show that in addition to having a long history the subject is really a serious one and that in the apparent diversity of use there is a unity of purpose. However, even if we had the competence to develop a satisfactory history it is outside the scope of our work. Instead we shall give a brief, but we hope suggestive, overview of the subject from the earliest times. It is to be hoped that someone who is an historian will provide a much needed history of the subject in the near future. The possibility of developing a satisfactory history of the idea of model seems real from the standpoint of a particular experience of the senior author. In the days when he was a professor of engineering at the Johns Hopkins University, he had the particular good fortune to be able to participate in a series of Tuesday night seminars on the history of ideas. These had been initiated many years before by professor A. O. Lovejoy who had made the history of ideas his life's work. Lovejoy's thoughts on the subject were clearly set forth in his William James lectures at Harvard [1].* Long after the old gentleman had retired from the Hopkins as professor of philosophy, he remained an incisive contributor to the seminars. The topic of modeling never came up but a cognate one did. It was the history of the idea of methodology, especially as applied to physics. Despite the fact that the senior author had attended these seminars and despite the fact that he began his career as a model tester at the U.S. Experimental Model Basin it had never occurred to him that the subject of models might have an ancient history and could be treated formally in a very general fashion. This is not so surprising however because it seems that the idea has not occurred to anyone else either, except in limited technical areas such as ship designing. Before leaving the Hopkins both authors of the present book had become acquainted, one as teacher, the other as student. As it turned out the junior author who was a doctoral candidate, did influence the senior author by encouraging him to conduct a long experimental study with small models. These will be described later in the book.

In order to orient ourselves with respect to the task before us it will be necessary to develop in some detail our overall view of the subject of models. It interests us and we hope it will do the same for the reader.

*Numbers in brackets refer to references at end of each chapter.

Our position is that the nascent concept of model goes back into pre-history and is an important aspect of evolving man. In arriving at such a conclusion we have been significantly influenced by Darwinian scholars but even more so by the essentially spiritual view of evolution espoused by Pierre Teilhard de Chardin [2]. The first concrete evidence of the model concept we consider to be such things as the cave pictures of early man, one example of which are the findings at Lascaux [3]. Later on we classify such models as iconic.

In its infancy the concept of model was certainly metaphorical. Here we involve philosophy and psychology, not our professions, but nonetheless we consider that to attain a proper understanding of the general subject it is necessary to contemplate such ramifications. The metaphorical nature of model is incisively treated by Max Black in his book <u>Models and Metaphors</u> [4]. Its thirteenth chapter which is entitled Models and Archetypes, we can strongly recommend to the reader. In the vein of the metaphor and in line with our historical view we consider the Book of Genesis to be an important example. Here we have a metaphorical model of the creation of the universe and the beginning of man. In terms of religion we see that the Hebrews made an important advance with their primitive world view. In this example we should appreciate that we are considering an early and particularly important model. More will be said later of models of the universe.

In the world, by the time we reach the ancient Greek culture we consider that the use of model has been seriously established but not yet consciously recognized. Modeling, like mathematics, had its primative beginnings in ancient times but did not reach a high state of conscious development until very recently.

In the twentieth century it is difficult to appreciate the elemental forces which shaped man and his thinking. For our purpose, which is to fully comprehend and use the concept of model, it may be well to think of some of the magical aspects of the development of the subject. While one can find other references, we chose to select a book entitled <u>History of Magic and Experimental Science</u> by Lynn Thorndike who did his graduate work at Columbia University at the turn of the century [5]. The period which he treats is from the time of the early Greek and Latin writers to the thirteenth century. While we recommend this book for an extensive exposition of a peculiar phase of our subject, we wish to state that the author never explicitly uses the word model. However, we will give one of many quotable statements in the history book. On page 19 Thorndike says, "Magic images were made of clay, wax, tallow, and other substances and were employed in various ways. Thus directions are given for making a tallow image of an enemy of the king and binding its face with a cord in order to deprive the person whom it represents of speech and will power. Images were also constructed in order that disease demons might be magically transferred into them, and sometimes the images are slain and buried." While some may dismiss this example contemptuously as superstition, we submit it is an excellent ancient

case of the use of the concept of model. There are various views one may take of that story with respect to modeling, but we simply stress here the use of the word image. In our later treatment of the subject, we show that image and its related term icon are important aspects of modeling. They serve many useful purposes. Thorndike's history ends at the time of Thomas Aquinas, which is the thirteenth century. While that historian does not say so, Aquinas seriously used in a systematic manner an important aspect of modeling, which is called analogy. While it would be improper for us to digress here in terms of the writings of Aquinas we may refer to an excellent recent study by George P. Klubertanz entitled St. Thomas Aquinas on Analogy [6]. It may be appropriate to give here a pertinent quotation from his book. He says, "In the thought of St. Thomas, also, the problem of multiplicity is one of the reasons for analogy, and a significant part of what he says about analogy is devoted to predication. To this extent, and in this sense, an investigation of analogy is more sophisticated and more supple than its corresponding doctrines in other philosophies." A further discussion of the subject of analogy in context with physical science can be found in a book by Mary B. Hesse entitled Models and Analogies in Science [7].

By the sixteenth century the very personification of modeling was evidenced by the person of Leonardo da Vinci. The vast depth and span of his accomplishments are indicated in his Notebooks [8]. The enormous scope of this genius covered the gamut of thought and performance from art to philosophy. He is one of the few universal giants of the intellect who have graced the history of man. It is not our purpose to record his many accomplishments but only to emphasize the face that Leonardo was one of the few at his time who seemed to be conscious of the enormous power of the model. An illustration of our point may be cited from an episode involving one Giovanni. Leonardo said of him, "But his whole intent was to get possession of these two rooms in order to get to work on the mirrors. And if I set him there to make my model of a curved one he would publish it." Leonardo also made routinely what he considered to be models of equestrian statues as well as of other objects of art. Not only did he deal with the image type or what we prefer to call the iconic model but he also profusely produced working models of a military, aerodynamic, hydraulic, and mechanical nature. It is obvious that he was inspired by modelistic thinking. We consider that his work can be used to demonstrate many points concerning the evolution of the concept of model in the long history of man. Leonardo was approaching the time when there occurred a veritable burst of productive activity in all fields of human endeavor. He was approaching the era of Galileo and Newton, both of whom had clear insights into the subject of models. Galileo actually used physical models of beams to study beam deflection theory and Newton introduced the mechanical principle of simulitude of which we shall treat more in a subsequent chapter.

As professions that conspicuously use models it may be useful to make a distinction between the use by physicists and the use by engineers.

Also we may examine that distinction in terms of the long histories of both fields. For the purpose we will present material from several historical texts. It is necessary to do this in order to glean an idea of the use of models from the earliest times and to indicate how that history contributes to the subject today. First, however, we will try our hand at a definition of engineering which, despite its faults, does provide a crucial view that distinguishes it from physics. In order to more effectively accomplish this we refer to a book entitled Personal Knowledge by Michael Polanyi [9]. Although the reader may find this book generally very interesting and useful from the standpoint of the development of human knowledge, we will confine ourselves to Polanyi's definition of machine. We refer specifically to page 328 et sequitur of his book. Here Polanyi maintains that a machine is characterized by an operational principle. He gives as examples the clock, typewriter, boat, telephone, locomotive, and camera. He further says and we quote, "A physical and chemical investigation cannot convey the understanding of a machine as expressed by its operational principles." At another point he implies that machine qua machine cannot be reduced to physics and chemistry. We do not wish to pursue the matter further here, but we do urge the reader to refer to Polanyi's book and make a careful study of his views. It can be very enlightening. In the light of these remarks we define engineering as that profession which is essentially concerned with the invention, design, construction, operation, and maintenance of machines, which include structures and systems, in order to control man's environment for his sustenance, protection, comfort, and pleasure. Physical science on the other hand, is an observational profession. It observes the phenomena of the physical universe and formulates its laws. We admit, however, that physicists must use instruments and machines for that purpose as will be pointed out in the following pages.

We might be tempted to go back to pre-historic times and observe what is known about the australopithecines, especially as to how they probably provided themselves with food, shelter, and protection. However, we will be satisfied for our purpose with those books which begin with the civilizations of the Nile Valley and Mesopotamia. While the reader may find other references which give historical information about engineering we will suggest several readily available books. The first provides an historical review of civil engineering and was written by Hans Straub [10]. It may be recalled that this branch of engineering is primarily concerned with structures, but it also has to deal with hydraulic problems. A famous example of the latter is seen in the ancient Roman Aqueducts. Straub discusses many of the canals, roads, bridges, buildings, ships, and harbors of the ancient world. It takes little imagination to see that these colossal structures had very modest beginnings during the evolution of man and of civilization. In fact Straub says, "The small irrigation canals, and the primitive water engines driven by man or ox, may be of casual invention, perfected by generations of peasants. But the major engineering works, such as the ancient canal between the Nile Delta and the Red Sea mentioned by Herodotus and later repeatedly repaired (e.g., under Ptolemy Philadelphus),

or the great dams and reservoirs, must have been conceived by single persons or teams of persons of outstanding engineering ability." It seems very odd to us that Straub does not apparently see that the small irrigation canals served as models for the canal near the Nile Delta. This aspect of modeling we wish to emphasize throughout our book. We honestly wonder what Straub thought was the source of "outstanding engineering ability." Early man must have exercised his nascent inventive capacity. Observing the possibilities, he must have tried the first crude working model to concretize his ideas. From these humble beginnings the greater engineering works grew. Unfortunately, Straub presents little material from the ancient and medieval worlds, but rapidly proceeds to the time of Galileo Galilei. In discussing Galileo's crucial contributions to the theory of structures known at that time, he shows a picture of the model of a cantilever beam which was profoundly studied by the great man. Despite the occasional indirect reference to models, Straub seems to be mainly concerned in recording how he understands the theoretical aspects of civil engineering to have been developed. As might be expected there is in the history of mechanical engineering more opportunity to discern the power of the model in developing a profession. We refer the reader to a history of that subject by A. F. Burstall [11]. He covers the story of mechanical engineering from prehistoric times until the Nuclear Age. Even in his preface, the author makes an interesting point about models. He says, "While teaching the history of engineering to university students it has been found that lantern slides and cinema films are no substitute for models that 'work'. Models of such simple machines and mechanisms formed the basis of some of the diagrams in this book. . . ." It may be stated that there are 291 illustrations in the book. Many are of historic engines and machines. The model aspect of invention, design and use are demonstrated by many examples. The author observes that the wedge, lever, and wheel were in common use before 3000 B.C. Further, we know that all of the elemental machine principles were known by Aristotle. No additional principle was introduced until Blaise Pascal introduced his famous principle of hydraulics in the seventeenth century. From that principle Pascal produced "a machine for multiplying forces." Burstall shows a model of Pascal's machine on page 172 of his book. It is well-known that this is a forerunner of the hydraulic press. The Froude hydraulic brake is illustrated in Figure 211. It may be recalled also that it was the Froude father and son, William and Robert, who became famous for the introduction of ship models to determine the speed and power for the prototypes.

As an example of the development of an engine over a period of approximately a millenium and a half, we may choose the steam engine. It starts with a model of a steam powered device which was invented by Hero of Alexandria. A contemporary model of that device is shown as Figure 54 in Burstall's book. He lists it as Hero's aeolipile. Recent scholars place the time of Hero as late as the third century A.D. However, some still claim that he lived in the period 150-100 B.C. In

any event, the principle of the first simple steam engine was known somewhere around the beginning of the Christian era. It served no practical purpose but may have been related to religious rituals as were so many things of that time. A vast period of time intervened before any attempt was made to improve Hero's primitive steam reaction turbine. By 1698 Thomas Savery produced the first, though inefficient, steam engine which was practical. Newcomen in 1705 introduced improvements in design and his engine was used for pumping mines. However, it was not until 1763 that James Watt began a series of experiments and made inventions that revolutionized the field. He made working models with which he performed many experiments, leading to basic patents. The story of the success of the steam engine during and after the industrial revolution is well-known.

The importance of the model in the history of the evolution of machines is not particularly stressed by Burstall, but it is our thesis that the use of the model was and is crucial. We believe this fact is now becoming more recognized in the twentieth century.

For contrast with the presentation by Burstall we recommend History of Western Technology by Friedrich Klemm [12]. In his book there is an excellent set of references for source material as well as a good general bibliography. Here again the model is not explicitly mentioned as a generative device for progress in technology, but even a casual review of the book will demonstrate the uses and value of the model. To illustrate the tone of the book we will give two direct quotations. In the preface the author says, "At that period, from 12,000 to 20,000 years ago, man lived in a world of magical identity between objects and their images, between matter and innate forces. Thus, representation of an animal imprisoned in a trap or transfixed by a spear was regarded as equivalent to actual possession of the animal. The actual snaring or slaying of the wild animal in open country was in some sort merely the completion of the magically achieved seizure. We are dealing with the hunting witchcraft and with the magical technique of a period when symbol and object were still regarded as one, as were also this world and the next." The reader may recall our earlier reference to Thorndike and his history of magic and experiment. He will find that magic played a role in man's development long after Klemm considers as its period of dying. The second reference which we wish to make to Klemm's book is on page 60, where St. Augustine is quoted from his classic City of God in praise of Creation. We quote in part as follows, "What varieties has man found out in buildings, attires, husbandry, navigation, sculpture, and imagery! What perfection has he shown in the shows of theatres, in taming, killing, and catching wild beasts! What millions of inventions has he against others and for himself in poisons, arms, engines, stratagems, and the like! What thousands of medicines for the health, of meats for the throat, of means and figures to persuade, of eloquent phrases to delight, of verses for pleasure, of musical inventions and instruments! What excellent inventions are geography, arithmetic, astrology, and the rest! How large is the capacity of man, if we should stand upon

particulars!" We consider that in this outburst the mind of Augustine, who lived in the fourth century A.D., is rapturously engaged with a vision of metaphor, image, and model.

A final scholarly work, which we recommend, concerns the advance of man technologically from prehistoric times to the time of the industrial revolution and is entitled A History of Technology and Invention [13]. Its editor is Maurice Daumas. Here we find careful scholarship with many references to sources. The profuse illustrations are intriguing and they again emphasize the role of the model in human thought and invention. We cannot go into the details of this work nor can we reference histories of other branches of engineering at this time. The reader no doubt knows that the histories of these other branches necessarily cover much shorter periods of time than do those of the ancient professions of civil and mechanical engineering. However, it must be admitted by anyone that the contributions from engineering in the fields of electricity and chemistry are not only impressive but they are essential to the world of the twentieth century. Furthermore, they provide superb examples of models and modeling. The development of the electric dynamo parallels in excitement and importance the development which we sketched for the steam engine. The pilot plant of chemical engineering shows in many ways the role of models. Finally, in connection with our definition of engineering, we wish to stress that the word systems covers, in part, all of that phase of electrical engineering which may be termed transmission systems, communications, and systems analysis. Modern high speed computing owes its existence to electrical engineering and here we have some beautiful examples of models used in the development of the subject.

Paralleling the growth of technology and engineering is the growth of physics. We will try to make clear our understanding of the fundamental difference between the two professions. Our main interest in the distinction, as treated in this book, relates primarily to the concept of model. We reiterate that technology and engineering are concerned with making physical things whereas physics is concerned with observing the functioning of the universe and its parts. No doubt the reader will see that overlapping of function occurs as in all human activities it must. However, little reflection is required to understand that one is concerned with the operational principle of the machine and the other with the operational principle of the universe. One produces machines, structures, and systems while the other discovers laws of physics. Before contrasting further these important matters we will examine briefly the history of physics as we previously did for technology and engineering.

There are a large number of histories of physics, but we recommend a brief but useful one for our purpose by Edmund Hoppe, published in the Handbuch der Physik [14]. Here we have a treatment in three parts; from ancient times to the seventeenth century, from the seventeenth century to the middle of the nineteenth century, and then from the middle of the

nineteenth century up to the twentieth century. In the first part Hoppe treats of the contributions of the ancient Babylonians, Egyptians, and Greeks. After these he examines the work of the Arabs and the growing physical studies in Christian Europe. In the second part he concentrates on the radical developments from the time of Galileo to the pre-atomic age in four distinct sections. Finally, the third part is devoted to what Hoppe calls modern times but of course, there is included none of the great work by physicists in the twentieth century. Hoppe refers to the three fundamental quantities of physics as space, time, and mass. These are directly related to mechanics, heat, magnetism, electricity, and optics. The laws of the subdivisions of physics are the special objects of study by physicists.

Man apparently demonstrated a great deal of curiosity about physical phenomena from his earliest times. Means of livelihood were obviously related to this curiosity but also related were what might be called magic and religion. With fire early man could become an artisan but also, sooner or later, he was constrained to ponder its relations to the sciences of heat and thermodynamics. How does fire work? A long evolution of thought about operational principles or laws was begun and it has not terminated to this day. In our previous discussion of Hero's fire-activated device we stressed the machine qua machine principle. It is also clear, as Hoppe points out, that the laws of pressure and heat-generated pressure become causes of puzzlement which eventually lead to the formulation of laws.

As everyone knows, astronomy was of great interest to ancient man. It was impossible for him to look at the heavens and not have his curiosity aroused. The obviously chaotic first appearances very gradually began to manifest some signs of law and order. Particular heavenly bodies were finally identified along with parts of the paths of some of them. Of course, all of this is nascent physics but because it is confined to the heavens the special designation of astronomy was used. The history of astronomy is long and distinguished. From this part of physics we may take a classic example to demonstrate the evolution of our knowledge of laws just as we started with Hero's engine and observed its development into the effective steam engines of the eighteenth century.

We can start with the ancient views of astronomy and trace the notion of the motion of heavenly bodies up to the time of the Copernican revolution in astronomical thinking. Copernicus, who lived in the fifteenth and sixteenth centuries, set the stage for the development of the laws about planetary motion. Based upon the strenuous observations of Tycho Brahe, Kepler was able to enunciate the famous three laws. Finally, Sir Isaac Newton made it possible for one to calculate the paths of the planets and analytically determine Kepler's laws on the basis of his law of universal gravitation together with his three laws of motion.

we repeat that our effort is only to contrast the bases of engineering and physics for the purpose of our study of models. Both professions use models, but one is for the development of machines while the other is for the detection of laws. Notwithstanding this fact, we appreciate that in the division of labor there is overlap and we now wish to acknowledge that fact. For that purpose we refer to the four volume work on the history of science edited by René Taton [15]. Here again the history is developed from ancient times into the twentieth century. The superb essays which constitute this encyclopedic work will be a source of delight to any serious reader. An acknowledgement is made in the general preface to those responsible for the accomplishment. We feel constrained to quote from that preface as follows, "So gigantic a task could never have been completed without the devotion of its many eminent contributors, nor without the prior spade work of such ardent pioneers as Paul Tannery and George Sarton, who were the first to plead the cause of the general history of science and to plead it so eloquently."

The peculiar dichotomy between man's need to do in order to survive and his need to know partly to satisfy his insatiable curiosity is treated in many places in the history edited by Taton. In a chapter entitled the "Dawn of Science" the first sentence says, "No history of science can ignore the achievements of prehistoric man." And further on the author says, "On these clues, sparse though they are, we have to base our entire knowledge of the first gropings of nascent science. Some prehistoric men must have handed on their knowledge by word of mouth, and it is undoubtedly due to their teachings that proto-historical and ancient science was born." We see the gradual developments of the need to do and the need to know. In Taton's monumental work we have not only historical treatment of physics but also of other sciences such as mathematics, astronomy, geology, biology, and chemistry. Also, there is developed a history of the relationship of these sciences to applied fields such as engineering, medicine, and navigation. The sense of contrast between the engineer and the scientist is exemplified in the part devoted to science in the Greco-Roman world. There it is stated, "Unfortunately, Straton's theories were far less well-received by his fellow scientists than by those Alexandrian engineers who, from the third century B.C., devoted much of their energy to the practical application of scientific discoveries. The most famous of them were Ctesibios, his disciple Philon of Byzantium (third century A.D.), and Heron of Alexandria (first century A.D.). In addition to their practical work they also produced theoretical treatises . . . ." The last sentence underlines the overlap in interest. Here we have men who produce practical results and also theoretical studies.

Before leaving the encyclopedic history which is edited by Taton, we wish to especially refer to the last volume which covers accomplishments of the twentieth century. We draw attention to this particular volume for two reasons. First, because it gives a beautiful treatment of the divisions among the sciences and secondly, because it gives

magnificent pictures which illustrate objects studied by science and particularly, machines which are used by science. The machines include the gigantic modern telescopes, large microscopes, cyclotrons, and electronic computers. An excellent photograph is given of the first model of an electronic computer, completed in 1942. We mention these matters for several reasons. They include the fruitful collaboration of scientist and engineer, but also they emphasize the nature of machine qua machine and its use in permitting the scientist to gain further knowledge of the laws of the physical world.

We complete our historical references with one by S. Sambursky [16]. It is limited in its historical range but it is a quite scholarly treatment. The period covered is in the first sentence to the preface. It is stated that, "The present book, in a way a continuation of my two earlier ones, describes the development of scientific conceptions and theories in the centuries following Aristotle until the close of antiquity in the sixth century A.D. From the copious literature of that period, and the works of the Aristotelian commentators in particular, We have selected and interpreted texts which are of interest for the comparative history of scientific ideas, with special emphasis on the epistemological foundations of physical theories." In Sambursky's little book is a scholarly treatment of the development of ideas about the physical world. Again we have a treatment of human thinking without reference to models, but it will not be lost on the reader of that book that one model after another is discarded as more acceptable theory is constructed. It is an example of the history of ideas to which we referred earlier. In this connection, the author says on page 117," . . . We will touch briefly on an aspect of scientific thought which is of general interest in the history of ideas and exhibits some features characteristic of Greek mentality." Perhaps, the term mentality may mean that the Greek scholars of that time had a special modelistic outlook. We shall not pursue these historical matters further but will conclude with some observations on their meaning as well as how we intend to further develop the subject of models.

The concept of model is surely not new as we have seen in our historical review. But what we think is new is the conscious use of the concept as an analytical tool which applies to all areas of thought and study. We consider that in the future the study of generalized modeling may constitute a discipline in itself. We hope, that there will come historians who will treat the history of modeling, the philosophers who will provide a philosophy of modeling as there have been those who developed the philosophy of mathematics, and the analysts who will develop the general principles of modeling which will be available to all areas of scholarship.

On the basis of our historical study it seems to us that the inception and growth of human knowledge have depended fundamentally on the idea of model. What we will have to say further in our book will involve the broadest possible interpretation of the concept of modeling.

As a consequence, we accept the task of attempting to portray the universal character of modeling and we will develop our thesis to such a degree as to make it useful to philosophers, to scientists, to engineers, to business men, to theologians and indeed to anyone concerned with the development of a branch of knowledge and practice. Our acceptance of such a task may seem brash and some may consider us presumptious, notwithstanding, however we consider that the time has now arrived for someone to exhibit the necessary boldness to cross all frontiers of knowledge in the quest of establishing the principles of modeling on a broad generalized basis. Then there will be those who can, if they wish, study the subject in a systematic manner as an autonomous discipline. We will no doubt be clumsy in our treatment and at times inept in attempting to carry out such a program but we feel that it is at once possible and necessary. No doubt, there are those who may be inspired by our humble attempt and will themselves do a much more effective job. We conceive of our effort as a developing one which will lead to an increasingly systematic growth of the science of modeling which will in turn, increase our powers to develop and control the world about us.

In the following chapters we will deal with the basic concept of model and the various types of models known at the present time. We hope to provide some suitable classification system and, also, to clearly indicate how the subject can grow and become increasingly effective in the analysis of myriad problems. We will briefly outline, as a part of our development, those phases of the subject that are well-known and also show how we may proceed to develop much more generalized models.

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CHAPTER 2

THE EVOLUTION OF THE CONCEPT OF MODEL INTO THE TWENTIETH CENTURY

It is the position of the authors that the evolution of the concept of model began in the earliest days of the existence of man on earth. In the last chapter, an attempt was made to justify our position by an examination of histories of technology, engineering, and science. From the early nascent condition in prehistoric times the gradual growth of the idea was traced toward the twentieth century. It was emphasized that the word model was never used in the very early days. The concept of analogy was systematically exploited by Thomas Aquinas in the thirteenth century in his Summa Theologica and the word model was actually used by Leonardo da Vinci in the sixteenth century. Technological development was slow; the conscious use of the concept of model was also slow in coming.

In 1687 the Newtonian scientific revolution began with the publication of the Philosophiae Naturalis Principia Mathematica by Sir Isaac Newton. Newton really was the giant of physics in his day. He left his mark on many phases of the subject and it can be said that he demonstrated a consciousness of the concept of model. His view of the heavens, particularly that of the planetary system, was clearly influenced by the notion of the mechanical model. Such an opinion is underscored by the fact that E. J. Routh, the famous coach for the English Tripos examinations, in his book on dynamics, credits Newton with the principle of similitude [1]. In explaining the mathematical nature of the principle, Routh says, "In other words, model is made of a machine, and is found to work satisfactorily, what are the conditions that a machine made according to the model should work as satisfactorily?" It is our conviction that with the introduction of the highly developed concept of model by Newton, a greater development of physical science was underway. The British school of empirical method was definitely established by this time. We are not alone in such speculation, for others have commented on the fact.

At the turn of the twentieth century, the great Poincaré made an incisive observation on the British kind of thinking as compared with that of the savants from the Continent. We will let Poincaré speak for himself [2]. He said, "For a Latin, truth can be expressed only by equations; it must obey laws simple, logical, symmetric and fitted to satisfy minds in love with mathematical elegance. The Anglo-Saxon to depict a phenomenon will first be engrossed in making a model, and he will make it with common materials, such as our crude, unaided senses show us them. He also makes an hypothesis, he assumes implicitly that Nature, in her finest elements, is the same as in the complicated aggregates which alone are within the reach of our senses. He concludes from the body to the atom." Poincaré goes on discussing such a speculation, but we need not quote further. Rather we recommend

that the interested reader study the excellent book for himself. Suffice it for us to say that the claim is somewhat exaggerated, especially in view of the use of mechanical models by such Latins as Galileo and Leonardo da Vinci. However, in our further comments on models in the present chapter, it can be seen that Poincaré has good reason on the side of his views.

What Poincaré might call the British mechanical habit of mind is illustrated by a particularly cogent example from physics. We refer to the idea of the aether. In this connection, we think it useful to consider briefly the history of the idea, which arose in physics about the time of René Descartes. Here we have another classical case of model influencing the entire development of physics. For our purpose we have available the superb history of the subject by Sir Edmund Whittaker [3]. We rely heavily on this work and will quote him rather freely. After a very penetrating view of the growth of science, beginning with Aristotle and moving through the Middle Ages, which brought Thomas Aquinas onto the scene, Whittaker states that his history of the aether properly begins with René Descartes.

The scientific development of the concept of aether began with Descartes. One of the existing problems of natural philosophy was to account for actions transmitted between bodies not in contact with each other. He considered that they must be effected by the agency of the only types of action between bodies which were perfectly intelligible, namely, pressure and impact. As a consequence he denied the principle of action at a distance. His principle was also maintained by the ancient Greek atomists, by Aristotle, and by Thomas Aquinas. It was denied by many others, including Duns Scotus and his followers. Space, in Descartes view, is a plenum, being occupied by a medium which, though imperceptible to the senses, is capable of transmitting force, and exerting effects on material bodies immersed in it -- the aether, as it is called. The word had meant originally the blue sky or upper air, and had been borrowed from the Greeks by Latin writers, from whom it had passed into French and English in the Middle Ages. Here we have a long history of an idea and certainly a use of the concept of model in evolution. When the notion of a medium filling the interplanetary void was introduced, aether was the obvious word for it. Descartes was the first to bring the aether into science, by postulating that it had mechanical properties. Whittaker very meticuously portrays the fortunes of the idea of the aether in the hands of Descartes and others of his time, but we do not consider it proper to follow the details further. Instead we recommend to our reader that he pursue this intriguing story on his own.

Because of the tremendous role played by Sir Isaac Newton, we feel it pertinent to sketch here his ideas on the subject. Essentially, his view was that all space is permeated by an elastic medium or aether, which is capable of propagating vibrations in the same way as the air propagates the vibrations of sound, but with far greater velocity. The

aether pervades the pores of all material bodies, and is the cause of their cohesion; its density varies from one body to another, being greater in the free interplanetary spaces. It is not necessarily a single uniform substance, but just as air contains aqueous vapor, so the aether may contain various 'aethereal spirits', adapted to produce the phenomena of electricity, magnetism, and gravity.

We can readily see that the aether has two of the most important properties of a model. It permits important dialogue and it assists in the development of knowledge. We will have a great deal more to say about these aspects of model in the next chapter.

Since the time of Newton the old model of the aether has had varying success and apparently received the death blow at about the time Poincaré, in 1900, asked at an International Congress of Physics held in Paris an important question. "Our aether", he said, "does it really exist?"

We cannot follow the matter further, but before we leave it we wish to quote from the preface to Whittaker's history. He writes, "As everyone knows, the aether played a great part in the physics of the nineteenth century; but in the first decade of the twentieth, chiefly as a result of the failure of attempts to observe the earth's motion relative to the aether, and the acceptance of the principle that such attempts must always fail, the word 'aether' fell out of favor, and it became customary to refer to the interplanetary spaces as 'vacuous'; the vacuum being conceived as mere emptiness, having no properties except that of propagating electromagnetic waves. But with the development of quantum electrodynamics, the vacuum has come to be regarded as the seat of the 'zero-point' oscillations of the electromagnetic field, of the 'zero-point' fluctuations of electric charge and current, and of a 'polarization' corresponding to a dielectric constant different from unity. It seems absurb to retain the name 'vacuum' for an entity so rich in physical properties, and the historical word 'aether' may fitly be retained."

In our opinion another rather superb use of the idea of mechanical model applied in physics came about in England as a result of the cooperation of Lord Rutherford and his young Danish colleague, Niels Bohr. We prefer to let the eminent L. de Broglie tell the story [4]. He writes, "The 19th century may be called the heroic age of 'classical physics' i.e., of the study of physical phenomena on the directly observable scale. It was the century in which the theory and practice of classical mechanics, acoustics and optics were set on firm foundation, in which the science of electricity made giant strides towards Maxwell's magnificent synthesis, and in which thermodynamics began to have repercussions in most other branches of physics. But despite all of its brilliant achievements, classical physics was suffering from a number of grave ills. Its victories were based on the achievements of mechanics in the 17th and 18th centuries." What de Broglie is concerned about here is the fact that the classical mechanics alone cannot account for atomic behavior. Quantum mechanics is required for the purpose. In

order to pursue the matter, we quote further from de Broglie as follows, "Quantum theory scored one of its greatest victories in 1913, when the young Danish physicist, Niels Bohr, who had worked with J. J. Thomson at Cambridge and with Rutherford at Manchester, applied it to Rutherford's nuclear model of the atom. In Rutherford's model, an electron moves in a circle round a much heavier nucleus and, according to classical physics, it would have to lose energy by radiation and hence spiral quickly into the nucleus. As a result the frequency of radiation (determined classically by the frequency of the revolution of the electron in its orbit) would increase continuously and give rise to a continuous spectrum, but in fact, a line spectrum corresponding to a number of discrete frequencies is observed. To explain these facts, Bohr made use of the quantum hypothesis and added the following two postulates to the laws of classical physics:

1. Atoms exist only in sharply defined 'stationary states', or 'levels', whose energy contents differ by fixed amounts; there are no intermediate states.

2. In stationary states the atom radiates no energy at all. Radiation occurs only through transitions between two stationary states. Light is emitted with an energy quantum equal to the energy difference of two stationary levels. The frequency of the emitted light is given by Planck's quantum condition:

$$h\nu = E_1 - E_2\text{''}.$$

We only repeat this old story in such detail because we consider that it represents an historic milestone in the use of the concept of model. We clearly see that there is a crucial Rutherford model of the atom and then the related quantum theory as applied by Bohr. With the model and theory it was possible to explain such things as atomic line spectra and to calculate Rydberg's constant from atomic constants. The rich viewpoint developed by Rutherford and Bohr led, in the hands of many others, to an illustrious series of triumphs in physics.

It is considered that our case histories in physics amply define the fruitful evolution of the model concept into the twentieth century, but we still think it will be instructive to the reader for us to recall some spectacular success stories concerning the use of the model in engineering during the nineteenth and twentieth centuries. We opine that these greatly influenced the wider use of models.

As mentioned in our introduction, the Froudes developed the technology of physical models in Britain to study the performance of ships. That was the beginning, in the nineteenth century, of an important research activity which has persisted until the present time. The towing tank which was used so effectively for the purpose is used systematically on an international basis today. An account of the

introduction and use of the methodology of towing tank analysis in the United States is provided in a later chapter.

During the latter part of the nineteenth century, there was interest in the study of estuarial flows and tidal action. Because of certain flow phenomena in the Mersey estuary in England about 1885, Osborne Reynolds, an outstanding British engineer, designed and constructed a model to a very small scale in order to study the matter. Along with many other pioneering achievements in engineering, Reynolds is credited with the first serious investigation of scale factors as applied to estuarial flow. As a consequence of his work, Reynolds was able to make some important deductions concerning sediment transport in waterways. Because of his great fame as scientist and engineer, a great deal of attention was increasingly given to the methodology and now large establishments are used for the study of such hydraulic problems. A very important one, which was originally used to investigate flooding on the Mississippi River, is the U.S. Army Engineers Waterways Experiment Station, at Vicksburg, Mississippi. At this Station, working models of important bodies of water in the United States can be seen at the present time. For further information about the original work by Reynolds, we recommend to the reader a very stimulating paper by A. T. Ippen [5].

In the twentieth century, after the invention of the airplane, it was only natural that the aeronautical engineers would design and use wind tunnels for the study of airplanes, by using small models, just as Naval architects were using small models for studying ships. For the interested reader who is not familiar with the subject of model analysis of ships and airplanes we recommend an excellent reference on the subject by L. I. Sedov [6].

About the time of World War I, the U.S. Navy was also using the small scale model of battleships to predict damage to underwater structure, which could be caused by mines and torpedoes. Furthermore, during the period between WWI and WWII, the Navy made extensive studies of the problem and used half scale models for the purpose of predicting damage to the then building battleships. The senior author, who was a civilian engineer with the Navy at that time, had the opportunity of studying all of the data on such models, which were in the Navy archives, preparatory to his formulating a critique on the subject. Very interesting information concerning damage caused by underwater explosions became available and useful damage criteria were formulated. During the same period of time of which we are writing, other structural problems were being studied by the Navy in terms of small scale models. Models of stiffened cylindrical submarine hulls were subjected to external pressure in order to predict the depth to which the vessels could safely dive. Also, small models of structures, which constituted important parts of Naval vessels, were subjected to both static and dynamic loads in order to determine damageability.

It can be appreciated, with little study, how the towing tank, the wind tunnel, and the structural laboratories, not only advanced the specific fields of ship and aircraft design, but also impressed on others the great value of the explicit use of models.

Another instructive use of models grew out of the need of the Navy to provide effective propulsion systems for its ships. An important part of such a system is, of course, the propeller, and the Navy has been performing experiments with small models of propellers since the turn of the century. We might outline the old method of performing such experiments, however instead we shall use the present occasion to examine a peculiar problem which arose in connection with such studies. By 1930 it was discovered that the standard tests of model propellers led to an essential difficulty. As is well known, one of the requirements for ship model testing is to maintain geometrical similarity between model and ship. As a consequence the model propeller had excessive pressure because atmospheric pressure is the same for model and prototype. As a consequence cavitation was inhibited on the model. The highly undesirable cavitation then occurred on the ship but was not predicted at corresponding speed on the model. The need for a way out of the dilemma led to the invention of the water tunnel, which is similar to the wind tunnel, except that the air is replaced by water. To contain the water a large doughnut shaped pipe is provided. In the line of the pipe a large spherical enclosure is installed at the uppermost point of the vertically placed doughnut. The propeller under test is installed within the enclosure and the horizontal propeller shaft runs from the propeller through a packing gland in the wall of the water tunnel to appropriate dynamometers which are situated outside. In this manner the required measurements of torque, thrust and work at predetermined speeds for the model can be made. In order to scale the pressure head on the propeller properly an air space is provided in the upper part of the test chamber, just above the free water surface which is maintained at the correct height above the model in order to satisfy similarity conditions. Now the investigator can attach a vacuum pump to the air pocket in the test chamber and pump down the atmospheric pressure to any predetermined value he pleases, nearly down to the vapor pressure of water at the given temperature if so desired. With this development of ship model experimentation it is quite simple to study the phenomenon of propeller cavitation. Also, the speed at which cavitation occurs on the ship can be predicted.

Inasmuch as flow around propellers is very complicated, it is not possible to satisfactorily calculate propeller performance on the basis of physical theory alone. The success of such ventures has really been prodigious and demonstrates again how physical models provide important technical knowledge.

There are various books which tell the story of physical model analysis as an aid to engineering structural design. In order to obtain a quick review of such a methodology we refer the reader to a brief but

enlightening treatment by T. M. Charlton [7]. There he can learn how Beggs, Gottschalk, and Reichhof used small physical models for the analysis of forces and deformations in beams and space frames during the first quarter of the twentieth century. We heartily agree with the author when he says that model analysis of structures utilizes the same assumptions as formal analysis, and offers nothing more in the end-product, but the user gains unique insight into structural behavior.

In addition to the extensive model analysis of structures used by the military forces, we direct attention to the impressive use by the civilian segment of society. For the purpose, we refer to several specific illustrations.

A type of structural analysis with models was conducted by M.I.T. in the early part of the present century, to determine the earthquake resistance of water supply tanks which are supported on tall columnar type structures. Small models which were geometrically similar to the prototypes were mounted on a horizontal platform which could be moved in its plane with motions simulating those taken from seismographs. As an aside it may be remarked that the engineers who conducted these experiments used the small resistance wire strain gages for the first time. It is readily seen that models of the type we are now discussing put an important research tool in the hands of structural engineers. The expense of such models is relatively low, desired alterations in design can be readily accomplished, and measurements of strains and displacements during excitation can easily be made. The particular M.I.T. model is mentioned as an early example of experiments from which important knowledge of dynamic structural response can be gained.

Suspension bridges, in the early days, were also studied in the design stage by means of small models. The Golden Gate bridge at San Francisco was extensively studied in this manner. Unfortunately, however, experiments with dynamic type loadings were not conducted, especially those in wind tunnels. Had this been done on the Takoma Narrows bridge a tragic failure might have been averted.

Before leaving the subject of structural models, we wish to cite the interesting technique of photoelasticity for the determination of stress fields by means of models. It may be recalled that with appropriately shaped flat plates of such substances as bakelite and standard photoelastic apparatus such stress analyses can be made. The theory and practice can readily be found in such treatises as the classic by Coker and Filon [8]. We will not review the details here. Instead we will cite an excellent application of the method to help solve an important specific problem. When the senior author was a graduate student at Columbia University he had the opportunity to assist in the performance of some very interesting experiments in photoelasticity in order to obtain an answer to an important question which came up in the field of elasticity. The problem arose in connection with a plate which was subjected to a uniform field of tension but had a small hole near an

edge. It concerned the essence of research which was then being conducted by Professor R. D. Mindlin, who subsequently published a paper which contains the details of the investigation [9]. Because of its possible interest to the reader and also because it so clearly demonstrates one aspect of the power of physical model analysis, we will provide a brief abstract of the paper. Mindlin points out that the theoretical problem was first studied by G. B. Jeffery as an application of his general solution of the two-dimensional equations of elasticity in bipolar coordinates. In the course of deriving the solution of what Mindlin called the 'tunnel problem', in which Jeffery's general method was employed, it was discovered that Jeffery had made an error in applying his results to the problem of present interest. One result was that compressive stress was predicted at the edge of the plate in the vicinity of the hole and in the direction of the plate edge. Such a situation seemed unlikely and of course much discussion resulted among elasticians. In a discussion of Mindlin's tunnel paper, J. H. A. Brahtz gave some results of a photoelastic study conducted at the U.S. Bureau of Reclamation for the purpose of comparison with Jeffery's solution. Brahtz convinced himself that the stresses along the straight edge of the plate were tension throughout, in contradiction to Jeffery.

Because the corrected mathematical solution revealed such pronounced peculiarities in the stress distribution it was considered desirable to extend the photoelastic studies of Brahtz to plates with holes very close to the edge. As a consequence, at least qualitative agreement was obtained between results of the corrected calculations and those of the experiments. In particular, the stress along the straight edge is tension at all points. The Mindlin project clearly emphasizes the value of the physical model when used in connection with theoretical analyses.

Similarity modeling and the use of dimensional analysis for a wide variety of engineering problems are treated in a recent book by Baker, Westine, and Dodge [10]. The authors point out that while their work was originally motivated by studies which they had made and courses which they had given in connection with the modeling of weapons effects, the field covered in their text is much larger. In fact they interestingly contrast the military applications and similar industrial applications. It may interest our readers if we quote from their preface concerning the matter. They state, "For example, shaped charges were developed because of a military desire to penetrate armor, but they can also be used by the oil industry for drilling. Loading plates with a shock wave can be applied to the important industrial process of explosive forming. The containment of explosion and fragments is a safety concern of public utilities. Explosive cratering can be applied to the digging of a new canal or to mining. Off-the-road mobility of vehicles is an engineering interest of among others, Caterpillar Tractor, John Deere, and International Harvester."

Especially interesting is their discussion of scaling laws for blast waves, which have been extensively studied by the junior author of the present book. These laws of modeling are, also, of interest in civil applications as well as in military. Since gas explosions and their interaction with structures cannot be fully treated by mathematical physics, many model experiments are required in order to develop necessary design information.

Another interesting portion of their book contains rather long discussions of the model studies of the rigid body motions of the Lunar Excursion Module and the Apollo Command Module. They clearly outline the research of NASA on the vehicular stability on landing on the surface of the moon and the tumbling motion incidental to landing on the surface of the earth.

We strongly recommend the book to the reader for a survey of a wide field of applications of dynamical modeling. It may be stressed that the type of modeling which they treat depends essentially on dimensional analysis and the principle of similitude. We explain the methodology in some detail in later chapters.

The modeling which was discussed in the present chapter so far relates to physical laws and engineering design. It follows a very definite pattern and by now has almost a classic form. At present there are many nascent forms of modeling which are developing. Some of them had their beginning in World War II. These involve the establishment of a new science called Operations Research and the invention of the electronic computer. From their inception to the present, there has been a prodigious expansion in the use of models in many different fields. The value of these two activities, systems analysis and high speed computing, is obvious. What may not be so obvious is the fact that their birth and growth are directly attributable to an extremely long prior history of modeling. It is our purpose in the next chapter to attempt to make this fact clearer. One reason is a desire to normalize history, but a much more important reason is the desire on the part of the authors to at least indicate the beginnings of a discipline of modeling qua modeling, which has a history and a philosophy. We consider that such a perspective, if established, especially in the colleges, will lead more effectively to the advancement of all professions.

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CHAPTER 3

CRITIQUE AND CLASSIFICATION OF MODELS

We have stated our opinion that the idea of model is as old as the human race, however we also consider that it was not treated consciously until more recent times. It seems that the first systematic use was made by Thomas Aquinas in the thirteenth century. It is well-known that he developed the concept of analogy in a meaningful manner to produce his prodigious Summa Theologica. By the sixteenth century scholars like Leonardo da Vinci were using the concept of model in a significant manner. But from that time until the twentieth century, although there were outstanding applications of the concept, it was not until World War II that modeling came into its own and was almost treated as an independent discipline. As we pointed out in the last chapter, the inception of Operations Research and the invention of the electronic computer laid the groundwork for a more extensive use of modeling. That activity is very much alive and still growing at the present time. While we wish to say more about this growth, we will first return to the classic history of natural science and its philosophy. It is very important to do so because by the middle of the twentieth century the study of methodology was at a very mature level and vitally significant things were being said about models and modeling.

As a start, in our quest for more enlightenment concerning the philosophy of science and models, we refer the reader to a fairly long article by Mary Hesse in the Encyclopedia of Philosophy [1]. At the beginning of her article she points out that the term "model" has become fashionable in the literature and philosophy of science, with the result that the many different senses of the term need to be distinguished before the philosophical problems connected with models in the sciences can be understood. She deals with the subject under the following classification system: logical models, replicas, analogue machines, analogy, mathematical models, simplifying models, and theoretical models. She further discusses predictivity and the meaning of theoretical concepts. Probably the most important feature of the whole article is the trenchant exposition of the function of model. Under function it is stressed that philosophical debate about models concerns the question of whether there is any essential and objective dependence between an explanatary theory and its model that goes beyond a dispensable and possibly subjective method of discovery. The debate is an aspect of an old controversy between the positivist and realist interpretations of scientific theory. Hesse gives as examples various outstanding cases, of which we have already mentioned several. Because of their great importance in the history of science we will repeat her entire list. It is as follows: application of Ockham's razor to scientific theories, the Newtonian-Cartesian controversy over the mechanical nature of gravitation, nineteenth century debates about the mechanical aether and the existence of atoms, and Machian positivism. In its modern

form the argument for the essential dependence of theories on models was first developed in 1920 by N. R. Campbell in Physics, the Elements. It seems that Campbell attacked the contemporary positivist view, expressed by Heinrich Hertz, Ernst Mach, Pierre Duhem, and others, that models are merely dispensable aids to theory construction and can be detached and discarded when the theory is fully developed. Campbell admits, as probably most serious scholars will, that theories are made up of three elements: a formal deductive system (hypothesis) of axioms and theorems; a "dictionary" for translating some of the terms of the formal system into experimental terms; and experimental laws such as the Boyle and Charles gas laws, which are confirmed by empirical tests and also can be deduced from the system of hypothesis plus dictionary. Campbell argues that the hypothetico-deductive form is insufficient to account for an explanatory theory as understood in science. He insists on an essential fourth element in theories - namely the analogy, which is exemplified in gas theory by the model of point particles, etc. Campbell has two arguments for his view that the particle model is essential. It is intellectually satisfying as an explanation of the empirical data but, more cogently, it draws attention to the dynamic character of theories and their use in prediction. Campbell further argues that without material analogy there is no rational, nonarbitrary grounds for predictions. Hesse says that there are objections to the position of Campbell and that formalist alternatives have been proposed by R. B. Braithwaite, and others. Because of the importance of the concept of model in physical theory we will refer to the work of Braithwaite.

A very serious study of model is presented in a book by Braithwaite entitled, Scientific Explanation [2]. Because of the rather esoteric manner in which he proposes the relationship of model and theory we will not repeat it here. However, the elements of the situation are model, theory, and calculus. In order to concretely expound his theoretical position, Braithwaite quotes Heinrich Hertz, on the matter and makes certain deductions therefrom. We do not consider it appropriate to pursue that matter further here, but do recommend that the reader consult the book. We cannot leave the problem however, without giving a few more quotations from Braithwaite that we think are important for us. He says, "we shall see that to think about a scientific theory by thinking about a model for it is an alternative to thinking about the theory by explicitly thinking about the calculus representing it." Further on he writes, "to think in terms of the model is therefore frequently the most convenient way of thinking about the structure of the theory .... Thus there are great advantages in thinking about a scientific theory through the medium of thinking about a model for it." We reject what we consider to be confusion in the first statement, but heartily endorse the spirit of the second. Our position comes clearly to us from the considerations of Polanyi to whom we referred in the Introduction. Especially do we subscribe to Polanyi's discourse on machine qua machine. We consider that man's essential insight follows the plan outlined by Polanyi and that ultimately it is subjective and

intuitional. That "picture" as Hertz calls the model is an intuition which is obviously essential to our comprehension of modeling.

Another incisive philosopher to whom the reader may refer is Israel Scheffler. In his Anatomy of Inquiry he not only develops his own thought but relates significantly to the work of such scholars as Willard Van Orman Quine and E. Nagel [3]. Scheffler does not explicitly deal with the concept of model for our purpose but does treat solidly of many aspects of the philosophy of science so that we can whole heartedly recommend him to the reader. His selected general bibliography is excellent for one interested in scientific method in the twentieth century. Most of his effort in the text is on such standard things as deduction, induction, explanation, significance, and confirmation. He is also concerned with applications to history and psychology of which we shall have more to say later. Of particular interest to us is a long quotation he has taken from E. Nagel. We select from that quotation to underscore some of our own positions. He quotes from Nagel as follows, "an analogy between an old and a new theory is not simply an aid in exploiting the latter but is a desideratum many scientists tacitly seek to achieve in the construction of explanatory systems. Indeed, some scientists have made the existence of such an analogy an explicit and indispensable requirement for a satisfactory theoretical explanation of experimental law. .... the lack of marked analogies between the theory and some familar model is sometimes given as the reason why the new theory is said not to offer a 'really satisfactory' explanation of those facts. Lord Kelvin's inordinate fondness for mechanical models is a notorious example of such an attitude. He never felt entirely at ease with Maxwell's electromagnetic theory of light because he was unable to design a satisfactory mechanical model of it." Further on he quotes Nagel as saying, "Theories based on unfamiliar models frequently encounter strong resistance until the novel ideas have lost their strangeness, so that a new generation will often accept as a matter of course a type of model which to a preceding generation was unsatisfactory because it was unfamiliar. What is nevertheless beyond doubt is that models of some sort, whether substantive or formal, have played and continue to play a capital role in the development of scientific theory." In this latter quote we wish to particularly stress the implied role of history in the development of an idea. It can be seen that the evolutionary aspect of knowledge should be a caution to both the old and the new generation.

At this point we would like to refer again to the work of Professor Nagel of Columbia University. He is a formidable adversary for his positions in the philosophy of science and we strongly recommend his great work which is entitled The Structure of Science [4]. This eminent scholar who is well familiar with linguistics, the subtleties of symbolic logic, and the general apparatus of modern analysis can also present straightforward stories of the rise of man's knowledge and in some measure predict the probable destiny of it. We have already quoted him in connection with our remarks about Scheffler. Before passing him by we would like to specifically refer to his four patterns

of explanation. They are: the deductive model, probabilistic explanations, functional or teleological explanations, and genetic explanations. The first he says is a type of explanation that commonly occurs in the natural sciences, though not exclusively in those disciplines, and has the formal structure of a deductive argument. With regard to the second he says that many explanations in practically every scientific discipline are prima facie not of the deductive form, since their explanatory premises do not formally imply their explicanda. Nevertheless, though the premises are logically insufficient to secure the truth of the explicandum, they are said to make the latter "probable". Probabilistic explanations are usually encountered when the explanatory premises contain a statistical assumption about some class of elements, while the explicandum is a singular statement about a given individual member of that class. With regard to the teleological explanations he says that in many contexts of inquiry - especially, though not exclusively, in biology and in the study of human affairs - explanations take the form of indicating one or more functions that a unit performs in maintaining or realizing certain traits of a system to which the unit belongs, or of stating the instrumental role an action plays in bringing about some goal. Such explanations are commonly called "functional" or "teleological." Finally he says about genetic explanations: Historical inquiries frequently undertake to explain why it is that a given subject of study has certain characteristics, by describing how the subject has evolved out of some earlier one. Such explanations are commonly called "genetic".

Finally, with respect to Nagel, we would like to cite his statement of the three major components in theories. They are (1) an abstract calculus that is the logical skeleton of the explanatory system, and that "implicitly defines" the basic notions of the system; (2) a set of rules that in effect assign an empirical content to the abstract calculus by relating it to the concrete materials of observation and experiment; and (3) an interpretation or model for the abstract calculus, which supplies some flesh for the skeletal structure in terms of more or less familiar conceptual or visualizable materials.

In addition to our reference to Nagel we would like to recommend to the reader the views of Leonard K. Nash as expressed in his book entitled The Nature of the Natural Sciences [5]. We particularly recommend his Chapter VIII which is devoted to Theories and Models. On page 241 Nash endorses the fourth point of Campbell to which we referred previously. He quotes Campbell as follows: The explanation offered by a theory .... is always based on an analogy, and the system with which an analogy is traced is always one of which the laws are known...... Thus our theory of gases explains the laws of gases on the analogy of a system subject to dynamical laws.

An example of a treatise on the philosophy of science which avoids a cogent treatment of the concept of model is the second edition of Scientific Inference by Sir Harold Jeffreys [6]. Sir Harold, whose

life work was in cosmogony and geophysics, was mostly concerned about the probabilistic side of science. It is interesting to observe the rather perfunctory manner in which he treats model. A lesson to learn from this example is that highly successful scientists can write about certain aspects of the scientific method, such as probability, sampling, errors, mensuration, and mechanics, without having any real feel for the history or the psychology of the subject.

A much more satisfactory treatment of the working spirit of science is to be found in a festschrift, honoring Joseph Henry Woodger, entitled Form and Strategy in Science [7]. Here we have a set of essays on various aspects of the subject which should delight the reader. The various parts of the book are philosophy of science, logical analysis of theory structure, models in science, and analytic biology. While our professional fields do not include embryology, one of the principal areas of interest to Woodger, we consider that biology, in general, illustrates the widening paths of the model concept. In the early part of the century, Woodger studied and wrote on axiomatic method in biology. In the festschrift Bonner writes on analogies in biology, Mays treats probability models in the thought and learning processes, Will Lewontin discusses models, mathematics and metaphors. In the last mentioned article, there is an incisive treatment of model which indicates familiarity with the Philosophy of Science by A. R. Rosenblueth and N. Wiener. He quotes these authors on what we think is an important point in the theory of models, although to some it may appear trivial. Rosenblueth and Wiener say, "The best material model of a cat is another, or preferably the same cat." This is a statement of what we refer to later in the present book as the identity model. It should be clear to anyone treating the generalized model analysis there is a need for an identity element. Before leaving Lewontin we should say that he provides sections on both the deterministic and the stochastic models. As is well-known the physics and mathematics of biology have only recently developed to any very serious stage. It appears that certain kinds of progress in biology will now be rapid.

Digressing even further than biology we would like to examine briefly the relationship of philosophy and model to theology. To some it may appear that we are getting a bit far fetched. However, we would like to reiterate our original objective which was to cross all boundaries in human knowledge and particularly emphasize the model concept for all of the various areas. While we have observed that Thomas Aquinas contributed substantially to the model concept in the thirteenth century by powerfully applying the idea of analogy, our present viewpoint on the subject will be determined almost exclusively by recent events. The principal scholars to whom we wish to refer for our purpose are Teilhard de Chardin, mentioned in our introduction, and Alfred North Whitehead, particularly his book Process and Reality [8]. With this approach we wish particularly to emphasize what we said in the beginning, that man's view of the universe is conditioned by his history and particularly his evolution. A possibly simplistic view, but one which

we must take in order to proceed with our main study of model, is that the world is either static as in many old religious views or it is dynamic and changing. The consequences of either position may be apparent to the reader. It is the latter position which characterizes what is called process theology. For present purposes we refer the reader to a series of essays which provide the basic writings by key thinkers in this major modern movement. They can be found in a book entitled Process Theology [9]. Our aim can be best expressed by a quotation from the introduction by Cousins. He says, "The shift in scientific models should be seen in the larger context of a shift in experience in our culture as a whole. The relation between scientific models, culture, and theology is a recurrent theme among process theologians. No one has explored this theme more consistently and in greater depth than Bernard E. Meland. In selecting material for this present volume, we have chosen a piece in which Meland gives a general statement of his position. In this selection, entitled Faith and the Formative Imagery of Our Time , Meland claims that the thinking of a people moves within a set of images that illumines the meaning of terms and sets limits to their understanding. This imagery is bound up with their life experience, the sensibilities of the age, and scientific constructs. Meland observes that we are in the midst of a change out of which new metaphors, peculiar to our time, are forming. In this process, not merely our imagery but our experience is undergoing change. He traces the shift in the scientific world view from Newtonian mechanism to twentieth century science and into the cultural ethos of the atom bomb and space travel. In a similar vein, throughout his writing Teilhard de Chardin traces the impact of cultural forces - of industrialization, technology, communications, and the expansion and convergence of world population - on the shaping of the consciousness of our time."

For the last time we wish to refer to theology to gather some impetus with regard to our treatment of model. In order to accomplish our objective we refer to a very challenging little book entitled Models and Mystery [10] by I. T. Ramsey, Professor of Philosophy at the University of Oxford. The reader will find this a very rewarding book. From it we have taken a term to designate one of our temporary classifications of model. We also agree with the general tone of Ramsey's work and think it enlightening to quote from his first chapter, Models in the Natural Sciences and Theology." He says, "... my thesis will be that our various disciplines despite their necessary and characteristic differences, nevertheless have a common feature of great significance, a feature which is often overlooked and frequently misunderstood: the use they make of models. It is the use of models that each discipline provides its understanding of a mystery which confronts them all, ...". A reader will find that Ramsey struggles valiantly with older philosophies of science and the metaphorical view of Max Black. Indeed it is from these two sources that there is born in his mind the idea which he calls disclosive model. Such a model he says permits dialogue and generates knowledge. It is the generality of his position which we hope to use later in what we will stipulate is our classification of models. To emphasize this point-of-view we wish to further quote from

Ramsey. In his final paragraph he says, "Models like metaphors, enable us, I have said, to be articulate, and both are born in insight. But it is in an insight which, viewed as a disclosure, reminds us that in such insights the universe is revealing itself to us."

At this juncture we would like to give the classification of models which we propose in the present book. Before doing this, however, we wish to examine what may appear to be excessive classification in an ingenious paper entitled the Modeling Process by G. Arthur Mihram [11]. He discusses somewhat at length what he properly calls a taxonomy of models. Then he provides a table which shows twenty-four classifications. On the top of the table he has two items called material and symbolic. The material is divided into replication, quasi-replica, and analogue. The symbolic is divided into descriptive, similar, and formal. Such a division provides six columns. Then on the side of the table he divides into static and dynamic. The static and dynamic he further divides into deterministic and stochastic. Such a division provides four rows. The six columns times the four rows gives the twenty-four classes which he claims for his type of taxonomy.

We are impressed with the try by Mihram but consider that his classification is premature. For the present we will be satisfied with our most modest taxonomy. Our list is as follows: iconic, analogic, similitudinous, Newtonian, extended Newtonian, and disclosive. We treat all of these at great length in the following chapters. For the present we would like to provide an example, from our own experience, which we call a disclosive model. In order to clarify our meaning we will proceed to give a somewhat detailed treatment.

During the last several years the authors conducted an investigation to determine the value of physical models for studies of the human cardiovascular system [12]. A series of flow experiments were conducted on several different models. The first model was a fairly elaborate one designed by the cardiologist Simon Rodbard. After operating it for a brief period of time, it was decided to design and construct a somewhat simpler one as the basis for a planned series of increasing complexity in the spirit of Julia Apter's theory of biological systems analysis [13]. Consequently, a model of the systemic circuit using a single pump, was designed, constructed, and tested. Further experiments, including some on bleeding, were made on that model. The next, and final elaboration of the model, was one which consisted of the twin cardiovascular circuits, the systemic and the pulmonary.

There are various means available for the study of physiological systems. In the literature, R. R. Rushmer and O. A. Smith, Jr. have provided a list [14]. It will be useful to repeat it here. They said that, "In general, physiological problems have been approached by scientific methods derived largely from the field of physics. In virtually all areas of physiology, investigation is carried out on at least five levels:

a) mathematical formulations,

b) physical models,

c) excised tissues or organs,

d) organs _in situ_ in anesthetized animals, and

e) human subjects and patients.

The use of physical models for physiologic phenomena is a challenge and a stimulation. The present status of the subject indicates great progress in the future. The physiological models are difficult to deal with but they are essential to a proper understanding of functioning of organs and systems in the animal body. Such systems are the most complicated encountered by man in his study of the physical universe.

We consider that the only ways open to controlled studies of a biological organism is on the prototype itself, on a similar organism, or on a physical model. Obviously the physical model may be a crude approach as a first approximation, however it does provide some important advantages. Measurements of important physical variables may be made at all locations, experiments may be repeated as often as desired, alterations can readily be made and effects intelligently studied. The idea of Apter that one should consider a possible series of models which may be considered to approach closer and closer to the prototype we consider not only a good one but an essential one. A model is different in at least some ways from its prototype. The fact is obvious, but when one considers such a complex prototype as a physiological organism one may improperly consider that no model is appropriate. Any such position is completely untenable if one is to make progress in the study of biological activities. However, although we may be willing to admit differences between model and prototype, we must insist that some physical features are demonstrably similar.

Because of the importance we attach to the concept of series of physical models we would like to consider the idea a little further. To be somewhat formalistic let us consider a sequence of models $\{M_i\}$ where i is a real number not necessarily an integer. Then we may consider that the sequence converges on some prototype M as i becomes larger in some methodical fashion. We may for definiteness consider that the smaller values of i represent the cruder models. As i increases we may assume that the model evinces more and more properties in common with the prototype. The value of the simple idea of a sequence enables the investigator to deal consciously with the notion that his problem is essentially of a certain type and that there is no other manner in which it can be conceived. The reader may have already observed that there seems to be an infinite number of ways that the model may be constructed. To emphasize the importance of this fact we will refer to a famous proof which was given by Henri Poincaré.

The proof may be stated as follows: If one mechanical explanation for a phenomenon can be given, an infinity of others can also be constructed. The proof consists in noting that the number of equations relating the coordinates of position and momentum of the masses in the hypothetical model with the experimentally determinable parameters of the phenomenon is greater than the number of such parameters. It then follows that the coordinates of the model can be chosen at will, subject only to the requirement that they satisfy some assumed law for them which is consistent with the equations. In detail the argument is as follows: Let the parameters which can be determined experimentally and which specify the phenonomen under investigation be q_1, q_2, ..., q_n. These parameters are related to one another and to the time t by laws which we may suppose can be expressed as differential equations. Now suppose that there is a model consisting of a very large number p of molecules, whose masses are m_i and coordinates of position are x_i, y_i, z_i ($i = 1, 2, \ldots\ldots\ldots, p$). We assume that the principle of conservation of energy holds for the model, so that there is a potential function V of the 3p coordinates x_i, y_i, z_i; the 3p equations of motion for the molecules will then be

$$m_i \frac{d^2 x_i}{dt^2} = - \frac{\partial V}{\partial t}$$

with similar equations for y and z, while the kinetic energy of the system will be:

$$T = \frac{1}{2}\sum m_i (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2)$$

so that

$$T + V = \text{constant}$$

The phenomenon will then have a mechanical explanation if we can determine the potential function V and can express the 3p coordinates x_i, y_i, z_i, as functions of the parameter q.

But if we assume there are such functions, so that

$$x_i = \phi_i(q_1, \ldots, q_u)$$

$$y_i = \psi_i(q_1, \ldots, q_u)$$

$$z_i = \theta_i(q_1, \ldots, q_u)$$

the potential function V can be expressed as a function of the q_i alone, the kinetic energy T will be a homogeneous quadratic function of the q_i and their first derivatives $\dot{q}_i$, and the laws of motion of the molecules can be expressed by the Lagrangian equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} = 0$$

$$(k = 1, 2, \ldots., u)$$

accordingly, the necessary and sufficient condition for a mechanical explanation of the phenomenon is that there are two functions

$$V(q_1, \ldots., q_u) \text{ and } T(\dot{q}_1, \ldots., \dot{q}_u, q_1, \ldots., q_u)$$

satisfying these requirements with the obvious proviso that the laws of the phenomenon can be transformed so as to take the indicated Lagrangian form. Such functions can be specified if and only if

$$T(\dot{q}, q) = \frac{1}{2}\sum m_i(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2)$$

$$= \frac{1}{2}\sum m_i(\dot{\phi}_i^2 + \dot{\psi}_i^2 + \dot{\theta}_i^2)$$

where $$\dot{\phi}_i = \dot{q}_1\left(\frac{\partial \phi_i}{\partial q_1}\right) + \dot{q}_2\left(\frac{\partial \phi_i}{\partial q_2}\right) + \ldots. + \dot{q}_u\left(\frac{\partial \phi_i}{\partial q_u}\right)$$

and similarly for $\dot{\psi}_i$ and $\dot{\theta}_i$. But since the number p can be taken as large as one pleases, this condition can always be satisfied, and indeed in an infinite number of different ways. H. Poincaré, Electricité et Optique, Paris, 1890, pp. ix -xiv.

Although we have reproduced the demonstration of Poincaré that an infinite number of models is possible for a mechanical system, we do not wish to imply that any mathematical technique is available for the actual construction of a system of converging models. Rather we consider that the possible approach for a model study of any prototype is always going to be intuitive in accord with the Polanyi principle for the invention of a machine.

Before departing from the subject of our investigation of cardiovascular models as examples of disclosive models we would like to indicate that spin off from the main study of models may come about.

For the purpose we refer to two papers by the senior author. One of them, done jointly with J. C. C. Liu, was about elastodynamics pumps [15]; the other, done jointly with Lee Wan, was about large deformations of elastic tubes [16].

With respect to the elastodynamic pump we concluded that it could be used as an alternative to the De Bakey heart pump, as a heart booster. Also, we concluded that the findings from the study of large deformation of elastic tubes might encourage further research which may determine the most effective form of constitutive equations for further investigating the problem of flow of liquids in elastic tubes, which is so important in current physiology and biomechanics.

Finally, with regard to physical modeling of biological systems, we may stress the fact that such investigations lead to the development of special instrumentation with which to sucessfully measure important physical quantities. As an example we refer to the flow meter we had to develop in order to measure average velocity and volumetric flow in a pulsating flow system, like the cardiovascular [17].

The disclosive model, a type of which we have just been illustrating with the cardiovascular system, is obviously a most general classification. We consider that it provides the most important features claimed for it by Ramsey. It permits serious scientific dialogue and furthermore, assists in the development of knowledge and the reduction of mystery. The other five types of models in our taxonomy are much more completely defined and we will deal with these in ensuing pages. First, however, we must consider some special problems which are vitally important to all considerations of modeling. In the next chapter we consider the essential concepts of space, time, and matter. Then we will review the elements of dimensional analysis. Finally, we will define and illustrate extensively the iconic, analogic, similitudinous, Newtonian, and extended Newtonian types of models.

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CHAPTER 4

SPACE, TIME, AND MATTER

We wish now to consider a special aspect of the theory of models. If we are to count, measure, and quantify with respect to models it is essential to consider the concept of number applied to the trinity of space, time, and matter. First, however, let us examine a bit the concepts of space, time, and matter themselves. The title of the present chapter is something that finds itself on some books dealing with physics and especially relativity. In our treatment of the subject of modeling the notions of the natural sciences are important but they are not exhaustive with respect to all of the concerns of mankind. Many useful models exist outside the field of physics and the other natural sciences. In the end we wish to show how space, time, and matter apply in a very generalized fashion. First, however, it is considered that we must examine the position of man within what we may call the physical universe in order to orient our thinking and effectively come to grips with the many different situations in which man finds himself.

With regard to what we may call the real universe it is necessary to locate man with respect to size, duration, and inertia. To us this need seems to be essential in order to make any reasonable judgments concerning activities which are so important to man's very existence.

Whether the world can be viewed as three dimensional in space with time as separate and distinct or whether the world should be considered as a space-time continuum as required by relativity is not our immediate concern. We think that at first we should intuitively consider the relationship of man with respect to two space regimes. The material everyday world of man appears to lie between a microcosm and a macrocosm, between the world of the microscope and the world of the telescope. We seem to have two different space worlds, the world of the stars and the world of particles, between which man with his lilliputian size finds himself. Along this line of thinking we note that Huxley had Eddington saying that man is almost precisely halfway in size between the atom and the star [1]. It is our opinion that such a fact conditions man in all that he thinks and does.

Obviously, the subject which we are now considering touches in a serious manner both psychological and metaphysical thinking. As we have said before and now repeat, we ourselves are engineers. We recognize that there are professionals who competently deal with such esoteric subjects in an authoritative manner, however, because we have dared to accept the challenge to cross boundaries in the realm of ideas for the purpose of studying the generalized concept of model we must make our peace with the situation. In lieu of professional competence

in all fields, which no one possesses, we rely heavily upon others as guides in our search. Always there is the danger that such guides may be misrepresented, nevertheless, if one decides to integrate the intellectual world at large for the purpose of generalizing the concept of model, that is a risk that must be taken. The reader may recall that we have stressed intuition as our guide. Our simple philosophy is that intuition is everyman's guide. No doubt the philosopher, the mathematician, and the physicist also use intuition in order to discover and to invent. Criticism, logicizing, and mathematizing come later. These are severe and necessary disciplines but they do not and cannot substitute for inventiveness.

As an example of what we have in mind we recall for the interest and the information of the reader the history of the development of ideas in such a subject as mathematics. It is an intriguing story which is treated in various books, but for our present purpose we refer to the Philosophy of Mathematics by S. Körner [2]. There one will find that mathematics appears to be seen from three different points of view. They are the logical, the formal, and the intuitional. Leibniz sought the content of mathematics in logical relations between propositions and concepts. On the other hand Kant anticipated the guiding principles of two modern movements in the philosophy of mathematics which are usually called formalism and intuitionism. With the former is associated the name of Hilbert and with the latter that of Brouwer. It may be emphasized that for Kant the role of logic in mathematics is precisely the role it has in any other field of knowledge. He held that theorems follow from axioms according to the principles of logic, but that axioms and theorems are not themselves principles of logic, or any applications of such principles. For him they are descriptive and describe the structure of perceptual data, namely, space and time. Hilbert, following Kant assumes that there is something which is presupposed in logical inference and in logical operations. There are concrete objects which are intuitively present as immediate experience. These underlie all thought. One of the fundamental convictions of the intuitionist is that mathematics is wholly autonomous and self-sufficient. Its methods do not require the guarantees which logicists and formalists allege to provide. The intuitionist says that the assumption that mathematics requires the aid of extensive logical theory or rigorous formulation arises only where the subject is not fully understood. Both formalists and intuitionists follow Kant and reject Leibniz with regard to the philosophy of mathematics. The intuitionist constructs in pure intuition and does not require prior guarantees of existence. We cannot pursue the subject further here but do wish to emphasize that a certain tension does exist within the body of mathematics. We also strongly subscribe to the position that logic per se is not mathematics and it is not physics or engineering either.

One concerned with the history of the development of the concepts space, time, and matter must go back to the pre-historic period. However, with regard to the beginning of what we may call the modern era, special attention must be given to Leibniz and Kant. For a brief story

of each we refer the reader to Volume II of Windelband's History of Philosophy [3]. The treatment by Windelband covers the Renaissance, Enlightenment, and Modern periods. A particularly scholarly treatment of the first era, the Renaissance, is provided by the eminent writer Cassirer in his book, The Individual and the Cosmos [4]. We would further suggest a reading of another work by Cassirer. It is his Essay on Man [5]. In his treatment of the subject of special interest to us there is included an essay entitled The Human World of Space and Time. Here we have an historico-philosophic development. As an overview of his treatment, which is of particular interest to our general thesis, we quote him as follows, "In the boundless multiplicity and variety of mythical images, of religious dogmas, of linguistic forms, of works of art, philosophic thought reveals the unity of a general function by which all these creations are held together. Myth, religion, art, language, even science, are now looked upon as so many variations on a common theme - and it is the task of philosophy to make this theme audible and understandable." The stage for all of these manifestations of man is defined by Cassirer in his statement that "Space and time are the framework in which all reality is concerned. We cannot conceive any real thing except under the condition of space and time."

A truly inspired tracing of a combined humanistic and scientific treatment of the evolutionary process of man's thinking from the time of Galileo's Dialogues (1632) and Newton's Principia (1687) to the current view of nature as a dynamic process is given in Barbour's Issues in Science and Religion [6]. Towards the very end of his book Barbour says, "Time is constitutive rather than incidential; the temporality of process has been one of our recurrent themes."

Having completed our preliminary statement with respect to the subject with which we are concerned, it seems that it may be instructive to review briefly the treatment of space, time, and matter in a well-developed discipline such as physics. In it we have an impressive model for all other pursuits in which man may wish to apply quantitative methods.

As a first try maybe we can do no better than transcribe an article on space-time by Albert Einstein in the fourteenth edition of the Encyclopedia Brittanica. The following presentation is an attempt to summarize Einstein's thoughts. We hope that a minimum of erroneous statement enters our interpretation.

The theory of Relativity has brought about a fundamental change in the scientific conception of space and time. A study of some of the facts leading to such a theory are important for our understanding.

All our thoughts and concepts are caused by our sense-experiences. On the other hand, however, they are products of the activity of our minds; they are thus in no wise logical consequences of the contents of

these sense-experiences. If, therefore, we wish to grasp the essense of a complex of abstract notions we must investigate the mutual relationships between the concepts and, also, we must investigate how they are related to the experiences. We must always consider that the concept-systems of science have grown out of those of daily life.

Initially we are concerned with the meaning of "where", that is, of space. It appears that there is no quality contained in primitive sense-experience that be designated spatial. The concept "material object" must be available if concepts concerning space are to be possible. It is the logically primary concept. It leads fundamentally to the position-relationships of bodies. The general laws of such position-relations are the concern of geometry. In the history of pre-scientific thought we find an evolution of growth from naive space concepts up to the inception of the Euclidean point, straight line, and plane as self-evident things. The meanings of the concepts and propositions of geometry became uncertain only after non-Euclidean geometry had been introduced.

A serious difficulty arose in the old interpretation of geometry because the rigid body of experience does not correspond exactly with the geometrical body. However, it is not advisable to give up the view from which geometry derives its origin. For our thought processes such a model definitely has its uses. Norwithstanding it is essential to examine the foundations of geometry. For analysis of physical problems in the large it is found that greater generality than the Euclidean geometry is required. The non-Euclidean geometry is essential for the full expression of physical law. After the introduction of the concept of time in our theorizing we come to the need of a four-dimensional space-time of non-Euclidean character. Such a geometry is built up on the metrical invariant ds^2, where ds^2 is given in terms of the coordinates x_i and the metrical coefficients g_{ij} as follows:

$$ds^2 = g_{ij}dx^i dx^j$$

The variability of the functions g_{ij} is equivalent to the existence of a gravitational field.

The generalized concepts of space curvature as developed by Gauss and Riemann now become essential for the full development of the mechanics of relativity. A special branch of analysis is required. Tensor analysis provides the par excellent treatment of relativistic problems. The physical laws are studied as invariants under very generalized types of transformations of coordinates. It may be recalled that Felix Klein once defined geometry itself as the study of invariants under transformations.

Einstein goes further to discuss time, clocks, and simultaneity of events as they relate to relativity. He distinguishes the special and general theories. Details of the subject could be followed further, but we rest here because our only intention is to recall how modern physics arose from pure sense-experiences to its highly developed state of mathematical analysis. For those who may wish to pursue the subject further we strongly recommend the classic treatise on Space, Time, and Matter by Hermann Weyl [7]. All of the material that we have been lightly skimming are rigorously treated along with a formal presentation of geometry and tensor analysis. Weyl says that space and time are commonly regarded as the forms of existence in the real world, matter as its substance. It is this position which we wish to emphasize in connection with our general thesis about models. In fact we insist that a transference of the analysis from the field of physics to other fields depends mainly on the generalization of the concepts of matter and substance. More will be said by us on this subject later.

In order to increase the availability of references for our readers we wish to suggest several additional books. First, there is the paperback by Max Born, in English, which is published by Dover Publications [8]. Born is truly a physicist of eminence who has written a non-mathematical type of book for the serious reader. In underscoring that fact we might do well to quote his work. He says, "in our time science, and physics in particular, has become a fundamental part of our civilization, and the number of people who wish to grasp its essence has grown immensely. Now, rereading my old book, I get the impression that its way of presentation should appeal to a considerable number of people, particularly to those who, without knowing higher mathematics and modern physics, remember something of what they learned at school and are willing to do a little thinking. I believe that they could gather from a book of this kind more than a vague feeling about grand, but dark and abstruse, mysteries of nature; they might really obtain an understanding of modern scientific thinking."

The second reference we wish to cite is a technical non-popular treatment of the theory of relativity by the Russian scholar Petrov in an English translation entitled Einstein Spaces [9]. Again we quote the author. Petrov says, "The methods of investigation comprise experiments followed by hypotheses and assembly of data defining four-dimensional Riemannian geometry with signature (---+). Naturally, contemporary mathematicians cannot be satisfied with the methods of analysis used by Einstein, Hilbert, and the other founders of general relativity; this explains the present tendencies to devise up-to-date experiments and to apply new mathematical methods to investigate new topics." He further says in his Foreword that, "This book is devoted to an investigation of the various spaces which form the basis of the theory of relativity, and the generalization of these to an arbitrary number of relativity theory a series of fundamental monographs such as

Landau and Lifshitz's The Classical Theory of Fields and Fock's Theory of Space, Time, and Gravitation, we have consciously tried to limit ourselves to a number of questions not considered in these works, which should be of interest to both physicists and mathematicians. This explains why the treatment is given a mathematical emphasis and the special attention paid to 4-spaces with a Lorentz-type signature."

Finally, we refer to one of the monographs which Petrov cited. We have examined the English translation of Russian Academician V. Fock entitled The Theory of Space, Time, and Gravitation and consider that it should be added to our list of references [10]. It is a highly professional piece of work and therefore sufficiently mathematical. Again we think it best to quote directly from the preface to the first edition. Fock says, "The aim of this book is threefold. Firstly, we intended to give a text-book on Relativity Theory and on Einstein's Theory of Gravitation. Secondly, we wanted to give an exposition of our own researches on these subjects. Thirdly, our aim was to develop a new, non-local, point of view on the theory and to correct a widespread misinterpretation of the Einsteinian Gravitation Theory as some kind of general relativity." We think it is important also to quote from the preface to the second edition. There Fock says, "The author's views on the theory are explicitly formulated in different parts of the book and are implicit in the reasoning throughout the whole text. Their general trend is to lay stress on the Absolute rather than on the Relative. The basic ideas of Einstein's Theory of Gravitation are considered to be: (a) the introduction of a space-time manifold with an indefinite metric, (b) the hypothesis that the space-time metric is not rigid but can be influenced by physical processes and (c) the idea of the unity of metric and gravitation. On the other hand, the principles of relativity and of the equivalence are of limited application and, notwithstanding their heuristic value, they are not unrestrictedly part of Einstein's Theory of Gravitation as expressed by the gravitational equations."

The quoted material which we have just presented, including the article by Einstein, are purely technical physics and have very little philosophical overtones. We now consider that some material of a more philosophical and historical nature is desirable in order to place the discussions in a proper context with our more general purpose. In order to accomplish this we will cite several texts of that type.

Our first reference for the purpose is The Measure of the Universe which is a history of modern cosmology by J. D. North [11]. The author provides a fairly substantial historical introduction to modern cosmology and then analyzes some conceptual problems. In terms of the presentation by North, which we heartily recommend to the reader, we would like to single out for presentation here his categorization of model universes on page 130 and then his comments on cosmological theories and models which begin on page 310.

For present purposes we take the liberty to quote directly from North concerning the number and nature of cosmological models. North says, "The number of possible relativistic models is thus large, but from our point of view it is not necessary to provide an exhaustive account of them. For the purposes of later discussion, however, we list seven distinct forms for the function R(t):

(i) Contracts to minimum, after which it expands monotonically to a de Sitter state. (Models exist for $K = +1$ and $\lambda_e > \lambda > 0$).

(ii) ('Eddington-Lemaître model') Expands asymptotically from the Einstein state to a de Sitter state. ($\lambda = \lambda_e$, $K = +1$).

(iii) Einstein state ($\lambda = \lambda_e$, $K = +1$).

(iv) Expands monotonically from singular state. Point of inflexion. (Cases for $\lambda > 0$ and $K = 0$ or < 0. Also $\lambda = \lambda_e$, $K = +1$). This last was Lemaitre's favored model, namely that beginning what he called a 'primeval atom').

(v) Expands monotonically from singular state but the curve has no point of inflexion (Cases for $\lambda = 0$ and $K = -1$ or $K = 0$, The latter is the Einstein-de Sitter' model).

(vi) Limiting case, separating examples of case (IV) (monotonic) from examples of case (vii) (oscillating) ($\lambda = \lambda_e$, $K = +1$).

(vii) Oscillatory models (Cases for $\lambda = 0$, $K = -1$ or 0 and $\lambda_e = \lambda$).

North then presents the seven cases as curves in an R,t diagram, where R is radius of the universe and t is time. The physical parameters of importance in the above classification are curvature, pressure, density, and time.

The reader may be interested in examining North's critical discussion of cosmological models. It is our purpose to draw attention to the analysis because we disagree markedly with his position about models. The section of his book under which the material is treated is entitled <u>Cosmological Theories and Models</u>. A direct quote of importance to us is as follows, "Models, ..., are in no sense logically necessary adjuncts of the main theory." He also says, "they [models] suggest ways in which the main theory may be extended or otherwise modified to advantage. They thus assist in the growth of theories." We do not completely disagree with North but we think our quarrel is on the basis of a dangerous ambiguity which he introduces concerning models. It is very interesting that a highly intelligent man seems to be confused in this matter. We will attempt to clarify our position in the following. Now we refer the reader to the position taken by Campbell as discussed in Mary Hesse's treatment of the subject which we previously referenced. Campbell

establishes the point that, because of a certain property, model is not identical with its related theory. North himself at one point admits the fact that model is 'didactic'. From our point of view the epistemology of the situation is that the model is not logical in a technical sense but rather it is intuitive and inventive. The logical theory works along with it to generate reliable knowledge. The matter is somewhat remniscent of Polanyi's idea about machine qua machine. To comprehend machine one must have an insight which is not reducible to logic.

If one wishes to pursue the philosophical aspects further we suggest Philosophical Foundations of Physics by Rudolph Carnap [12]. He treats laws, measurement, structure of space, causality, determinism, and statistical aspects. We wish to emphasize his notion of measurement of quantitative concepts and for that purpose we will quote directly. Carnap says, "If the facts of nature are to be described by quantitative concepts - concepts with numerical values - we must have procedures for arriving at those values. The simplest such procedure, as we saw in the previous chapter, is counting. In this chapter, we shall examine the more refined procedure of measurement. Counting gives only values that are expressed in integers. Measurement goes beyond this. It gives not only values that can be expressed by rational numbers (integers and fractions), but also values that can be expressed by irrational numbers. This makes it possible to apply powerful mathematical tools, such as the calculus. The result is an enormous increase in the efficiency of scientific method." On first glance, to some, this statement may seem trivial. We know it to be fundamental and essential to the comprehension of mathematical analysis in the applied sciences. Mathematical analysis, as well as geometry have little analytical value without the aid of algebras and their arithmetics. We shall go more into the fundamental nature of number in the next chapter where we treat the subject of dimensional analysis.

Before leaving the purely technical subject of physics we wish to call attention to histories of the three important physical concepts of space, mass, and force by Max Jammer. They are the history of the theories of space in physics [13], the history of the concept of mass [14], and the history of the concept of force [15]. All of these books are scholarly and they competently present their topics for the period which begins with the Greek philosophers and ends at the present time. Presently we will add to these a reference to an important recent treatise on time, but then the concept will be greatly generalized so that it does not apply to physics alone.

Now after having surveyed in some detail the generation of a body of physical knowledge, beginning with Einstein's article from the Encyclopedia Britannica, we come to a time at which we wish to introduce our ideas about the meaning of model. The reader will recall that many scholars working in physics recognize the value of model but think of it as dispensable because of what they consider to be the dominant role of theory. They seem to think of theory and model as somehow related but

that in the end the concept of model is really dispensable. One philosopher, at least, to whom we have referred, that is Campbell, apparently had reason to flatly deny that model is dispensable. He considered that it is an essential feature in the development of knowledge. We wish to indicate our proof that model is essential in the same manner as theory by presenting our conception of the process of learning, especially as related to the physical sciences and engineering.

We can accomplish the purpose of proving our ideas about the necessary relationship between model and theory by examining the sciences of communications, control, systems, and operations research. First recall the details of Einstein's article on space-time. He commenced with some important considerations of sense-perception and ended with his notions of relativity. His study did not involve a conscious application of what we shall henceforth call the concept of theory-model dyadic thinking. We shall consider the hyphenated expression theory-model as an essential dyad. It is a sort of Siamese twin combination. You simply cannot have one without the other. We hope that some ingenious person will design a Greek derivative to represent properly the important hyphenated expression. For the present we shall simply refer to it as dyad of the thinking process which results finally in an important body of knowledge. Einstein began his study as would a Greek philosopher of old or of Thomas Aquinas in the thirteenth century. They insist that the human senses are the sole source of knowledge. The Angelic Doctor insisted on such a position even at the risk of losing his position as the theological doctor of the Church. Modern psychologists also proceed from the same initial point but now have a great deal of sophistication in their subject. We too must begin our development with sense-perception as the initiator of knowledge and outline our conception of the essential system that finally provides the completed knowledge. For our purpose we consider the mind-body as a system which exists in an environment which is capable of feeding information to it. Through the senses of the body information is fed to the mind by the way of the brain. We do not wish, at this point, to be diverted from our purpose by any controversy concerning the meanings of mind, body, and brain. We consider that the reader has sufficient understanding of the matter for us to proceed quickly to the further development of the theory-model concept without undue confusion.

So we simply express the process as follows. The human being constitutes the mind-body system to which input comes through the senses from the surrounding environment. In the mind there is the model-theory feature for coping with the incoming sense information. Perception is possible. The input generates a model in the mind. Here we see the essential role of the model. Now a question arises as to the faithfulness of representing the real world or prototype by means of the model. At this juncture theory comes into play. It actually is the critical faculty for evaluating the model. The output of the mind-body system is the model. With the thought dyad, or model-theory, the environment is now examined for conformity with the model. The theory is the essential

means for accomplishing the critical analysis. We can always assume that there will be some lack of conformity, some error. Here we encounter the feedback aspect of the process. The error, or lack of conformity, is detected by again using the senses in the act of comparing model and prototype by means of theory. The model may be crude or accurate the first time it comes into being, but in any event there will surely be some discrepancies. The person involved may accept the model as satisfactory and considers that it is an appropriate representation of the prototype or he may consider otherwise and reject it. As any systems analyst knows, a process of model refinement could continue as long as the person desired.

Now recall an idea, that we previously presented, about a model sequence whose limit is the prototype. We said that a sequence of models $\{M_1\}$ is considered to approach the prototype M as i is increased. We did not explain, however, how the convergence was to be accomplished. Now we can see how the fundamentally important theory portion of the thought dyad provides the means. The theory, just like logic in the logico-mathematical argument of Kant, occupies the role of critic with respect to the model.

At this point the reader may begin to wonder, if he has not already done so, what is the proper attitude to assume about theory. Is it supposed to be considered absolute or perfect. Of course not. Therefore one may entertain the notion of altering the theory to improve its nature. We may stress now, even if we have not done so before, that experiment is really the means used with the senses to examine the environment and its objects. As the reader well knows the cry always is that the theory must agree with the crucial experiment. We agree. Experiment or experience is the final court of appeal for the theory-model process. By its means both theory and model can be realistically modified. By now we consider that it should be clear what the role of model, of theory, and of experiment, is in the knowledge generating process.

We promise to prove our assertion about the learning process. What can the criteria be for such an attempt? Our only reply is that we will refer to what the psychologists are doing today in this field. Our feeling is that many of them would agree with our position. Maybe some of them have already enunciated what we have described above. So much the better for our position. We would welcome experts on our side. However, we stress the point that even if there exist theories of knowledge which we have just outlined we consider that no one so far has attempted to explain the role of model and the role of theory in the knowledge generating process in the way we have. From henceforth we wish to conceive and define model in the fashion which we have outlined. We consider that we have fully justified the essential position of the concept of model in human reason and we further consider that such a position enables us to apply the idea and its categories in the future.

Going now to the field of psychology we wish to examine the crowning achievement of the great Jean Piaget to whom the senior author listened with such great satisfaction many years ago at the tricentennial anniversary of Harvard University. For those who may not know, we can say that Piaget spent a long lifetime studying the nature of the learning process. One of the fascinating fields to which he devoted a great deal of time is the field of child psychology. We now refer to the last, or certainly nearly the last, book of his life. It is entitled Biology and Knowledge [16]. We prefer to let Piaget speak for himself from his preface. He says, "The aim of this work is to discuss the problem of intelligence and of knowledge in general (in particular logico-mathematical knowledge) in the light of contemporary biology. It is therefore a gathering of interpretations rather than of experimentation. But this theoretical essay is the work of an author who has been engaged for forty-five years in psychological experimentation in development and who therefore intends to adhere as closely as possible to the facts." The reader may find part of the substantiation for our thesis on models in the last chapter of Piaget's book which is entitled Conclusions: The Various Forms of Knowledge Seen as Differentiated Organs of Regulation of Functional Exchanges with the External World. We feel constrained to quote from the first lines of of this chapter. Piaget says, "Having reached the end of our analysis, we shall find it useful here to take another look at our main hypothesis as set out in section three. What is amounts to is, on one hand, the supposition that cognitive mechanisms are an extension of the organic regulation from which they are derived, and, on the other, the supposition that these mechanisms constitute specialized and differentiated organs of such regulations in their interaction with the external world." We think that the final work of Piaget is so important in itself and, also, so important to our thesis that we wish to have him state his own conclusions. He says, "On the whole, I think that I have justified the two hypotheses which were linked together in my main thesis in section 3: that cognitive functions are an extension of organic regulations and constitute a differentiated organ for regulating exchanges with the external world. The organ in question is only partially differentiated at the level of innate knowledge, but it becomes increasingly differentiated with logico-mathematical structures and social exchanges or exchanges inherent in any kind of experiment.

There is nothing unusual about these hypotheses, I know, and I am sorry that it should be so. Nevertheless, they are hypotheses which must constantly and more extensively be explored, because, strangely specialists in epistemology, particularly mathematical epistemology, are too much inclined to leave biology out of account, while biologists, as a rule, completely forget to ask why mathematics is adapted to physical reality."

Because of the importance of psychology to our theory of models based on the notion of sense-perception we wish to provide the reader with several additional references.

Of great importance to our position on models is a brilliant article by Wolfgang Metzger entitled the Phenomenal-Perceptual Field as a Central Steering Mechanism, which is Chapter VII, beginning on page 241, in a book entitled The Psychology of Knowledge, edited by Royce Rozeboom [17]. We urge the reader to study the entire book and particularly the article by Metzger. He provides a diagram on page 253 which represents what he calls the relations between phenomenal world, transphenomenal world, and critical phenomenal world according to the critical realistic view. We think our idea about mind-body system as related to model-theory dyad is greatly substantiated by Metzger's arguments.

One may well ponder the writings of the earlier scientist, Hermann von Helmholz, on the subject of the physical basis for perception. A recent publication may serve the purpose of the reader. It is entitled Helmholz on Perception: Its Psychology and Development.

The final reference which we may suggest as a general survey of the subject of sense-perception is a standard textbook by Eleanor J. Gibson entitled Principles of Perceptual Learning and Development [18].

One may wonder why we have chosen a title of space, time, and matter under which to define and expound upon the idea of model. We may answer by saying that it has been in that set of ideas that we best saw an opportunity to most effectively deal with the meaning of model. Physics is so well-developed and structured, as well as being a subject in which the logico-mathematical approach can so incisively be applied, that one is easily led to use it as the ground for developing a more generalized concept of model and demonstrating its role in human thought. We have reviewed cosmological space rather extensively and discussed its related science of geometry. The concept of matter we have not examined in any detail but have referred to Max Jammer's book on mass which he shows is closely related to the concept of matter. On occasion we have discussed particles in physics and in mechanics in particular. We may equally well have referred to cosmological masses in galaxies. It appears that further development is not necessary at present and, in any event, we wish to reserve the term matter to cover all of those things which man thinks about exclusive of space and time. Further developments of the treatment of matter will occur later in the book.

We do wish to say a bit more, however, of the concept of time which is so important to the meaning of the space-time-matter trinity. It turns out that there is an excellent general treatment of the time concept in a recent treatise entitled The Study of Time which has as editors Fraser, Haber, and Müller [19]. The book constitutes the Proceedings of the first conference of the International Society for the Study of Time, held at Oberwolfach (Black Forest) in West Germany. As an aside one might equally well suggest an International Society for the Study of Space and another for the Study of Matter. However that may be, we do wish to make some references to the present volume on time.

The widespread interest in the subject is indicated by the table of contents. The five principal divisions are: Time and the Physical Sciences, Time in the Life Sciences, Time, Philosophy, and the Logic of Time Concept, Time and Culture, and finally an application topic entitled Dysrhythmia. The book opens with a chapter on the history of the time concept. It begins with the statement that "The origins of our concept of time are shrouded in mystery, but from our knowledge of ancient civilizations and also of surviving primitive races it is reasonable to assume that the lives of our remote ancestors were far less consciously dominated by time than ours." Such a view is consistent with our opinion that increasing consciousness of man is an evolutionary process that follows the passage of time.

A particularly important chapter of the book for the purpose of our study of the concept of model is by John A. Michon, a professor from the Institute for Perception, RVO-TNO, Kampweg 5, Soesterberg, Netherlands, and the State University, Netherlands. His article is so important from our point of view that we will quote the author directly from his summary. He says, "For man as an information processing system, time is one of the experiential dimensions of information such as intensity, size, etc. Since as a processor man has a limited capacity there will be necessarily a trade-off between temporal and non-temporal information, which is open to quantification. .... Most contemporary models of time evaluation incorporate a - specific 'pulse counter' mechanisms to account for the internal clock by which time is measured subjectively. The rate of this internal clock is thought to be influenced by the information processed by the subject. In this paper an alternative formulation is defended: time evaluation is a cognitive reconstruction of contents of the interval. The latter formulation avoids the unnecessary assumption of the former. It explains the same phenomena equally well while moreover it can handle various matters that offer difficulties to models stated in terms of clock mechanisms." There are many readable and interesting articles besides this one but we must resist the temptation to quote further. We hope we have excited the curiosity of the reader sufficiently to study the entire book.

We conclude the chapter on Space, Time, and Matter with a brief summary of our main thesis. As a result of our study, we find that it is essential to link the concept of model and the concept of theory. The model arises in the mind-body system as a consequence of sense stimulation. Theory is the auditor for checking the correctness of the model. By a feed-back character of the system the model can be continuously corrected to conform to the prototype through the intervention of the theory as a criterion. In such a fashion the model or the sequence $\{M_i\}$ can be made to converge on the prototype M. A model is essential to the development of human knowledge. It is not incidental and dispensable as some have said. It is an essential part of the thought dyad -- theory-model. It is not a substitute for theory, nor is theory a substitute for it. It is on this firm basis that we propose to erect

particular model concepts, especially the widely usable disclosive model which we have previously discussed. In the remainder of our book we treat specific models in some detail. First, however, we have one more auxiliary topic which we wish to present in some detail. It is the important subject of Dimensional Analysis.

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CHAPTER 5

ROLE OF DIMENSIONAL ANALYSIS

Dimensional analysis is related to the elementary principles of physics and to the theory of scaled models. The latter subject is a portion of our larger approach to models and consequently we wish to give it some consideration in general, but more specifically because it relates intimately to our subsequent treatment of what we call similitudinous models.

Anyone familiar with the subject knows that there is a fairly substantial history and a great deal of early controversy associated with it. There have even been some quarrels about priority of claims with respect to some of the principles and so-called proofs of theorems. The great Fourier is called by some the father of dimensional analysis. We know definitely that Newton enunciated the principle of similitude and gave some thought to the subject in the seventeenth century. At the present time we do not wish to place ourselves in the business of establishing claims in a paternity case. Notwithstanding we do think that the history of the subject is interesting and one can gain some insight to important points by studying it. While we do not endorse all of the views in the very brief history by E. O. Macagno we can recommend it to the reader for perusal [1]. The author does treat some of the interesting points. Also, he provides a useful group of references if one wishes to read further.

In the early twentieth century a great deal of attention was given to dimensional analysis in this country by E. Buckingham and P. W. Bridgman. In 1914, Buckingham published a long systematic paper [2]. Following him in 1922, Bridgman published a short study of the subject of dimensional analysis, which in 1931 became the now well-known revised edition [3]. An experience of one of the authors of the present book can be used to illustrate what may be considered significant thinking in the period. Your senior author, as a young man, was planning a trip to the tricentennial of Harvard University in 1936. Before going he visited Buckingham at the National Bureau of Standards and had the pleasure of a long talk about dimensional analysis and model testing. At that time it turned out that Buckingham was very enthusiastic about the subject and desired to collaborate with Bridgman on an extensive treatise. Accordingly, your author was requested to tentatively take the matter up and determine the then existing feelings of the great Harvard physicist. The outcome was very interesting. Bridgman explained in some detail why he had written his book in the first place and why he did not propose to revise and extend it. It seems that the reason for such a response rests on the fact that Bridgman as a young man thought that an intense study of dimensional analysis would have some value in the philosophy of science and more importantly might shed some light on the foundations

of physics. In the meantime his enthusiasm for that viewpoint had cooled and so he did not wish to invest more time and energy on the subject.

From that interesting experience we learned several things. Bridgman was not very much interested in engineering models, even though his book ends with some examples about them, taken primarily from work by Buckingham. Also it is clear that the interest of Buckingham was mainly in engineering applications in the field of scaled models, even though his big paper was published in the Physical Review. One point that impressed Bridgman was that the number of variables which define a physical problem could be reduced by means of dimensional analysis. Of course this is true but we do not consider it a very important point in itself. In fact that viewpoint led Bridgman to state on page 53 of his book, "dimensional analysis would certainly not apply to most of the results of biological measurements, although such results may perfectly well have entire physical validity as descriptions of phenomena. It would seem that at present biological phenomena can be described in complete equations only with the aid of as many dimensional constants as there are physical variables. In this case, we have seen, dimensional analysis has no information to give. In a certain sense, the mastery of a certain group of natural phenomena and their formulation into laws may be said to be coextensive with the discovery of a restricted group of dimensional constants adequate to coordinate all the phenomena." It is quite clear that Bridgman is really interested only in the development of physical laws in classical style and not in model analysis, especially of an engineering nature. A careful reading of his preface will underscore the point. Nevertheless Bridgman's little book is still a good reference for beginners who wish to understand something about the principles of dimensional analysis.

Another more recent book, which although it is somewhat verbose, may be of interest to the reader. It is a treatise on units, dimensions, and dimensional numbers by D. C. Ipsen [4]. The beginner in engineering and applied science may find Ipsen's book of great value to him. It not only treats units in such subjects as mechanics, heat, and electricity, but also has something to say about dimensional analysis and similitude.

One can enthusiastically recommend a fairly recent book on many aspects of dimensional analysis and its applications by L. I. Sedov [5]. Not only does he present a forceful development of the classical parts of the subject, but in Chapter V we consider that he lays to rest Bridgman's pessimistic views about the application of the methodology to such complicated natural sciences as biology, by his own brilliant application to the somewhat recondite field of astrophysics.

A more modern exposition of dimensional analysis is given in a small book by H. E. Huntley [6]. Although it was originally published in the United Kingdom, there is now a nicely done paperback edition by Dover. The classical treatment is well presented but there is also a reference to new and important developments by Moon and Spencer.

We will discuss the work of these authors later. Huntley, also, provides a brief historical outline in his Chapter III. He assigns priority in the development of the subject in the following quotation. He says, "The method of dimensions, or dimensional analysis, had its origin in a principle referred to by Newton in his Principia (II proposition 32). This is the 'principle of similitude'. It was used by Newton at the time when he was laying the foundation of mechanics as a fundamental branch of science."

A recent book which is somewhat similar to that by Huntley may also be of interest to the reader. As a convenience, the so-called dimensional matrix is used systematically in applications of the method. There is, also, a large number of tabulated dimensionless parameters listed in Appendix No. 2. The book, which is said to be for engineers, is by J. F. Douglas [7].

Having introduced some of the more standard works on the subject we now turn to some of the more recent considerations which we think are important in themselves, but more especially because they seem to be leading to more fruitful developments of dimensional analysis and its applications in the field of certain kinds of models.

In 1949, Moon and Spencer published a paper entitled A Modern Approach to Dimensions, which we consider to be a landmark in the subject of dimensional analysis. That paper has been mentioned by such mathematicians and scientists as Garrett Birkhoff [8]. Because of the new departure and importance of the paper we wish to refer to some of its specific features. As a preliminary we wish to recall the dimensional ambiguity that arises in connection with the concept of energy and the concept of torque. In the usual dimensional scheme of elementary mechanics both terms are represented by the product of a force and a length. Now while this fact may not cause difficulty to anyone thoroughly aware of the quality difference between the two terms, it does point to the basis for logical difficulties which probably should be removed from the subject. In order to accomplish this, Moon and Spencer introduce the notion of 'vector' in place of what they assume is a scalar that is normally employed in rudimentary analysis. The reason for this is probably obvious. Since such terms as energy and torque cannot be distinguished on the basis of scalar magnitude alone, another element to designate the physical quality of the term should be introduced. Those authors then think up the possible analogy of the physical quantities energy and torque with what they call a vector. Now the reader should be advised that, as he already knows from classical mechanics, energy is a scalar and torque is a vector. So then how do the authors wish to refer to both of these terms as vectors. Well as so often happens in mathematics, the same expression is used to denote different things. The authors take great pains to clarify any possible ambiguity. We do not wish to go further into the matter at this time but would like to let the authors of that paper speak for themselves. In order to accomplish this we quote from their summary. They say, "The paper describes an investigation of some possible 'dimensional' systems for the designation of

physical concepts. The usual system, based on the primary concepts of length, mass, and time, results in a large number of ambiguities (Table II) and is therefore unsatisfactory. The introduction of an additional primary concept for heat and another for electromagnetism is an improvement, but many ambiguities still remain (Table III). The trouble is caused by our failure to distinguish between distances in different directions.

By introducing two fundamental lengths 1_r and 1_t (radial and tangential) in place of the usual 1, we eliminate the former difficulty and obtain distinct designation symbols for the concepts of energy and torque, for normal stress and shear stress, for pharosage (radiant energy per unit area) and helios (brightness)." It can readily be seen that with what problem the authors are concerned. We will proceed no further with this important matter concerning dimensional analysis but will rather refer the more curious reader to the paper itself.

An additional book and paper may be added to the list of references which are concerned with the current treatment of dimensional analysis. The book is by Julio Palacios and is simply entitled Dimensional Analysis [9]. It is a critical review of the classical expression of the subject and it takes up matters such as those which concerned Moon and Spencer. While it is a little pompous at times in the treatment of other authors, we consider that it may be a valuable reference for anyone interested in the general subject. The paper, which is by F. V. Costa, takes up the question of classical vectors that arise in the analysis. He even employs the expression directional analysis [10]. Because of its importance we will quote directly from the author's summary. He says, "The consideration of directions in directional analysis, as recommended herein, renders its applications even more difficult and more prone to error. But as the consideration of directions makes the solution of some new problems possible, such as the representation on the same scale of forces of different natures, the direct determination of the convenient distortion, and the selection of the suitable properties of the material to be used in a model, including its anisotropy , it seems worthwhile to investigate further the practical applications of the systematic approach suggested herein.

The information written about the consideration of directions by Ames and Murnaghan, Duncan, Palacios, and especially Huntley was of great use to the writer in preparing the paper. It will certainly be of even greater use to those interested in developing the subject and having a greater experience in the designing of models."

A book by W. J. Duncan entitled Physical Similarity and Dimensional Analysis is actually a straightforward application of classical dimensional analysis to engineering problems [11]. However, a different use of the term similarity occurs in a small book by A. G. Hansen [12]. Hansen, in the main, is really concerned with the solution of partial

differential equations associated with physical boundary value problems. But in addition to various techniques and methods such as group-theory he does devote a chapter to the application of dimensional analysis for his purpose. We recommend Hansen's book to the reader primarily because of the use of dimensional analysis in the solution of already derived differential equations.

Probably one of the most suggestive and incisive treatments of the subject of dimensional analysis and its applications in the theory of models is by Garrett Birkhoff. The reader will find there an important treatment of both old and new aspects of the subject. One of the most important features of Birkhoff's study is his strong emphasis on general transformation theory as related to model analysis. He senses the importance of his work but we must admit he does not go far in its development. However, he certainly does open up vistas for a future investigator. One point about Birkhoff's book that we do not understand, however, is his insistence on a method called "inspectional analysis" which was emphasized in a short paper by A. E. Ruark [14]. The assumption apparently is that one is already supplied with the differential equations which define the problem of interest and that inspectional analysis provides the investigator with some insights that perhaps he could not get in any other manner. We disagree with both Birkhoff and Ruark concerning the value of any such vague method, especially if one is considering physical models of a given prototype.

Finally, we would like to cite a reference that we consider is concerned with a new aspect of the subject. Here again we have the nascent development of new methods for solving a general group of problems. Our reference is to a paper in the journal Simulation [15]. It is by C. M. Woodside and is entitled Scaling Analysis and Dimensional Analysis of Simulation Models. Woodside says in his introduction that, "Scaling analysis is a tool for getting the maximum information out of computer simulation experiments which explore parameter changes. Its use and its theoretical basis are the same as dimensional analysis in experimental physics: however, the name scaling analysis emphasizes the freedom from physical dimensions of mass, length, time, etc., and also the origin of the artificial dimensions or Independent Scaling Factors introduced. Scaling analysis also appears to be a subset of similarity analysis for simplifying the solution of differential equations. Similarity analysis uses a very general class of scaling relationships on variables (as opposed to parameters) and has been applied to partial differential equations."

We have completed our brief critical survey of the technical literature on dimensional analysis and now wish to set forth some of the principal features of the classical theory in order to concretize the subject matter and prepare the way for applications in our chapter on similtudinous models. The so-called Pi theorem of Vaschy and Buckingham is the basis for analysis and so we will now state the theorem. All of the books on the subject, including that by Bridgman,

have their own peculiar ways of expressing the theorem but for our purpose we wish to refer to page 82 of Birkhoff's book [13]. He states the so-called theorem as follows:

"Let positive variables $Q_1, \ldots, Q_r$ transform by (2) under all changes (1) in fundamental units $q_1, \ldots, q_n$. Let $m \leq n$ be the rank of the matrix $||a_{ik}||$ defined by (2). Then the assertion that

$$f(Q_1, \ldots, Q_r) = 0$$

is a unit-free relation and is equivalent to a condition of the form

$$\phi(\pi_1, \ldots, \pi_{r-m}) = 0$$

for suitable dimensionless power-products $\pi_1, \ldots, \pi_{r-m}$ of the Q_i."

For our purposes we will rephrase the theorem in a pragmatic form as follows:

Defining a physical problem let there be:

1) a set of physical variables X_i, $i = 1, \ldots., n$ and

2) a set of fundamental quantities x_j, $j = 1, \ldots, m$ where $m < n$ and the physical variables X_i are expressible in terms of the fundamental quantities x_j.

Then suppose a physical problem defined by:

$$\phi(X_1, X_2, X_3, \ldots, X_n) = 0$$

An equivalent equation can be found as follows:

$$\psi(\pi_1, \pi_2, \pi_3, \ldots, \pi_{n-m}) = 0$$

where the π's are product-quotient numbers which are dimensionless with respect to the fundamental quantities x_j.

It can be seen that the number of variables in ψ is (n-m); that is a number fewer by m of fundamental quantities. This fact is an important aspect of the Vaschy-Buckingham theory. As Bridgman emphasized, the number of necessary variables required to define a physical problem is smaller than the number of physical variables which effectively relate to the problem. Such a reduction in itself has its value, however, our

main interest in the π_i dimensionless numbers is that they relate model to prototype in a straightforward manner in the classical model analysis.

Now in order to illustrate the application of the theory and to prepare for its use in our chapter on similitudinous models we will consider two physical problems from the field of mechanics. Both are well-known and occur in practically all texts on the subject.

Problem No. 1, Oscillation of a Simple Pendulum

Suppose the problem is one of determining an equation for the period of vibration T of a simple pendulum. We assume that somehow we know the list of physical variables X_i and that they are T, m, l and g, where T is the period, m the mass on the string of length l, and g the gravity constant.

To obtain an equation for T we assume that we can solve the equation:

$$\phi(X_1, \ldots, X_n) = 0$$

so that $T = f(m, l, g)$.

Then dimensional homogeneity requires that:

$$[T] = [M]^{\alpha}[L]^{\beta}[LT^{-2}]^{\gamma}$$

where L, M, and T are the set of fundamental quantities, that is, mass, length, and time. α, β, γ are real numbers to be chosen in such a fashion that the dimensional homogeneity condition is to be satisfied.

Hence we see that:

$$\alpha = 0, \ \beta + \gamma = 0, \text{ and } -2\gamma = 1.$$

Therefore

$$\beta = -\gamma, \ \gamma = -1/2$$

and we have: $$T \propto \ell^{1/2} g^{-1/2}$$

so that $$T = \text{constant} \sqrt{\frac{\ell}{g}}\,.$$

This is, as is well-known, the solution of the second order ordinary differential equation which defines the motion. For small motion it may be recalled that the constant is 2π.

Inasmuch as there are four physical variables and three fundamental variables there is only one π_i number which may be written:

$$\pi_1 = \frac{T}{\sqrt{l/g}} = \frac{Tg^{1/2}}{l^{1/2}}$$

or π_1 is any power of the quotient. It may then, also, be written for example:

$$\frac{T^2 g}{l}$$

If one wished to apply the result to a physical model problem to determine the period of the prototype from that of a small model then the number should be made the same for both model and prototype; that is:

$$\frac{T_m^2 g}{l_m} = \frac{T_p^2 g}{l_p}$$

where the subscripts m and p mean model and prototype respectively. The gravity constant g is the same for both and cancels out of the equation.

Hence if we measure the period T_m om the model, the predicted T_p for the prototype is:

$$T_p = \sqrt{\frac{l_p}{l_m}}\ T_m$$

It is assumed that lengths of both model and prototype are known.

This problem of the simple pendulum may seem to be trivial, however it does contain all of the necessary features to illustrate the application of dimensional analysis in model theory for scaled models.

A somewhat more complicated case is the Froude problem of ship resistance. We shall now analyze it as our problem No. 2.

Problem No. 2

It is desired to tow a model which is geometrically similar to the ship prototype and measure the towing resistance so that we may be able to predict the force necessary to propel the ship at corresponding speed.

Again we write down the list of physical variables as follows:

R = towing force

1 = length of model

v = velocity

g = gravity constant

η = viscosity coefficient and

ρ = density of liquid

The fundamental variables are M, L, T for mass, length, and time respectively.

According to the Buckingham π theorem there should be (6 - 3) or 3 dimensionless numbers π_1, π_2, and π_3.

Proceeding in the same manner as with the pendulum problem we may solve for the towing force R and by the condition of dimensional homogeneity obtain the value of exponents α, β, γ, δ, and κ in the following manner:

$$R = g^{\alpha}\eta^{\beta}\,{}^{\gamma}v^{\delta}1^{\kappa}$$

or $$MLT^{-2} = (LT^{-2})^{\alpha}(ML^{-1}T^{-1})^{\beta}(ML^{-3})^{\gamma}(LT^{-1})^{\delta}L^{\kappa}$$

whence $$1 = \beta + \gamma$$

$$1 = \alpha - \beta - 3\gamma + \delta + \kappa$$

and $$-2 = -\,2\alpha - \beta - \delta$$

Two of these exponents must be arbitrary and the remaining three may be written as follows:

$$\kappa = 2 + \alpha - \beta$$

$$\gamma = 1 - \beta$$

and $$\delta = 2 - 2\alpha - \beta$$

whence we may write:

$$\frac{R}{\rho v^2 l^2} = \left(\frac{gl}{v^2}\right)^{\alpha}\left(\frac{\eta}{\rho v l}\right)^{\beta}$$

$$= \left(\frac{gl}{v^2}\right)^{\alpha}\left(\frac{\nu}{vl}\right)^{\beta}$$

where $$\nu = \frac{\eta}{\rho}$$

Further we may, in general, write:

$$\psi\left(\frac{R}{\rho v^2 l^2}, \frac{gl}{v^2}, \frac{\nu}{vl}\right) = 0$$

Now in order to predict ship performance from measurements on the model we use the following three conditions:

$$\frac{R_m}{\rho v_m^2 l_m^2} = \frac{R_s}{\rho v_s^2 l_s^2}$$

$$\frac{gl_m}{v_m^2} = \frac{gl_s}{v_m^2}$$

and $$\frac{\nu}{v_m l_m} = \frac{\nu}{v_s l_s}$$

If the same liquid is used for model tests and for ship the physical constants ρ and ν cancel out in the above equations and an inconsistency of requirement results. We either have to agree to use different liquids for the model than the usual water for the ship or resort to some empirical means. The latter course was taken by Froude and we shall examine his method in our chapter on similitudinous models.

We consider that we have now examined the role of dimensional analysis in sufficient detail for the purpose of relating it to our general conception of model and therefore will now go to the detailed treatment of some classes of well-defined models to which we have already referred. The first of these is what we call the iconic model.

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CHAPTER 6

ICONIC MODEL

In our taxonomy of models the iconic model comes first. Of all the types of models with which we deal the iconic and the analogic are indeed the most ancient. In the next chapter we will deal with the analogic model; in the present chapter we wish to treat the iconic. The student of models does well to examine the history of his subject. He thereby learns about its evolution from the beginning and thereby increases his consciousness of the subject and the effectiveness of his use of it. As we noted in our introductory chapters the element of mystery is basic in any approach to modeling. In our view the very purpose of model is to reduce mystery. Someone wishes to know how a structure looks and we show him a small model of it so he can have some understanding. We may say that we thereby decrease the mystery. In a more profound sense we use the model for purposes of prediction of the future or examination of the past. The latter may seem a little odd but we will clarify the point presently.

First we wish to deal with the etymology of icon (in Greek εικων) and then examine briefly its ancient history and usage. In a sense the iconic model is the simplest in our classification, which also includes the analogic, the similitudinous, the Newtonian, the extended Newtonian, and the disclosive. It is clear that while the iconic and the analogic are ancient, the remaining types are really post-Newtonian. In a sense the disclosive model includes all of the previous types, but it will be seen that as it evolves in time it will take on a more definite and precise form of its own. It really is the open ended terminus of our present taxonomy.

One may use any good dictionary to get some idea of the meaning of icon, or ikon, which we use as the root word in our designation iconic model. For convenience we use the Oxford Dictionary and will now quote freely from it [1]. There we find that the icon is an image, a figure, a representation, a portrait, or an illustration in a book. It also may be an image in the solid or a statue. In the Eastern Church it was a representation of some sacred personage, itself regarded as sacred, and honored with a relative worship. In rhetoric it is related to simile. The adjective iconic means of or pertaining to an icon. It may mean of the nature of a portrait and hence is applied to the ancient portrait statues of victorious athletes. There is also a noun iconism which means a representation of some image or figure. This word also means imagery or metaphor. We can even speak of iconize which means to form into an image, to figure, to represent. Iconography means the description of any subject by means of drawings or figures. Finally, it means the branch of knowledge which deals with representative art in general.

There are two fields of knowledge which are rich in the history of icon. These are religion and archaeology. In pursuing the history of the idea of icon we should appreciate the fact that there are synonyms also. We may speak of image, sign or symbol. The first of these is closely associated with biblical and pre-biblical history. An exposition of this aspect of the subject is outside the scope of our present study but one may use many references to examine an important phase of the evolution of the concept. In particular we refer to a dictionary on the bible by John L. McKenzie [2]. In his book, starting on page 382, he has an article on image. For convenience, and because it relates to our small attempt at history, we quote him. He says, "A large number of divine images have been preserved from Egypt, Syria, and Mesopotamia; but outside of Egypt not many of the temple images of the gods have been preserved, and Palestine has revealed no image that can be certainly identified as such." McKenzie continues in an interesting vein but we must resist being distracted further from our main purpose. We leave it to the interested reader to follow the subject as it develops very vivid interpretations of the concept.

We would now like to look elsewhere for additional illustration. We consider that the field of archaeology will serve the purpose and we can do no better than refer to an old friend and colleague who used to teach at Johns Hopkins University. He was Professor W. F. Albright, a most distinguished orientalist and archaeologist. We fondly remember his attachment to and use of science and engineering in his scholarly work. Many times we discussed the use of metallurgy for the proper unrolling of the ancient copper type of Dead Sea Scrolls and carbon 14 for estimating the age of these scrolls. One of the works of Albright to which we wish to refer, for present purposes, is From Stone Age to Christianity [3]. In his introduction he says, "I still insist on the primacy of archaeology in the broad sense, including the interpretation of written documents recovered by archaeologists as well as the excavation and reconstruction of material culture. I continue to maintain, without reservation, that we must approach history as the story of man's total past, with just as rigorous a method as is used by natural scientists, and that within proper limits we must follow the general principles of logical empiricism." We stress here that all archaeologists, including Albright, are using the icon, the image, the symbol for the purpose of unlocking the secrets of the distant past. It may be seen that thereby they are reducing somewhat the mystery of our cultural origins which lay enshrouded in clouds of time. The archaeologists use all of the modern tools available to study the meaning of the ancient lore. We quote Albright further to emphasize this fact. He says in his introduction, "Our knowledge of human pre-history has been greatly enlarged and has become far more accurate in detail than it was in 1940 - 46 because of the great expansion of archaeological exploration and especially because of the extraordinary development of the new radiocarbon technique."

We must leave Albright now and look elsewhere for further substantiation of our premise that the idea of icon or image is firmly implanted in

ancient courses of man's thought. From a theological viewpoint a brief treatment of image is given in an article by Herbert Schade [4]. In his introductory sentence he says, "The image is a figure which is so constructed that it enables something to be really present. Hence the concept of image is not identical with that of a work of art. It is philosophically more comprehensive. In its theological form the concept is very close to that of a sacrament, since the sacrament likewise uses an outward sign to bring about the presence of another reality, grace. In the history of thought, the notion of image has been of paramount importance at one time: it was the point at which human minds diverged." He follows in a very interesting vein but we must leave it to the reader if he cares to pursue the subject further.

The synonymous word symbol has a large and scholarly exposition but we must satisfy ourselves with reference to an article on the subject by the scholar Jörg Splett [5]. He says in his introduction that, "Etymologically, the word 'symbol' comes from certain usages in ancient law. Two parts of a ring, staff or tablet served when they were brought together to identify legitimate guests, messengers and partners. Thus the word came to have the meaning of a 'treaty' and in ecclesiastical language could designate the common profession of faith, the fixed and obligatory formulas or creeds (the 'symbols') and then the instruments, images and acts in which the faith was expressed. Through the study of religion, psychology, art, literature, the theory of science and logic, the concept has been extended and varied so considerably that it can take in the operative signs of logical calculations (as in 'symbolic' logic)." Splett goes further to distinguish symbol and sign but we must leave it to the desire of the reader if he wishes to pursue the matter further.

To obtain a sense of the transition from ancient to modern usage we rely upon a treatment of the subject by Ernst Cassirer in his book <u>Cosmos in Renaissance Philosophy</u> [6]. We can do no better than quote him. On page 54 he says, "The opposite form of interpretation is found in that study of nature that leads from Cusanus through Leonardo to Galileo and Kepler. It is not satisfied with the imagistic and sensible force of the signs in which we read the spiritual structure of the universe; instead, it requires of these signs that they form a system, a thoroughly ordered whole. The <u>sense</u> of nature must be mystically felt, it must be understood as a <u>logical</u> sense. And this requirement can only be fulfilled by means of mathematics. Only mathematics establishes unequivocal and necessary standards against the arbitrariness and uncertainty of opinions. For Leonardo, mathematics became the dividing line between sophistry and science. Whoever blames the supreme certainty of mathematics feeds his mind with confusion. Whoever relies on individual words falls prey to the uncertainty and ambiguity characteristic of the single word, and finds himself entangled in endless logomachies. Only mathematics can give a purpose to these disputes in that it fixes the meanings of words and subjects their connections to definite rules. Instead of a mere aggregate of words, mathematics gives

us a strictly syntactical structure of thoughts and propositions." On page 55 Cassirer says further, "Thus from Cusanus' basic notion of 'indestructible certitude', which is proper to none of the symbols necessary and possible to the mind except the mathematical signs, we move in a continuous historical line and arrive at those famous fundamental and guiding principles by which Galileo defines the aim and the character of his research. And when the revelation of the book of nature is juxtaposed to the revelation of the bible, the process of secularization is completed. There can be no fundamental opposition between them since both represent the same spiritual sense in different forms, i.e., since the unity of the unity of the divine originator of nature is manifested in them. But if a disagreement between them should nevertheless seem to arise for us, it can only be settled in one way: we must prefer the revelation in works to that in words; for the word is something of the past and of tradition, whereas the work, as something at hand and enduring stands before us, immediate and present, ready to be questioned."

We could follow up to recent times with a welter of books such as Symbols, Signals and Noise by J. R. Pierce [7]. For our purpose we refer only to symbol in Pierce's book. It is related to our treatment of icon. However, the exploration of information theory with which he is primarily concerned we leave to later chapters. In his glossary Pierce chooses to define symbol as a letter, digit, or one of a group of agreed marks. He further says that linguists distinguish a symbol, whose association with meaning or objects is arbitrary, from a sign, such as a pictograph of a waterfall.

We would now like to consider examples of the various cases of the use of the icon or iconic model. As far back as the third century A.D., Origen, a Greek philosopher and theologian, resorted on many occasions to the simple model in order to explain relatively complicated matters [8]. On one occasion he said that if something that man was considering turned out to be enormous in size compared to himself he would have to imagine a small scale model in order to visualize it and the relationships of its parts. Such a consideration leads us to the story of Christopher Columbus who used a globe in order to think about the character of his proposed ocean voyage. While this is a very simple appearing example, familiar to all school children, the consciousness of man concerning modeling and hence globing is not so forthcoming. It must be recalled that it required eons of time for man to reduce the problem of the shape and size of his surroundings to maps and globes. In dealing with the problems of man's environment we must always realize that the auditory, gustatory, olfactory, optical, and tactile senses of man are severely limited. It is the sixth sense of model, theory, and experiment that enables him to increase the span of his "sensing". A little thought on the matter leads us to see how true this is. The size and shape factors associated with the optical field, for example, provide problems whose solutions can only be obtained with models. If an entity is too large, man must use a small model to appreciate the relationships of its part.

If it is too small he must use a large model. To visualize the earth he uses a globe and to visualize the atom he uses a Rutherford model.

The story of the voyage of Columbus and his representation of it by means of a globe suggests to us the important story of the cartographer. How ancient is the art of cartography? Where did it have its beginning? What is the history of its development? To answer these questions we refer to the History of Cartography by Leo Bagrow [9]. He says that early maps have been known to us for a much shorter time than many other products of civilization. He further says, "The earliest world map surviving from the ancient world - a Babylonian map of the 6th or 5th century B.C. - is of approximately the same date as the first known references to maps of Greek origin. And for several centuries after this there are no maps, but only literary allusions and fragments of plans. To trace the beginnings of cartography and its subsequent development we must therefore look at the primitive tribes of today, whose cartographic art has stopped at a certain point in its development. Here we may find evidence suggesting by analogy that the historical peoples who preceded the present Mediterranean races passed through the same stages of evolution." The history by Bagrow is both delightful and instructive. We find there for example that Nicolaus Cusanus almost completed an engraving of a map of central Europe in the fifteenth century. Also, Peutinger acquired in the sixteenth century the scroll of an old Roman military road-map. We cannot pursue further the story of cartography but we do wish to assert that it is an enormously important subject which basically uses the iconic model.

A more dynamic use of the map is shown in a series of articles in a book edited by McConnell and Yaseen [10]. From the titles alone the possible use of the map is suggested. They are as follows: A Test of Directional Bias in Residential Mobility, On the Arrangement and Concentration of Points in a Plane, Some Properties of Basic Classes of Spatial-Diffusion Models, On Rural Settlement in Israel, Model Strategy, Action Space Differentials in Cities, and Dido Data: Gigo or Pattern Recognition. These topics clearly demonstrate the use of the iconic model.

An extraordinary use of the iconic model is demonstrated in the field of geography. An excellent book which will serve to prove our point is Models in Geography, edited by Chorley and Haggett [11]. It consists of the Madingley lectures which are really a series of essays on different aspects of the subject by outstanding authorities. A super introductory article is presented by Chorley and Haggett. They provide a general description of models which they introduce by a quotable saying of Kaplan. It reads, "Models are undeniably beautiful and a man may justly be proud to be seen in their company. But they may have their hidden vices. The question is, after all, not only whether they are good to look at but whether we can live happily with them." Their book is certainly not limited to iconic models but it is that portion with which we are interested for our present purpose. The various topics

treated in the Haggett and Chorley Chapter are: Facts, Models and Paradigms, Classificatory Paradigms in Geography, and Towards Model-Based Paradigms in Geography. Other topics in the book which we see as related to iconic model are: Maps as Models by C. Board, Hardware Models in Geography by M. A. Morgan, Models in Geographic Teaching by S. C. Harries, and Network Models in Geography by Peter Haggett. There are also chapters on Models in Geomorphology, Models in Meterology and Climatology, Hydrological Models and Geography. In addition, there are also many other kinds of dynamical models with which we ourselves shall deal in later chapters. Also included in the Chorley and Haggett work are many excellent bibliographies. We consider this treatise a key reference not only for studies in geography but also for anyone who has a serious interest in the general subject of models.

By our definition the iconic model is the simplest. The analogic model is for the purpose of reducing mystery in one area of knowledge by analogic reference to another or predicts performance of a machine or structure by the mathematical analysis of its analogue. The similitudinous model is always used to predict operational characteristics of a prototype from experiments on a scale model. The Newtonian model invariably involves mathematical calculations of a more or less complicated nature in order to predict results. These various types of models we will examine in detail in the next several chapters.

While the iconic model may be considered relatively simple it is still enormously useful and universally used. It is so well known that some of our readers may have concluded that it is trite. Our reply is that we do partly for the sake of completeness but also because, despite the universality of its use, it is not used as much or as effectively as it should be. Also, it is not really understood as well as it should be.

The iconic model has many different forms. It may be a set of plans of a structure, a wiring diagram for an electrical device, or a photographic book of instructions for the assemblage of a machine. It may also be just a map which a stranger uses to familiarize himself with the layout of a city.

A very special and well-defined set of iconic models which are used in Engineering, Industry, and many different professions are as follows:

1. A mock-up

2. An arrangement diagram

3. An instructional aid

4. An industrial aid

5. A preliminary design and

6. An appearance model.

The literature is profuse with articles on these important examples of the iconic model, but we must limit our references to relatively few, taken somewhat at random.

The mock-up type of model is well known in engineering and still serves effectively for certain purposes. To introduce it into our present considerations, we use three different articles. In an article on magnetic mock-ups, R. A. Ibison, a human factors engineer of the Armament and Control Section of the Light Military Electronics Department of the General Electric Company, discusses the mock-up in terms of economical methods for studying control-panel layout [12]. However, he provides a list of advantages of models which we think applies to the whole field of mock-up methodology. With respect to his own field he says:

"Magnetic mock-ups have several advantages, since they:

1. Allow for realism, and ease of revision and adaptation.

2. Give the appearance of complete development.

3. Permit a range of use in conference and convention exhibits.

4. Cost little to mount and to adapt to other applications.

5. Take little time to construct, and

6. Make use of previously prepared materials."

In an article by George W. Michalec, of General Precision Laboratory, we find further specific values of the mock-up [13]. He says, "Compact lightweight designing of intricate electronic and mechanical equipment can be greatly aided by use of appropriate mock-ups. Full scale models, made of wood for ease of fabrication find application as preliminary design aids, as checks on accuracy of detail parts and an excellent means for developing wiring and cable harnesses. In addition, mechanisms having motion with several degrees of freedom, such as antennas and linkage computers, can have their operations and critical clearance checked."

The cost of mock-ups is considered by M. P. Matthew and in an article in Product Engineering he discusses the use of cardboard for the construction of mock-ups [14]. He thinks that such models have an economic superiority to those which are usually constructed with plaster, plastics, and wood.

Another important use of the iconic model is for the purpose of visualizing the layout of large industrial plants, ships, and aircraft. There is considerable discussion of this phase of modeling in an article by A. F. Stedman [15]. A further reference is by H. L. Sebalia, in

which he discusses general industrial plant arrangement [16]. Sebalia outlines the major points of value of such models.

In 1950 an anonymous article on working models was published in Product Engineering [17]. It points out that during WWII transparent models were used by the military services to teach and demonstrate the operation of altimeters and other instruments. Also, plastic models of machinery components as complicated as carburetors were made for the same purpose. A metallurgist from Watertown Arsenal, Arthur M. Ayvazian, provides a special article on training with models [18]. Among other things, he says, "When the Davy Crocket weapon system was approaching the production stage, it was decided to fabricate plastic training aids to instruct users and maintenance personnel in the basic concepts of its operation." The medical schools have long used models for the purpose of teaching students about the parts and the functioning of the human body. Even today we see on TV elaborate models which are used to enable neophytes to detect and familiarize themselves with cases of cardiac malfunction.

The models are also used extensively for solving problems in design and construction. In a paper by De Groat, we find a discussion of three-dimensional models of machines which are used to study layout and production lines [19]. In a similar vein, A. H. Jennings discusses models for use in construction and testing [20]. A design supervisor, Bruce L. Paton, thinks that models can be used as design tools [21]. He says that their use in the early stages of design provide better communication and understanding amongst the concerned technical personnel.

Prototype appearance models are discussed by Koepf and Ferrari [22]. They consider that such models have great value in serving to familiarize engineers, production department personnel, and marketing people with the features of a prototype. They are also often used to gain initial consumer reaction, and provide market pretest data.

It should be obvious to anyone who is at all familiar with the field that there is a huge literature on the iconic model. Our purpose here has been mainly to concretize our thoughts as they relate to the iconic model as it is used in engineering and modern industrial practice. It is hoped that we have provided a useful survey of the iconic model from its earliest days. In terms of the many illustrations which we have used, its basic definition should be clear. We must leave that type of model now and give some consideration to analogy and analogic models.

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CHAPTER 7

ANALOGY AND ANALOGIC PHYSICAL MODELS

At least for the purpose of clarity it is necessary to define and distinguish the concepts of analogy and model. So far we have attempted to make clear our general understanding of the term model. We have reviewed some of its history since the earliest times and we have suggested the dyadic concept of model-theory. In terms of specific examples, such as the Rutherford-Bohr model-theory, we have tried to illustrate our understanding of model and theory. We emphasized that in our opinion the two, model and theory, go together like Siamese twins. The model is not just a scaffolding for the theory. It is essential to our thought processes. Throughout our discussion of model the concept of analogy was ever in the background. It is in the analogy that one detects the similarities between the model and the prototype. A model is operational; an analogy is not. In an analogue computer, with which physicists and engineers are familiar, it is the computer which is really operational, not the analogue or analogy. However, if one did not perceive the analogy, the question of an analogue computation would not arise. As we concluded that the term model enters into the dyadic structure of model-theory so we also opine that analogy is an integral part of the dyadic structure of analogy-induction. There is a fundamental difference between these two dyads, however. The model is a construct which is either actual or conceptual. One constructs a model. One does not speak of the construction of an analogy. One immediately perceives an analogy if it exists. A particular example of analogy can be taken from the field of literature. In this case the analogy leads to the construction of a metaphor or simile just as in physics it leads to the model of the atom. We then are using physical construction as the basis for atomic model and literary construction as the basis for metaphor. An example of the simile is the literary construction which says, "The Assyrians came down like a wolf on the fold." The analogy is obvious. The Assyrians are to their prey as the wolf is to its prey, the fold. The man of letters observes the analogy and then constructs the figure of speech, the simile.

We will make an effort now to enlarge upon these introductory remarks and examine somewhat the evolution of the term analogy by means of the history of ideas. The reader may recall that we previously took some effort to emphasize the fact that Thomas Aquinas profusely employed analogy in the development of his famous theological treatises. Of course his prime interest was in religious matters whereas ours may be in the physical sciences. However, as we have suggested before, there is a history of ideas which shows that the human person has only certain evolutionary tools with which to develop its intellectual life. One of these is surely the concept of analogy. There is a particularly cogent need for it in the comprehension of theological matters. Accordingly, we will consider some of the thoughts of theologians. We do this

particularly because understanding is many times reinforced by the flow of ideas back and forth across the unfortunately necessary boundaries to the many different fields of knowledge.

A useful reference is an article entitled Analogy of Being by Jörg Splett and Lourencino Bruno Paul [1]. It would probably be of value to the reader to review their work, however for convenience and to strengthen our own story we will sketch their thoughts. They begin by saying, "The word analogy is generally understood nowadays to mean the characteristic feature of a term which, when applied to different entities or domains of reality, undergoes an essential change of meaning but without thereby losing the unity of its content. In an analogical term, therefore, the factors of common character and difference, similarity and dissimilarity in the things referred to, combine in the logical unity of a signification. Analogia entis (literally, 'analogy of that which is', though the expression is usually rendered in English as 'analogy of being') means that all that is shares in being but in a different way in each instance. Our knowledge of what is, is therefore, expressed by an is-statement which in each instance is different."

They then go on to tell the story that the Greek origin for analogy literally means "proportion", "correspondence", and in this sense was used by Plato. Following him, Aristotle accepted the possibility and existence of analogical terms which are founded on the similarity of a ratio or analogy of proportion. Then in the thirteenth century influenced by both Plato and Aristotle, Thomas Aquinas developed his doctrine of analogy. There was a great deal of use of that doctrine for theological purposes, without much change, until the Renaissance. In modern philosophy, especially since Kant raised the question in terms of a transcendental philosophy, the problem of analogy entered a new phase. Kant denied any possibility of cognition beyond the domain of phenomena.

Splett and Paul further discuss the brilliant speculative work of Hegel and his attempt to penetrate beyond the plane of analogy. There have been numerous discussions of analogy and Hegelian dialectic. Beyond Kant and Hegel they point to the trenchant analysis of Martin Heidegger. We recall that Heidegger made a most important inquiry into the meaning of Being. He reproached Western Metaphysics for its ignoration of the subject of Being. His concern with the problem of Being and the problem of language have prompted and contributed decisively to a new consideration of analogy and of the whole philosophy of Being. The relatively recent activity in the study of analogy is mainly in the philosophy of linguistic analysis and the philosophy of science. Finally, the conclusion is reached that analogical knowledge is not something derivative but is really a condition of the possibility of univocal knowledge.

In the same reference [1], Leo Scheffczyk gives a treatment of analogy which is peculiarly fitted to the religious sphere. It is called the analogy of faith. He points out that analogie fidei is

taken from the Bible, where it occurs in Romans 12:6, meaning "What is in proportion to faith." The notion is introduced to warn charismatics not to indulge their charism too exuberantly and to avoid heady enthusiasms (such advice might well be given to some scientists and engineers on occasion). The analogy of faith as understood here must be described as religious and existential.

Since the biblical notion involves the element of the normative, it was capable of further development in dogmatic thought, where, departing somewhat from its original meaning, it came to mean the ecclesiastical norm or rule of faith. Jerome translated the Greek term for analogy by mensura and Augustine by regula. The Church Fathers often applied implicitly the analogy of faith to the relationship between the Old and the New Testament. Here the analogy of faith is used as a unifying or integrating principle.

In scholastic theology of an Augustinian type analogy of faith also took on the character of a methodological criterion by which the unity of revelation and natural knowledge, of faith and reason, of the orders of creation and redemption was to be obtained.

So much for explicit reference to the analogy of being and the analogy of faith, which we think has a bearing on our general theme because of the epistemological source of all knowledge. We wish now to suggest some recent literature on these subjects because of their systematic development and scholarly nature.

The first reference is a book with subtitle, An Approach to Understanding Religious Truth, by John E. Smith [2]. His book consists of the Warfield Lectures for 1970 which were presented at Princeton University. His introductory paragraph seems almost prophetic and we wish to quote it. He says, "It is not likely that future historians will ever come to describe the mid-twentieth century as an age of faith. On the contrary, those who attend closely to the facts are well aware of the many forms in which it has been claimed that 'God is dead', that religion is an anachronism, and that man is left alone in the Cosmos. As the result of a series of upheavals - social, political, scientific, cultural - the world of the western nations has seemed to outgrow its past and, almost overnight, to have acquired a paralyzing sense of the inadequacy of many of its hereditary institutions. Traditional patterns of thought and action no longer seem fitting and from every side we encounter either a nihilism that smashes everything before it, or a frantic quest for new gods, new experiences, new bulwarks in the face of the loss not only of God but of man himself." Launched from the position of this almost Apocalyptic prophecy, the scholarly author proceeds with a trenchant analysis of the analogy of experience. We consider that his techniques and methodology fit well within our general scope.

Another systematic monograph entitled Reflections on the Analogy of Being is by J. F. Anderson [3]. It is a modern and concise statement of

the ancient subject in current terminology. The reader can round out his ideas of the relation of our general subject to the analogy of Being. Furthermore, the author provides eight pages of pertinent references from the works of Aristotle up to the fourteenth edition of the Encyclopedia Britannica and beyond.

Another title we recommend is Analogy and Philosophical Language, a very recent monograph, by David Burrell [4]. This work not only treats competently the ancient works, including ancient theology, but provides the language which will appeal to Everyman. He commences with a forthright statement - What Analogy Is and Why. He proceeds through the origins of questions in classical philosophy and treats in detail the contrasting Medieval positions. After carefully examining the subject as developed in that era, he discusses the important relationships of analogy, metaphor, and model. Finally, he concludes with a brief statement about the uses of analogy, including the scientific.

As a transition from the type of analogy which we have been discussing we direct the attention of the reader to Takeda's monograph on Kant and the Problem of Analogy [5]. This is a stunning modern treatment of Kant's views on analogy. It is technical and tough, but our principal reason for drawing attention to it is that the author provides a long chapter on the relation of Kant to Modern Physics. He gives some attention to the analogic models which we wish to treat in some detail. On page 160 Takeda cites the analogy between the equation of Jacobi:

$$\frac{1}{2m}\left[\left(\frac{\partial s}{\partial x}\right)^2 + \left(\frac{\partial s}{\partial y}\right)^2 + \left(\frac{\partial s}{\partial z}\right)^2\right] = E$$

and the equation of geometrical optics:

$$\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 = \frac{\nu^2 n^2(x,y,z)}{c_o^2}$$

Futhermore, he cites the analogy between the Principle of Maupertuis:

$$\delta \int_{t_1}^{t_2} 2T dt = \delta \int_{\rho_1}^{\rho_2} mv ds = 0$$

and the Principle of Fermat:

$$\delta \int_{P_1}^{P_2} \frac{ds}{u} = 0$$

The analogies are closely related to the correspondence between wave motion and particle motion noted by de Broglie. We shall have more to say about such analogies later.

Most of what we have been saying will possibly seem very specialized to the reader so we wish to provide now a concise overview of the subject as it relates to all of man's knowledge. For the purpose we rely on an article on analogy by Professor A. Wolf in the 14th edition of the Encyclopedia Britannica. The material is as widely divergent as biology and theology. Wolf begins with the usual observation that the concept began with the early Greeks. He points out that the term analogy appears to have been used in the sense of proportion, and so was confined to quantitative relationships. The relation between x and y is analogous to that between nx and ny. However, along with quantitative application of analogy there was already a qualitative use. In its qualitative aspect the term indicates similarity in any kind of relationship between two sets of terms.

He pointed out that metaphor is an example of analogy. Also, there is an analogy between wings of butterflies and wings of birds, despite the face that they are structurally very different. They are functionally very similar. He, also, says that the case of metaphors, the case of analogous organs in biology, and the analogous constructions in grammar show that the similarity of relationships between pairs or sets of terms is apt to appear as a similarity between things themselves. And so the term analogy has come to be extended to similarities in general, excepting that very close similarity which exists between members of the same recognized class of objects in respect of those qualities which are regarded as characteristic of that class. Thus, e.g., one would not think of potatoes as being analogous to one another.

Quantitative analogy is a basis of valid inferences. Quantitative analogy is also frequently made the ground of inference. Some thinkers regard analogical reasoning as one of the fundamental types of inference together with deductive and inductive inference. Analogy is accepted as a valid ground when due care is taken. People do draw inferences from analogy. The question is, however, whether such inferences are conclusive. Can reasoning from analogy ever be regarded as more than tentative, as equivalent to proof? The answer simply is no. Independent proofs by induction or otherwise are required. The fact of transmissability of light and sound, which is surely grounds for thinking them analogous, does not establish the property of polarization for both. Analogy may suggest hypotheses for inductive investigations, but it cannot prove anything. Analogy is a fruitful source of suggestions, of hypotheses, that is, of tentative inferences, but it is not a type of proof at all. Analogical inference is usually proved by induction. If not so proved it is merely a suggestion, which may indeed be true, but not yet established.

Of the paramount role of analogy as an auxiliary to inductive investigations there can be no doubt. The history of science affords abundant evidence. Some of these, which are classic, are as follows:

1. Descartes' perception of analogy between algebra and geometry led to his analytic geometry.

2. Galileo's observations of the moons of Jupiter led by analogy to the conception of the solar system.

3. Franklin's study of lightning and his careful enumeration of analogies between it and electricity led to the identification between the two. Of course, in order to prove his tentative inference he had to resort to the famous kite experiments, and

4. Rutherford's analogy between his model and the simple atom led to Bohr's theory of the atom which could then be tested by experiments.

Finally, we may note that a profession which methodically uses analogy is the law. In that field there is a strong tendency to cite cases and follow precedents rather than formulate laws or principles - flexible analogies being regarded as safer than rigid formulas in certain types of legal problems.

In biology the recognition of difference between resemblances of structure and resemblances of function arose gradually and was only firmly established by Richard Owen in 1843. He applied to morphological resemblances the term "homology" and to functional or physiological resemblance, "analogy". In Owen's words, "Homology" as the same organ in different animals under every variety of form and function (e.g., fore limbs of Draco volans and wings of a bird). Analogy, "as a part or organ in one animal which has the same function as another part or organ in a different animal (e.g., parachute of Draco and wings of a bird)." This distinction between homologous and analogous structures is fundamental; it underlies all morphological studies whether they are concerned with comparative anatomy in the old sense, embryology, or the classification of animals. In the light of our view of analogy we think that Owen should have chosen another word which would equally well have made the desired distinction.

We now wish to give some consideration to analogy in the strict context of mathematical reasoning. There are several sources for such a purpose but for various reasons we limit our reference to the work on the subject by that highly competent mathematician, G. Polya. Polya was specifically interested in the nature of the development of mathematics. As a consequence he produced his first book on the subject under the suggestive title How to Solve It [6]. He says that his later two volume treatise on the subject grew out of it [7]. The author also points out that these books are related to that well-known

book on Analysis by himself and G. Szegö. The book How to Solve It, although it is rather verbose, can readily be recommended even to secondary school students who are seriously interested in mathematics. The other volumes are on a more sophisticated level and require greater maturity on the part of the reader in mathematics and philosophy.

While Polya, like ourselves, is obviously no philosopher he does cogently present the essential basis for invention or discovery in mathematics. Some readers may be surprised as to what Polya has to say about conjecture and simple guessing in the development of his subject. We would, however, prefer that he emphasize more what Polanyi chose to consider the intuitive approach. The reader may recall that Polanyi stressed the meaning of insight in the field of invention of machines and curiously stated that this insight or intuition could not be reduced to mathematics, physics, or chemistry. Oddly the same situation arises in the purely mathematical field. Invention or discovery in mathematics is impossible without this pure insight. However, it must be clear that in the field of mathematics as in the field of machinery no invention is possible unless the would-be inventor is steeped in the lore of the subject. As Newton said in his answer to the question as to how he discovered the law of universal gravitation, he was always thinking about it. Also, Louis Pasteur commented on the role of chance in discovery by saying that it was always present but hastened to add that the would-be discoverer must be thoroughly prepared.

What we have just been attempting to emphasize is the role of chance, quessing, and conjecture in the field of discovery and invention. From what we have said so far in the present book it must be obvious that we consider analogy and model to play essential roles. Although Polya does not go so far as to suggest the idea of the dyadic model-theory and the dyadic analogy-induction it is clear from his books that he is thinking along these lines. In the preface to the second volume of his treatise entitled Mathematics and Plausible Reasoning he says that inductive and analogical reasoning are particular cases of plausible reasoning. We think he has inadvertently introduced a possible confusion by the statement. He should have said that analogy is a means of invention and induction is a means of proof. Notwithstanding this, his entire treatment of the subject substantiates our position. Before proceeding to our last use of analogy we would recommend to those who have not done so that they carefully study all three of Polya's very rewarding books.

The final subject we wish to discuss in the present chapter is what we choose to call analogic computation. It is really the basis for the analogue computer. It involves the field of what we presume to call analogic physical models. Before examining several specific cases we wish to make reference to a well-defined literature which is associated with engineering. There are various good textbooks on the subject. One of these by G. Murphy was published in 1950 [8]. That author provides a wide variety of examples so that the interested person

can readily gain a complete understanding of the subject. He says for example that the torsional stresses developed in a shaft may be predicted from measurements made on a distended soap film having the proper boundary and the vibrational characteristics of a mechanical system may be ascertained from observations made on properly designed electrical circuits. It may be observed that the membrane analogy was discovered by Prandtl as far back as 1903. Murphy stresses that from measured data on one physical system corresponding data may be deduced for the analogue. We may, also, stress that for purely mathematical treatments it may turn out that a solution for a given physical problem may not be as readily obtained as for its analogue. For example, the equations for the torsion of a shaft with rectangular cross-section can be more readily solved by considering the rectangular soap film analogue. We wish to emphasize that the sole consideration that relates a given problem to its analogue, in the sense we are now contemplating, is that the defining mathematical equations are identical. In fact, in Murphy's treatment which begins with Chapter 14, Principles of Analogies, he has a series of chapters headed Analogies from Second-Order Ordinary Differential Equations, Analogies from Second-Order, Partial Differential Equations, Analogies from Fourth-Order Differential Equations, and Analogies from Four-Term Algebraic Equations.

The literature of the subject of analogy of the type which we have just been discussing is very extensive. Murphy, along with two other authors, has published a book on engineering analogies which contains approximately 2000 references [9]. Another useful reference which came out in 1959 is entitled Analog Methods by Karplus and Soroka [10]. An interesting feature of their book is a diagram on automatic computers which shows in branch form the analog and digital types. The computers are discussed at great length and a large segment of the literature on analogies and computational machines is cited. A systematic review of electroanalogic methods with extensive references is provided in a series of review articles by T. J. Higgins [11]. Finally, we would like to cite a more recent book entitled Field Analysis which was edited by D. Vitkovitch [12]. The various authors provide chapters on calculation techniques and many electro-mechanical problems. Vitkovitch himself has an extensive chapter on the electrolytic tank analogue.

Before ending this restricted, although very important phase of the general subject of analogy, we would like to review some features of a few specific problems associated with engineering analogies. A most instructive illustration of the physical analogy is the simple oscillator.

The equation of the simple oscillator, a one-degree-of-freedom system may be written as follows:

$$\frac{d^2\Theta}{dt^2} + p^2\Theta = 0$$

where Θ = oscillatory function

p = frequency, and

t = time.

This is a second-order ordinary homogeneous differential equation which represents free oscillations. It turns out that many analogous physical systems of radically different nature are represented by the equation. One may, and should, put the equation in dimensionless form. As we have remarked on other occasions the mathematical equations are intelligible only in fields of pure numbers, which are generally complex. It is never difficult to reduce physically representable equations to such dimensionless forms.

It may be instructive to review some radically different physical systems which are represented by the equation of the simple oscillator. Five different physical systems of the required type may be listed as follows:

1. Mass on a linear spring.

2. Simple pendulum (small motion).

3. Compound pendulum (small motion).

4. Water in a vertical U-tube.

5. Basic inductance-capacitance ac circuit.

It is obvious that case no. 1 and case no. 5 are radically different in physical make-up yet their oscillatory natures are identically represented by the same differential equation. For this reason measurements on no. 5 can predict corresponding measurements on no. 1 and vice versa.

Another classical case is the analogy of the plane strain in a prismatic bar and the transverse deflection of a flat plate of the same shape as the cross-section of the bar. The plate is assumed to be loaded only along its boundary. It may be recalled that the differential equation is:

$$\nabla^4\phi = 0$$

where ∇^4 is the biharmonic operator which in rectangular cartesian coordinates is:

$$\frac{\partial^4}{\partial x^4} + 2\,\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}$$

In concluding the present chapter we wish to emphasize the fact that the last treated topic, on analogies, which is eminently useful in obtaining solutions to certain physical problems is severely limited with respect to the domain of the general analogical principle. We have attempted to use the concept of analogy to view the entire intellectual world, including theology as well as mathematical physics. It is our position that the analogic knowledge gained from all the different fields tends to reinforce each other. Accordingly, it is our purpose to stress this thesis on every occasion. A closely related subject to the analogical analysis of mathematical physics as applied in engineering is the principle of similitude which we now wish to describe.

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CHAPTER 8

SIMILITUDINOUS MODELS

In our chapter on dimensional analysis, we associated that subject with a type of model which we call similitudinous. It may be recalled that we were led to that designation by Newton's principle of similitude which is really based on the ideas of dimensional analysis and scaled model. It is the purpose of the present chapter to more fully explore the definition and to provide some particular examples of it.

Peculiarly, there seems to be no definite term in the literature to designate such models. The reason for the situation seems to be that the subject simply grew somewhat at random. It was, however, related to a very definite set of ideas and it never seemed to occur to anyone that ambiguities could arise with respect to the concept of model. Now, however, since the field of models in general has become so diversified and is still growing it appears necessary to coin a word to designate each distinct branch of modeling. In our present book we are attempting to provide a suitable taxonomy for the entire subject and hence have a need for naming each class.

Since 1950 a fairly large number of books on dimensional analysis and modeling have been published and we will now refer to several of the more significant ones to show why there is a need for a distinctive word to designate the class of models which we have been calling similitudinous. We will take the references in chronological order.

In 1950 Glenn Murphy published a very useful book for engineers called Similitude in Engineering [1]. It deals with two different kinds of models according to our classification. One is what we call the similitudinous model and the other is what we call the analogic model. It may be interesting to provide the classification which he gives in his first chapter. It is as follows:

"(1) Geometrically similar. The model is a scale reproduction of the prototype.

(2) Distorted. The model is a reproduction of the prototype, but two or more scales are used, i.e., one scale may be used for length and breadth and a different scale for depth or height.

(3) Dissimilar. There is no direct resemblance between model and prototype. For example, the characteristics of a vibrating mechanical system may be predicted from observations made on an electrical circuit."

It is obvious that the last type, which he calls dissimilar model, is really our analogic model. The first two types are examples of what we call similitudinous model. Murphy uses scaled models and analyzes these in terms of dimensional analysis.

G. Birkhoff in his little book on hydrodynamics very incisively considers the case treated by Murphy under his second item, distorted model [2]. It is here that one gets a very provocative intoduction to affine and other types of transformations as related to the subject of models. Birkhoff has a single chapter on modeling and dimensional analysis. He begins by observing that although engineers have adequate laboratories, they are usually preoccupied by special practical problems concerning one type model whereas the academic scientist, preoccupied by the difficulties of logical exposition, too often ignores experimental reality. In a very optimistic manner he claims that he will narrow this gap, but what he really does is relate dimensional and inspectional analyses to model theory, provide some perceptive observations on transformation theory and then winds up with an exposition of some standard model problems which are well-known in the literature. In our opinion what Birkhoff is really doing is simply treating the similitudinous model.

W. J. Duncan, in a small book, discusses what he prefers to call similar physical system, geometric similarity, kinematic similarity, and dimensional analysis [3]. His key words are similarity and similar. In our opinion these usages substitute for the concepts of scale model and principle of similitude. Hence that author is also dealing with similitudinous models. He winds up his book with examples of models like those analyzed by Admiral D. W. Taylor in his book entitled _Speed and Power of Ships_.

H. L. Langhaar, likewise provides a book on standard dimensional analysis with applications to engineering model tests such as those for ship propulsion [4]. He also discusses similarity and model testing without seeming to see the need for a definite classification of his models.

L. I. Sedov who has published a truly excellent treatise uses the words similarity and dimensional methods in his title [5]. His Chapter II is entitled _Similarity, Modeling and Various Examples of the Application of Dimensional analysis_. Again we find emphasis on the words similar and similarity.

Finally, in 1973 Baker, Westine, and Dodge published a book which they called _Similarity Methods in Engineering Dynamics_ [6]. The significance of this book is that it has a large number of good examples of similitudinous models taken from many areas of engineering and industry.

What we have been trying to say with this brief review of the literature is that the authors are essentially applying dimensional analysis, the principle of similitude of Newton, and scaling procedures with regard to physical models and their corresponding prototypes. On the basis of this fact we consider that our proposed term similitudinous model serves to designate that which all of these authors are so assiduosly treating.

For similitudinous modeling it is assumed that there is a prototype and its small scale models of a physical nature. For example, we may have a steamship and its small model which is geometrically similar. We stress the fact that geometrical similarity usually obtains, however before we leave the subject we will show that complete gemetrical similarity is not always required. Fundamentally dimensional analysis is the key to relating the model to the prototype. It is assumed that experiments will be performed on the physical model and that from data so obtained the performance of the prototype can be predicted. It is immediately clear to anyone who attempts to do this that there must be some definite method whereby a proper correspondence can be obtained between model and prototype.

In our chapter on dimensional analysis it may be recalled that we reviewed the ship-model problem. We listed all of the physical variables that we considered part of the analysis and also recognized that the fundamental quantities for such a mechanical problem are mass, length, and time. From the Buckingham-Vaschy Pi theorem we saw that there are three dimensionless coefficents. These are as follows:

$$\frac{R}{\rho v^2 l^2} = \text{Specific resistance number}$$

$$\frac{gl}{v^2} = \text{Froude number}$$

and

$$\frac{\nu}{vl} = \text{Reynolds number.}$$

Because of the invariancy of the functional form relating these three variables it was observed that each should be equal for model and prototype. Now if we use the same liquid for floating model and ship the coefficent of visocosity will be the same for both. Hence the coefficient will cancel and we have a condition on the Reynolds number as follows:

$$\frac{1}{v_m l_m} = \frac{1}{v_s l_s}$$

or
$$v_s = v_m \frac{lm}{ls}$$

where the subscript m stands for model and subscript s stands for ship.

Furthermore since g, the gravitational coefficient, is the same for both we have a condition on the Froude number as follows:

$$\frac{lm}{v_m^2} = \frac{ls}{v_s^2}$$

or
$$v_s = \sqrt{\frac{ls}{lm}}\, v_m$$

The two conditions on the velocity are obviously incompatible and to avoid this we must use two different liquids. Such a requirement is highly impracticable. As a consequence, William Froude, after whom the Froude towing tank in England is named, suggested his well-known methods for analyzing ship models and predicting performance of the prototype.

The essential feature of the method is to separate the total towing resistance, or drag, into two parts, the skin friction resistance and the wave making resistance. It turns out that the skin friction can be calculated reasonably well for both model and ship, based on frictional plane experimental data. From the total towing resistance on the model the calculated skin resistance is subtracted. Then the residual portion, presumed to be caused by the waves which are generated, is projected to the value for the ship at corresponding speeds. The latter speeds are determined from the equality of Froude number for model and ship which gives, as we have seen:

$$v_s = \sqrt{\frac{ls}{lm}}\, v_m$$

with the model towed as a given speed v_m, the corresponding speed of the ship is v_s. Then the residual or wave resistance, obtained as we have just explained, is projected to the corresponsing value for the ship by means of the Pi number for specific resistance as follows:

$$R_s = R_m \frac{\rho v_s^2 l_s^2}{\rho v_m^2 l_m^2}$$

and using the Froude condition for corresponding speeds we obtain corresponding wave resistances as follows:

$$R_s = R_m \left(\frac{l_s}{l_m}\right)^3$$

which shows that the wave making resistance varies as the cube of linear dimensions. Finally, the total ship resistance at the corresponding speed is this value of R_s plus the calculated frictional or skin resistance at the same speed. It is then clear that if we have obtained a complete curve of total model resistance as a function of speed in a towing tank we can then construct the curve of total resistance of ship as a function of speed by performing the calculation for each point on the curve for the model as we did for a single value of v_m.

While the procedure outlined above may seem quite straightforward, we must hasten to insist that, as for all experiments with physical problems, there are a whole host of technical details that must be attended to before success is attained. In order to emphasize this fact, the senior author would like to refer to an experience in the early days when he was beginning his scientific engineering career. He had the good fortune to be working withCaptain Harold E. Saunders of the United States Navy at a time when that Construction Officer was making an extensive study of three geometrically similar ship models. In connection with his research Captain Saunders took the occasion to explain the entire methodology used in model analysis at the U. S. Experimental Model Basin, then at the Navy Yard in Washington, D. C. The story was effectively presented in a long article which was published in the Journal of the Society of Naval Architects and Marine Engineers [7]. He not only discussed the standard methods of analysis used at the Experimental Basin but also reviewed critically the Froude technique. In addition to a study of the performance of ship hulls he included results of so-called self-propelled experiments. A large portion of the report contained the results of an investigation of geometrically similar propellers. Saunders extensively reviewed the role of frictional resistance, usually called skin friction, and critically commented on its determination by means of friction plane experiments. He also thoroughly explained his experiments on three geometrically similar models and what he hoped to discover with their aid. It must be obvious to the reader that every experiment introduces numerous questions that do not arise in the purely theoretical analysis by means of dimensional analysis. For example, the ratio of size of model to size of towing tank must be considered. Reflection of waves from the sides of the basin are an important source of possible error. Errors can also arise from measuring equipment and carelessness on the part of personnel. Of course these latter considerations arise in connection with any physical experimentation, but they have their own peculiar aspects in work with similitudinous models.

Earlier we mentioned that the similitudinous model is used sometimes for model and prototype which are not geometrically similar in all respects. The reason for this is that there are cases in which dynamical similarity is sufficiently attained for all practical purposes even though the model and prototype may be geometrically dissimilar in some respects. Dissimilarity in the case of the simple pendulum is quite obvious. The only requirements are that the lengths of suspension strings be kept in constant ratio and the masses of the bobs be known. The exact size and shape of the latter are not really important in the determination of the period of a simple pendulm. If one conducts experiments on such a pendulum he very quickly observes that such is the case. In order to examine the case of the geometrically dissimilar model and prototype we will now consider in some detail a more complicated problem. For the purpose we may examine the problem of the deflection of a long cantilever beam subjected to a concentrated load applied statically to its free end.

For the physical situation which we are considering it is assumed that deflections are relatively small and caused solely by bending moments. The latter assumption implies that any shear induced deflections are small enough to be ignored. Applying dimensional analysis, the list of pertinent physical variables are as follows:

P = load at free end of beam

l = length of beam

E = Young's modulus of elasticity

I = moment of inertia of cross-section about centroidal axis

d = deflection of beam.

The fundamental quantities for such a mechanical system are mass, length, and time. According to the Buckingham-Vaschy Pi theorem there should be five minus three or 2 dimensionless numbers. At this stage one may question whether inertia effects are important in the determination of deflection. Should force be written as a fundamental quantity in place of mass and time? In order to conduct the analysis and see where it leads let us first assume that both mass and time should be considered. Then we may write:

$$d = P^{\alpha} l^{\beta} E^{\gamma} I^{\delta}$$

and
$$[L] = [MLT^{-2}]^{\alpha}[L]^{\beta}[MLT^{-2}L^{-2}]^{\gamma}[L^{4}]^{\delta}$$

so that
$$1 = \alpha + \beta - \gamma + 4\delta$$

$$0 = \alpha + \gamma$$

and $$0 = -2\alpha - 2\gamma$$

From the last two equations we have:

$$\alpha = -\gamma$$

and $$\beta = 1 - 2\alpha - 4\delta$$

so $$d = P^{\alpha} l^{1-2\alpha-4\delta} E^{-\alpha} I^{\delta}$$

$$= (P\ell^{-2}E^{-1})^{\alpha} (\frac{I}{\ell^4})^{\delta} \ell$$

The general deformational equation for model and prototype is then:

$$\phi(\frac{d}{l}, \frac{P}{El^2}, \frac{I}{l^4}) = 0$$

We see that there are apparently three dimensionless Pi numbers. However if the model and prototype are geometrically similar, fixing the scale ratio automatically satifies all geometrical conditions such as I, a geometrical quantity, varying as the fourth power of length. In order to determine the deflection at the end of the prototype from the measured deflection at the end of the model we have:

$$\frac{d_m}{l_m} = \frac{d_p}{l_p}$$

where subscript m stands for model and subscript p stands for prototype.

Also, $$\frac{P_m}{E_m l_m^2} = \frac{P_p}{E_p l_p^2}$$

This condition relates the load on model and load on prototype. Now if the two systems are geometrically similar it follows that:

$$\frac{I_m}{l_m^4} = \frac{I_p}{l_p^4}$$

Now suppose we wish to solve the problem if the cross-section of the model is not geometrically similar to that of the prototype. It turns out that we can readily accomplish this. Instead of a separate modulus E and moment of inertia I we have reason to suspect that the deflection is controlled solely by a stiffness factor which is the product of E and I. Let S be the stiffness factor and so:

$$S = EI$$

Now the list of physical variables may be written

P = load at free end of beam

1 = length of beam

S = stiffness of beam

and d = deflection of beam

Let the fundamental quantities for such a mechanical system be F for force and 1 for length. Then according to the Buckingham-Vaschy Pi theorem there are four minus two or 2 dimensionless Pi numbers. To find these we have:

$$d = S^{\alpha} 1^{\beta} P^{\gamma}$$

and $$[L] = [FL^2]^{\alpha}[L]^{\beta}[F]^{\gamma}$$

so $$1 = 2\alpha + \beta$$

and $$0 = \alpha + \gamma$$

whence $$\gamma = - \alpha$$

and $$\beta = 1 - 2\alpha$$

so $$d = S^{\alpha} 1^{1-2\alpha} P^{-\alpha}$$

$$= \left(\frac{S}{P1^2}\right)^{\alpha} 1$$

and the deflection equation for model and prototype is:

$$\phi\left(\frac{d}{1}, \frac{S}{P1^2}\right) = 0$$

To compare model and prototype we simply write:

$$\frac{dm}{l_m} = \frac{dp}{l_p}$$

and

$$\frac{Sm}{P_m l_m^2} = \frac{Sp}{P_p l_p^2}$$

The predicted deflection on the prototype will be given in terms of the measured deflection on the model in the first equation for known l_m and l_p. Then corresponding loads are according to the second equation for given stiffness factors S_m and S_p. These results all conform to the well-known theory of small deflection of a cantilever given by the differential equation:

$$EI\frac{d^2w}{dx^2} = P(\ell-x)$$

and the boundary conditions:

$$w = 0$$

$$\frac{dw}{dx} = 0 \text{ at the fixed end.}$$

where w = deflection

and x = distance from fixed end to any point.

We are using the so-called bending moment form of the beam equation, which is of the second order, and consequently has two boundary conditions. The solution of the differential equation subject to the boundary condition shows that the deflection at the free end is given by:

$$d = \frac{Pl^3}{3EI}$$

and this result is in conformity with the dimensional analysis results above as it should be.

A complete investigation of the structural deflection problem by means of dimensional analysis is given in the textbook by G. Murphy [1] The reader will find a great deal more information there, but our present purpose was simply to emphasize the fact that the theory of similitudinous model applies equally well to geometrically similar systems and to certain geometrically dissimilar systems.

We hope that we have made it clear what we consider the similitudinous model to be. Summarizing, one may say that a scale model is tested in order to predict the performance of its prototype. Dimensional analysis is the essential means for relating model and prototype. The two may be geometrically similar or under certain restrictions the two may have certain geometrical dissimilarities. The entire theory is predicated on Newton's principle of similitude. Before concluding we would like to emphasize the signal importance of such models. They permit the development of preliminary design knowledge with relatively low cost. They suggest valuable design modifications in the early stages of a program of development. Experiments can be quickly conducted and many important measurements can be made with relative ease. The authors are familiar with the great effectiveness with which towing tanks and wind tunnels have been used, but they also know of many cases in which the powerful similitudinous model was not used when it should have been.

At this point in the development of our taxonomy of models we now have specified three types: the iconic, the analogic, and the similitudinous. The next important type we wish to consider we define as the the Newtonian model. In the next three chapters we wish to define and thoroughly discuss this very useful type.

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CHAPTER 9

NEWTONIAN MODELS

The models discussed in the immediately preceding two chapters are well defined and generally well established in science and technology. In order to further generalize those concepts and increase the effective usage of the term model, it is proposed to associate the notion of modeling explicitly with that body of knowledge known as Newtonian dynamics. A recent treatment of the subject by J. L. Synge clearly delineates its mathematical and physical aspects [1]. It is generally agreed that Newtonian dynamics describes non-relativistic mechanical phenomena excellently. The contrast with relativistic mechanics is clearly portrayed in another work by Synge [2].

In accord with the spirit of the present work, it is proposed to use the idea of model rather freely and talk of mathematical model, physical model, and their interaction, as related to the phenomena of nature. In order to clearly understand the meaning of such a position it is proposed to briefly sketch the history of Newtonian mechanics. Statics will be entirely omitted, but the treatment will be extended to include the dynamics of deformable media which will be extensively illustrated with examples in the next two chapters.

Brief History of Dynamics

Although the gradually evolving history of the subject of mechanics goes back at least to Aristotle, who introduced the very term mechanics and who struggled with the difficulties associated with the concept of inertia, the period of the greatest importance for our purpose covers the sixteenth through the nineteenth centuries [3]. That is the period during which Kepler (1571 - 1630) introduced the now famous three laws of planetary motion in his works <u>Astronomica Nova</u> and <u>Harmonices Mundi</u>; Galileo (1564 - 1642) clarified the notion of inertia of physical bodies and described his dynamical experiments in his Dialogues [4]; and Newton (1642 - 1727) provided the basic three laws of motion and the law of universal gravitation in a systematic fashion in his <u>Principia</u> [5]. The celebrated general formulation and extension of Newtonian dynamics, with complete analytical development using differential equations was introduced by L. Euler (1707 - 1783) in his <u>Mechanica sive Motus Scientia</u> and further developed by J. L. Lagrange (1736 - 1813) in his justly famous book <u>Mécanique Analytique</u>.

It is always well to remember that our basic knowledge of the physical world comes from experimental observation. The present historical note shows no exception. Using experimental data taken by Tycho Brahe (1546 - 1601) during very tedious observations on the motions of the planets, Kepler was able to formulate his three laws

concerning those motions. Assuming the actual sun-planet system to be the prototype, Kepler devised the kinematical model. Following after him, Galileo was able to demonstrate clearly certain kinetical features of the motion of falling bodies near the surface of the earth. As a consequence of his experiments, he was able to shed light on the nature of inertia of bodies and its relation to the acceleration of such bodies moving in physical space under the influence of gravity.

While Kepler and Galileo clearly laid the groundwork for the magnificent development of the subject in the future, it remained for Newton to set down axiomatically his three laws of motion of particles and the law of universal gravitation.

The essential features of the first two of these laws were known by Galileo, however the third law and the axiomatic statement of all three laws as a basis for dynamics belong to Newton. Also, he extensively applied the laws by means of geometrical methods to mechanical and astronomical problems. The more general and powerful analytical methods were subsequently developed by Euler, Lagrange, and others.

For convenience of reference and to relate to subsequent discussion of the subject, the laws are presented according to Brouwer and Clemence [6].

Newton's Laws of Motion and Law of Gravitation

The three laws may be stated:

I. Every body continues in its state of rest, or of uniform motion in a straight line, unless compelled by an impressed force to change that state.

II. The rate of change of momentum is proportional to the impressed force, and takes place in the direction in which the force acts.

III. To every action corresponds an equal and opposite reaction.

The Newtonian law of gravitational attraction is: Every two particles in the universe attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

The law of gravitation is stated for particles of matter only, and does not apply to bodies of finite dimensions. It can be shown, however, that bodies with spherical symmetry attract each other as if their masses were concentrated in their respective centers. For bodies whose distribution of mass differs from spherical symmetry, if distances between the bodies are large compared with their dimensions, the mutual attraction approaches that which would apply if their masses were concentrated in their respective centers of mass.

The second law of motion can be written as an important set of differential equations. If a force with components (F_x, F_y, F_z) expressed in a 3-dimensional Cartesian coordinate system acts on a particle of mass m, the equations of the path of the particle may be written as follows:

$$\frac{d}{dt}(m\frac{dx}{dt}) = F_x$$

$$\frac{d}{dt}(m\frac{dy}{dt}) = F_y$$

and

$$\frac{d}{dt}(m\frac{dz}{dt}) = F_z$$

where t is the time and (x,y,z) are Cartesian coordinates which locate the particle with respect to a frame of reference at that time. It is obvious that these three equations may be represented vectorially as a single equation as follows:

$$\frac{d}{dt}(m\overline{V}) = \overline{F}$$

where $\overline{V}$ is the velocity and $\overline{F}$ is the force expressed as vectors.

Using the vector equation and polar coordinates, the motion of a planet around the sun can be fully determined and the three planetary laws of Kepler analytically verified. The analysis nowadays is an exercise for beginning students in physics and engineering as can be seen in any good textbook on dynamics such as that by Synge and Griffith [7].

While Newtonian dynamics remains the standard basis for computations in celestial mechanics, the equations of motion as generalized by Lagrange can be applied to such subjects as electrical circut theory. Moreover, the development of the subject by Euler, resulting in the so-called Eulerian equations of motion, permits applications of the basic theory to analysis of motion of rigid bodies such as the gyroscope.

In order to extend the original Newtonian theory to cover the case of continous media, it is only necessary to observe that the terms on the left in the three differential equations of motion, given above, represent the inertial force while the corresponding components on the right represent the resultant external force that acts on the particle. Such is the case whether the particle is an isolated mass point or an elemental volume of a continous medium. It is for this reason that the model may be called Newtonian, whether the system is one of isolated particles or of continuously distributed matter. It can be argued that although scientists like Euler, Lagrange, and Cauchy extensively developed

the Newtonian theory and applied it definitively in the study of the motion of deformable media, nevertheless it was Newton who provided the initial genius and effort to the ever-developing subject.

The differential equations of the deformation of continua subjected to forces were first developed by Navier (1785 - 1836) and Cauchy (1789 - 1857). The first important treatise on the subject, including dynamical problems, was introduced by G. Lame (1795 - 1870) [8]. Since his time the dynamical theory has been extensively developed and applied until today it might be considered one of the best models of scientific study of physical phenomena.

An extensive list of literature on continuous media is given in a study of classical field theories by Truesdell and Toupin [9]. The period of bibliography appended to their work covers the period from 1678 to 1960.

A very good single reference on the subject of the motion of particles and rigid bodies is the treatise by Whittaker [10]. A treatise on the deformation of solids which is still very useful is the classic by Love [11]. A similarly high level work on fluid mechanics is the book by Lamb [12]. More modern mathematical treatment of these subjects is the book on dynamics by Goldstein [13], that on elasticity by Green and Zerna [14], and finally a presentation of hydrodynamics by means of vectors by Milne-Thomson [15].

The theory of dynamical deformation of continous media which we wish to classify under Newtonian models is developed in 3-dimensional Euclidean space in the references just cited. Inasmuch as the examples which will be presented in the next chapter include beams, plates, and thin shells, it may be useful to complete the present brief history by giving a sketch of the development of the dynamical theory of such special but important cases for structural elements.

The beam equation was developed by Euler before the 3-dimensional theory of deformable solids was introduced. He had to rely upon an assumption of the proportionality of the bending moment applied at any point of a beam to the change of curvature which is caused by the loading. Very interestingly, the differential equation for the transverse displacement of the neutral axis was developed by means of the calculus of variations, as suggested by Daniel Bernoulli, a representative of a famous family of scientists. Bernoulli himself was the first to derive the differential equation for the lateral vibration of beams and he used it to study the modes of motion. In fact, after Euler integrated the equation, Bernoulli performed a series of experiments with physical models and observed the normal modes of vibration and their corresponding frequencies. As a consequence of these developments, the lateral

vibration of prismatic bars and beams is still studied by means of what is called the Bernoulli-Euler theory.

It was natural that such approximate stress resultant theories should be extended to the 2-dimensional case of thin plates and it turned out that another member of the famous family, Jacques Bernoulli (1759 - 1789), obtained a crude approximation to the plate deformation equation. His findings were based on Euler's study of the deflection of elastic surfaces.

Another example of physical experimentation leading the way in the development of a science is the study of E. F. F. Chladni reported in his 1802 book entitled Die Akustik. In a famous set of experiments, Chladni by covering a plate with fine sand and exciting vibrations was able to show the existence of nodal lines for various modes of its motion and to determine the corresponding frequencies. Similar experiments will be discussed in the next chapter for plates with stiffeners attached. After observing Chladni's experiments on the vibrating plates, Napoleon suggested to the French Academy that a prize be awarded to anyone who might develop a satisfactory mathematical theory of plate vibrations. As a consequence, Sophie Germain made several tries to develop the required equation and never fully succeeded but finally did do sufficient work to win the prize. One of her judges, Lagrange, subsequently obtained a very good approximation based on Sophie Germain's work [16]. In 1820 Navier discovered the correct 2-dimensional equation for the plate subjected to lateral loading. Its use in conjunction with the inertia force term used by Lagrange in his equation provided the equation for the study of the lateral vibration of plates used until the present time.

One further extension of the study of the deformation of structural elements, which is needed for the next chapter of the present book, led to the equations for the extensional and then the flexural vibration of thin shells. Lord Rayleigh (1842 - 1919) may rightly be called the first investigator to seriously study the theory of vibration of shells. He clearly observed that two kinds of vibrations characterize the motion of vibrating shells, the extensional and the flexural. Notwithstanding his discoveries in this field, however, it remained for A. E. H. Love (1863 - 1940) to substantially develop the subject of the deformation of shells. In fact, Love's original work in the theory of elasticity began with a study of thin shells. Both for a summation of our knowledge of the mechanics of continua, including thin elastia, plates, and shells, no single reference is available even today, which exceeds in importance Love's important treatise on the subject [11].

The Newtonian model which serves as a sub-structure for the evolution of the highly developed mechanics of solids also serves a similar role for the study of fluids. No single person can serve more effectively for our pursuit of knowledge of the flow of fluids than G. G. Stokes (1819 - 1903) who initiated the golden era of mathematical physics at Cambridge University. His name is associated with that of Navier in connection with the equations of flow of viscous fluids. His work is very important to the study of Newtonian model experimentations with fluids in Chapters 11 of the present book. A very important observation of Stokes on the relation of the equations of motion of solids and of fluids is cited by Timoshenko [16]. Stokes says, "There seems no line of demarcation between a solid and a viscous fluid." That remark relates incisively to the subject of rheology which was recognized as a distinct discipline in the twentieth century by E. C. Bingham.

Logic and Rheology

It may be categorically stated that every science has its logic. Of course it is not implied that such a statement is as clear and simple as it sounds. However, since the fact is important to the subject of modeling as a science it is recognized at this point, although the full development of such a thesis is far beyond the scope and intent of the present book. One should always be on guard against confusion of the role of discovery and the role of logicizing. It may be burden of man that discoveries must be made before they can be subjected to the light of logic. An excellent example of a subject that illustrates the everlasting problem of the apparent dichotomy is mathematics. One may obtain a quick insight into the nature of the problem by reading a book such as the one by E. T. Bell [17]. While it is highly anecdotal and many times downright trivial it does provide a rapid overview of the development of the relationship of logic to mathematics. It will at least be apparent that birth pains will arise with which one must live.

For the present time the subject of mechanics in general and dynamics in particular are being systematized and presented axiomatically. Such a development has great value, but from a didactic viewpoint can be very destructive. A rather recent example of the point is clearly demonstrated in an excellent paper by Truesdell and Toupin [18]. There is no doubt that the development of a science follows at least two paths, the subjective-intuitive and the technically logical. Chronologically one must precede the other and it is no mystery which has precedence. It should be admitted that the problem has its annoying features and that the history of a subject rarely shows the logical development one prefers from a position of hindsight. Leaving the question of axiomatic development, one may turn to the question of

terminology which also is influenced by historical development. For the present purpose, a single term is of great interest. It is the name for a science which really includes all of the previously developed sub-subjects such as elasticity and hydrodynamics. The need for such will readily appear to anyone with a little study. The name finally introduced by E. C. Bingham in the twentieth century was Rheology. As can readily be seen from its Greek derivation it is the study of flow and deformation of matter. The practical considerations that show the logical development are clear enough. Material things are the determining factors as usual. Since movement in water and bending of iron beams were problems of such importance to man for so long it was only natural that such a fact conditioned the development of the sciences associated with them, hydrodynamics and strength of materials. By the twentieth century, the demand for knowledge of flows of material such as beer, paint, and even biological fluids became paramount.

Now our present interest is not so much in having an all-inclusive term for the subject of the deformation and flow of such diverse materials but rather the key point is the understanding of that term. For the purpose of clarifying the position, return to the three equations of motion of Newton. Our position has been that those equations are central to the notion of Newtonian model as we define it. Also, an examination of them may shed some light on the real meaning of rheology. It may again be recalled that each equation has a term on the left which represents a component of inertia force and the term on the right which represents a component of the resultant force acting on the particle. As one may recall, the particle was generalized to include an elemental volume of the continua. Now in accord with Newton's third law, that infinitesimal mass of matter exerts a force of resistance to deformation. We submit that this is the key idea in rheology and emphasizes the fact that the subject is fundamentally concerned with the various causes and nature of that resistance. It is not possible to proceed further at the present time with discussion of such an enormous subject. However, for those interested, who are not already familar with the subject, reference may be made to the three volume work edited by Eirich [19] and also, a lengthy article by M. Reiner [20].

The law of the internal resistance to deformation for any given material may be expressed by what are nowadays called constitutive equations. These equations are central to the subject of rheology. The earliest development of such relations was made by Robert Hooke (1635 - 1703) who worked with mechanical springs, as a suitable physical model, and devised a simple law which essentially meant that stress is proportional to strain. Such a modest beginning led to the development of constitutive equations for an elastic body in three dimensions. These studies then led finally to the extensive work on anisotropic elastic materials by W. Voigt (1850 - 1919). His study of anisotropic constitutive equations is fully presented in his great book, "Lehrbuch der Krystall Physik" which is still an important reference for those

studying crystal physics. The much more general laws of internal resistance to deformation or flow are contained in a vast literature of which the works by Eirich and Reiner, already mentioned, are two instances.

It is considered that this brief history of dynamics of particles, rigid bodies, and deformable media will serve as a basis for understanding our meaning of the expression Newtonian model and its use in the next tow chapters.

Definition of Newtonian Model

A precise definition is a desideratum that is rarely attained. It is not expected in the present case. However, as a general classification it may be possible to make it unambiguous and useful. It will certainly be possible to determine that some models are not Newtonian in accord with our attempted definition. For example, those cases which in no way reasonably relate to the Newtonian laws of motion explicitly are excluded from the definition. On these grounds the iconic model as previously defined would be outside the range of the definition. However, some of the cases under analogic model might be considered examples of the Newtonian model. For it, there was an explicit defining feature which was required for that type of model. We may recall that it involved two physically different systems whose behavior is defined by the identical mathematical equation. The defining equation may happen to be derivable from Newton's law, however it is not the essential fact in determing the classification and the interest is not really related to the Newtonian theory. Similar remarks may be made concerning the similitudinous models but here again the defining features are unique and require no specific application of the Newtonian laws, although these laws obviously may be involved in the dynamics of the motion of the models.

In accord with these remarks, it may be said that the model is to be considered Newtonian if the three laws are explicitly involved. The meaning will be clear in the application discussed in the next two chapters.

It will also be clear how the Newtonian model is, in general, different from the very general type called disclosive model, which will be treated later in this book.

Although we limit the expression Newtonian model to types for which the Newtonian laws explicitly apply, it may be useful to apply the term model itself to various aspects of the total picture, using different kinds of modifying adjectives. Specifically one may refer to the geometrical model, the kinetical model, and the physical model. The distinction is not trivial and, in fact, may prove very useful in studying problems. In order to illustrate our meaning a single case will be examined in some detail.

For the purpose the motion of a conical pendulum will serve very well. It may be recalled that such a pendulum is nothing more than a small mass which is attached to one end of a weightless inextensible string, the other end of which is attached to a fixed point. The mass is assumed to move in a horizontal circular path with uniform speed. In such an event we have a steady state phenomenon in which the massive particle moves in a circular path, and the supporting string sweeps out a cone.

Now one may have his interest fixed on anyone of several aspects of the phenomenon. Also, one may speak of the geometrical aspect, the kinematical aspect, or the kinetical aspect. The actual pendulum may be referred to as the physical model. The geometrical model is simply the set of geometrical features such as the straight line of the string, the circular path of the particle, and the curvature at any point of the path (which of course for a circle is constant). Furthermore, the kinematical model introduces the time and relates to such matters as velocity in path and to any acceleration that may exist. Because the particle is moving in a circular path with constant speed it is easy to prove that it is always accelerating toward the center. All of these facts are kinematical considerations. By Newton's first law it follows that because the particle is not at rest or moving in a straight line it has a force impressed upon it. In fact, the second law tells us how much the force is because from the kinematical model we already know the acceleration. The force to produce the central acceleration is supplied by the horizontal component of the force in the string.

From this simple illustration it can be seen that one may speak of a kinematical model, a kinetical model, or a physical model. Also, it is clear that there is interaction between the models, which depends on the laws of Newtonian mechanics.

Much more elaborate examples of what we call the Newtonian model will be discussed at length in the next two chapters.

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CHAPTER 10

NEWTONIAN MODEL EXPERIMENTATION WITH SOLIDS

We will now illustrate the Newtonian model with a series of more complex examples than the conical pendulum which was treated in the last chapter. Various aspects of the model will be referred to as geometrical, kinematical, or physical. The definitions of these terms which are given in any good unabridged dictionary would be satisfactory for our purpose. However, for greater definiteness of understanding, they will be briefly defined and discussed in what follows.

The geometrical model is the shape or configuration of the structure, machine, or system under consideration. It includes boundary surfaces with their associated displacements, normals, and curvatures. All of these elements can be presented as functions of the coordinates of the points in the body which are determined with respect to some suitable frame of reference. They are considered to be known, or at least knowable, when the body is at rest or at a specifc time if it is in motion.

The kinematical model is one of pure motion. It consists of the displacements, velocities, and accelerations of all the points of the body as functions of time. Of special interest for vibratory motion are the normal mode shapes and their corresponding frequencies.

The kinetical model consists of the forces and stresses in the body and on its surfaces. These are assumed to produce the displacements, velocities, and accelerations of the points in the body. As a consequence it may be said to be interacting with the kinematical model. The constitutive equations, or stress-strain relations, and the differential equations of motion may be said to be the manifestations of the interaction. The equations of motion are derivable in accord with Newton's laws of motion and as a consequence furnish us with the reason for calling the model Newtonian.

The physical model is the actual or the scaled structure, machine, or system which typifies the class of objects of interest and which is subjected to experimentation in a laboratory. On such a model physical variables such as displacements, strains, and velocities are measured for the purpose of studying performance and comparing experimental results with predictions based on the solutions of the differential equations of motion subject to initial and boundary conditions.

Of special significance to any study with models is the prototype. It is the archtype of whatever we are investigating and must always be kept in mind. It is of fundamental importance with respect to any model which we may consider, whether Newtonian or not. Examination of good dictionaries will reveal that it is an original on which a

thing is modeled. Futhermore, it is an individual or complex that exemplifies or serves as a standard of the essential features of a group or type. In engineering the meaning of the term is always clear. For example, a prototype airplane is the full-scaled piloted flying model of a new type of airplane. In general, the prototype is the first full-scaled working model of a new type or design of furniture, machinery, or vehicle. It has always been common practice in military service to speak of the prototype of a tank, gun, or warship.

All of these matters will now be illustrated in some detail with investigations which have been conducted on impact on structures, vibration of structural elements, and loading of solids with couple stresses. For each of the topics which will be treated, the constitutive equations or internal laws of resistance to deformation, as well as the equations of motion will be given. The solutions of the latter, subject to initial and boundary conditions, will be discussed. These subjects are usually categorized as theory. Experiments, also, will be described in some detail along with results. References will be furnished for the complete treatments to be found in the technical literature.

Very many examples of what we call Newtonian models exist, but we shall limit ourselves to those problems with which we have been personally involved. We do this not only for convenience, but also because of our greater first-hand knowledge of the specific cases.

Longitudinal Impact on Bars

A brief but applicable discussion of the subject of impact on structures, with which we are now interested, is given in the three volume Handbook on Shock and Vibration by Harris and Crede [1]. Another useful reference of considerable depth is the book on the subject by W. Goldsmith [2]. In these sources it will be found that the material which will be treated in the following pages is presented in considerable detail and related to the general subject of the dynamics of structures.

The first model for our present consideration was simple in shape but was subjected to sufficiently intense loads to produce plastic flow and finally failure by rupture of the material. It consisted of a long metallic bar of circular cross-section provided with adapters at its ends for transmitting desired loads. Specifically, the bar was subjected to a dynamic loading by arranging that one end was in effect fixed and the other end was caused to move with constant velocity as shown in Fig. 1. As a consequence, a strain wave was caused to move from the loading end towards the fixed end.

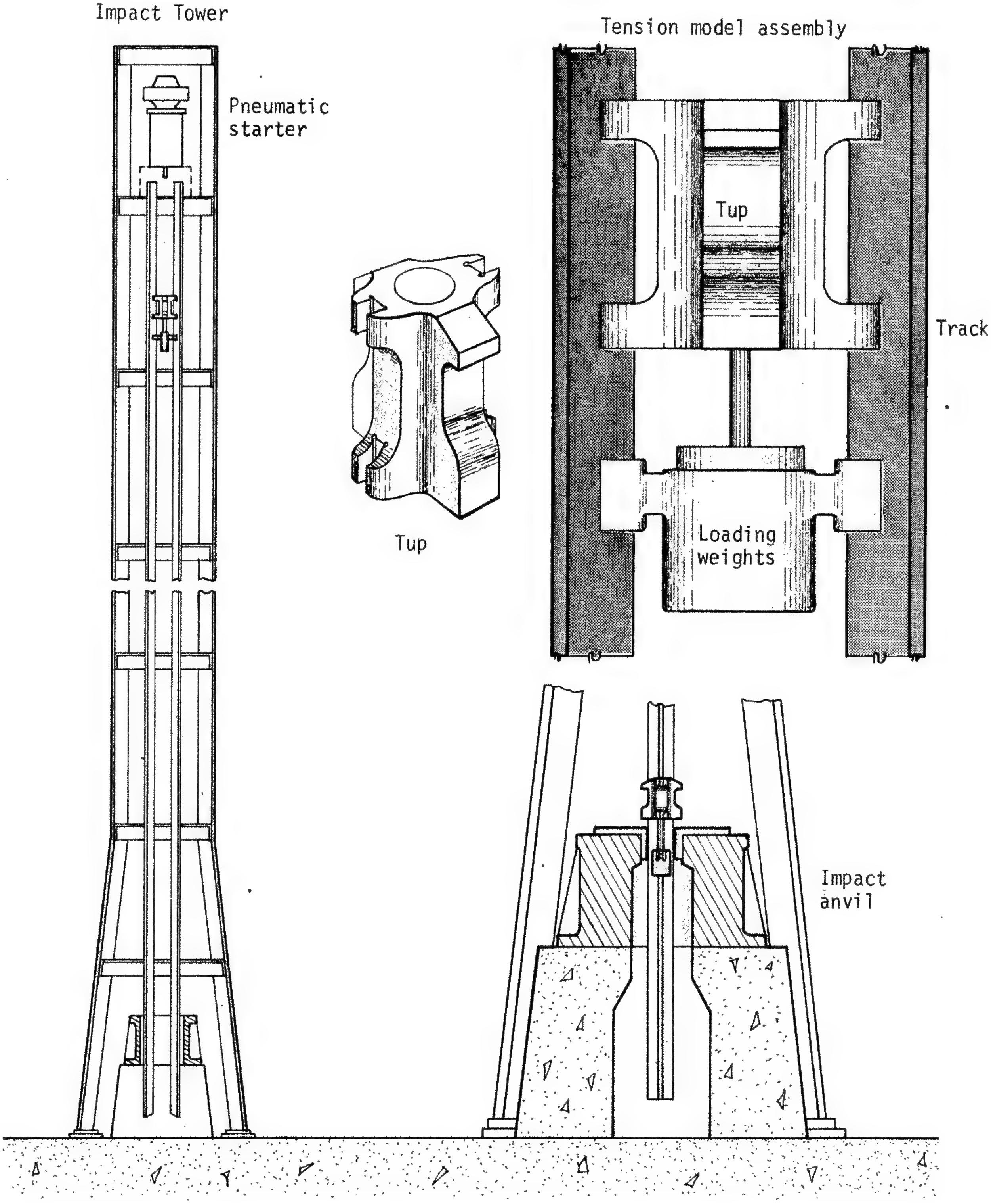


FIG. 1. Tension impact machine and tension model assembly.

The longitudinal impact problem for prismatical bars, within the elastic range, was first studied by Navier. It was more effectively treated subsequently by Saint Venant and Boussinesq [3]. For the elastic case, they developed the equation of motion (in accord with Newton's Laws) as follows:

$$\frac{\partial^2 u}{\partial x^2} = C^2 \frac{\partial^2 u}{\partial t^2}$$

where

u = longitudinal displacement

x = coordinate of a point

C = velocity of strain wave ($C^2 = \frac{E}{\rho}$)

E = Young's modulus of elasticity

t = time

ρ = mass density of material

Currently, this differential equation is referred to as the one-dimensional wave equation. It is quite satisfactory providing the elastic limit of the material is not exceeded. Since our model was subjected to stresses which exceeded the yield stress, it was necessary to resort to another differential equation which includes the phenomenon of plastic flow.

An important advance in the treatment of the problem for which large strains exist was contributed by Theodore von Karman. He used the same Newtonian equation of motion, but replaced the constitutive equation. He suggested for cases in which the proportional limit is exceeded the modulus E be replaced by the appropriate slope from the usual engineering stress-strain curve. On such an assumption the equation of motion becomes:

$$\frac{\partial \sigma}{\partial \varepsilon} \cdot \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

where

σ = longitudinal stress

and ε = strain at the particular stress σ

The other quantities are then simply as before.

The proposed equation is obviously non-linear and rather difficult to solve in general, however it provides a very important insight to the theory of impact and on its basis a remarkably important result can be obtained. It was predicted that a "critical velocity" for tensile impact on prismatic bars must exist. As had been known quite some time before von Kármán made his suggestion, a critical velocity had been discovered during tensile impact experiments [4]. This is another example of the experimental knowledge preceding the theory. Various reasons had been given by experimentalists as to why a critical velocity existed, but none of them were really satisfactory. In the paper to which we have just referred, carefully controlled experiments which reveal the critical velocity are described. There also, the experimental results are related to the von Karman theory. The energy absorbed by the model up to rupture is plotted against the velocity of the lower end of the bar. Clearly, after a certain velocity of impact is reached, the energy which the model absorbs significantly decreases. The geometry of the model after failure also points unquestionably to a critical velocity. The apparatus with which the experiments were performed is completely described in U. S. patent No. 2,763,148 granted to W. H. Hoppmann II in 1956. It is shown schematically in Fig. 1.

We consider that our first illustration shows that the Newtonian model can be signally successful. Anyone with special interest in problems of the type just discussed should refer to the publications which have been mentioned.

A different type of impact which further illustrates the Newtonian model will now be described.

Transverse Impact on Beams

While the problem discussed in the previous section relates essentially to impacts sufficiently intense to cause plastic deformation and finally rupture of material, the example now to be considered consists of a beam which is transversely loaded by an impact force in such a manner that the yield stress is never exceeded. To produce the impact force, the beam is subjected to a collision with a mass moving with a given velocity in a direction which is orthogonal to that of the length of the beam as shown in Fig. 2(a).

The general theory associated with the problem has been given by S. Timoshenko [3]. It is also discussed in considerable detail by W. Goldsmith [2]. The Newtonian equations of motion as extended by Euler for the case of beams are of fundamental importance. As related in the brief history of dynamics given in Chapter 9, the designation Bernoulli-Euler was given to the theory of the transverse motion of a beam. For our present purpose, the Newtonian model also includes a moving mass which strikes the beam. Therefore, in such a case the dynamic system under study consists of both a moving mass and a beam. Newton's equations of motion apply not only to the beam, but also to the moving mass. How one manages to perform an analysis on such a two-body system is described completely for a damped, elastically supported beam in a previous paper [5]. There the contributions of many investigators have been acknowledged and evaluated. The differential equation of motion may be written as follows:

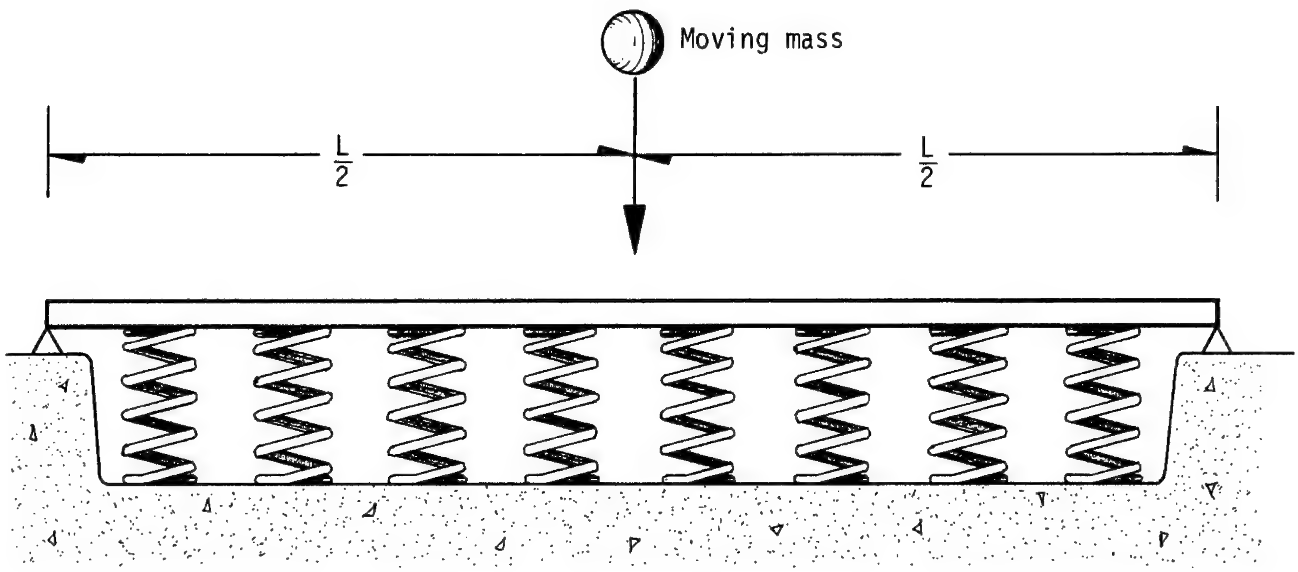
$$\rho A \frac{\partial^2 w}{\partial t^2} = \mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV} + \mathrm{V}$$

where

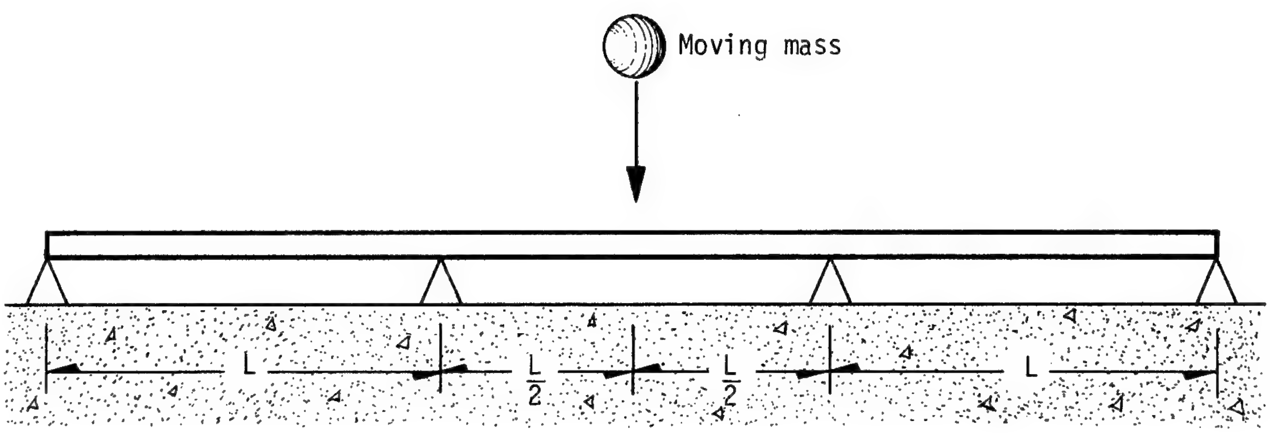
$$\mathrm{I} = EI \frac{\partial^4 w}{\partial x^4} = \text{internal elastic resistance}$$

$$\mathrm{II} = C_1 I \frac{\partial^5 w}{\partial x^4 \partial t} = \text{internal damping resistance}$$

$$\mathrm{III} = C_2 \frac{\partial w}{\partial t} = \text{external damping resistance}$$



(a) Damped elastically supported beam.



(b) Multispan beam.

FIG. 2. Impact of a mass on a beam.

IV = Kw = elastic foundation resistance

V = F(x,t) = impact force applied to beam

and

w = displacement, E = Young's Mod., C_1 = coefficient, ρ = mass density,

I = moment of inertia, C_2 = coefficient, A = cross-section area.

x = distance, K = Found. Mod.

The term on the left in the differential equation represents the Newtonian inertia force at any point along the beam given by coordinate x and at any time t.

The effects of damping and elastic foundation can be eliminated simply by putting C_1, C_2, and K equal to zero.

For certain values of the physical parameters, strains and displacements which result from the impact were calculated and presented in the paper. Hence we have information on the geometical, kinematical and kinetic models. Important aspects of the effects of damping and elastic foundation were discussed.

Subsequently, the theoretical analysis was extended to include the case of the multispan beam [6]. Again, results were obtained for the geometrical, kinematical, and kinetic models. As interesting as these were, it was imperative to investigate the corresponding physical models and see how well experimental results conform with theoretical predictions. Consequently, one-, two-, and three-span model beams were constructed and subjected to impact loads from masses moving with predetermined velocities as shown in Fig. 2(b). The beams, experiments, and results have been fully described in a previous paper [7].

It may be noted here that a very important physical phenomenon was observed as a result of the experiments. The measured strain rose abruptly to a significant maximum just after the collision and the curve of strain as a function of time agreed very closely with calculated results. Furthermore, it was clear that damping has very little power to reduce the initial maximum of the strain curve. The calculated results, even though they represent the summation of many normal modes, provide a reliable prediction of maximum strain caused by impact. The sharp strain increase just after impact and the peculiar shape of the strain-time curve which was reported in the paper was discussed later in a very interesting paper by H. H. Emschermann and K. Ruhl [8]. They conducted an extensive series of impact experiments to verify our results. Anyone interested in the details may do well to study their paper which includes not only strain data from steel beams but also many results from photoelastic studies of the problem.

Developments of the models which we have just been discussing can be made to include the effect of axial loads or of concentrated masses attached rigidly to the beams. Both of these cases are shown in Fig. 2(c) and Fig. 2(d) respectively. The theory of the beam response to a transverse impact load when it is carrying a compressive axial load is presented in a previous paper [9]. Subsequently the physical model was studied and the effect of axial load clearly shown [10].

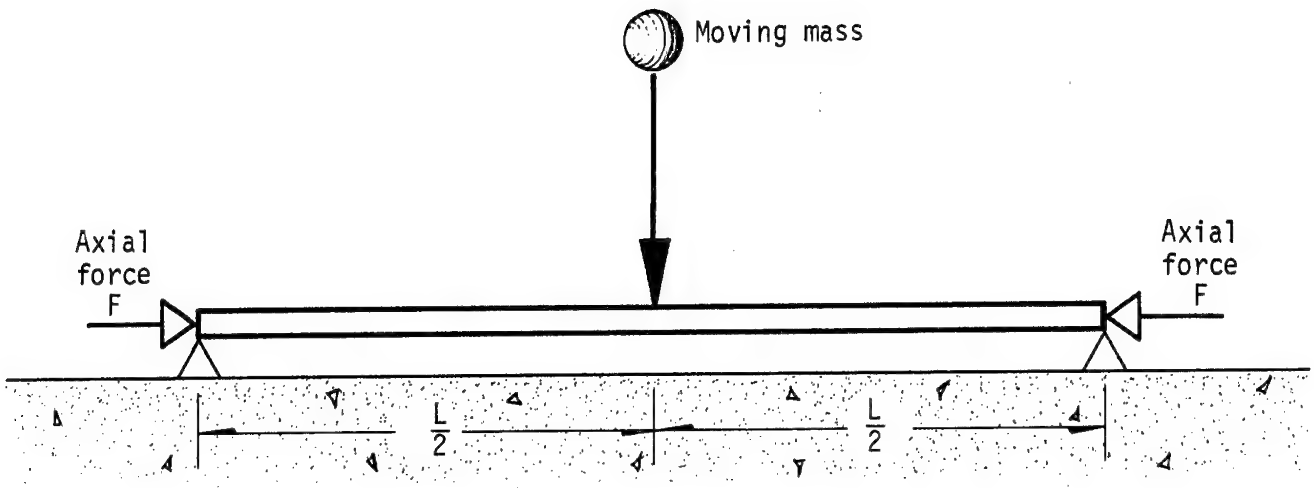
The effect of a concentrated mass on the vibration of a beam was thoroughly investigated and results for theory and experiment published [11]. The condition which must be satisfied for the beam-mass system to respond as a single-degree-of-freedom system is determined. How the response radically changes as the ratio of the attached mass to the mass of the beam decreases can be clearly seen.

The model method was also applied by us in an investigation of the vibrations of systems of elastically connected parallel beams and of systems of elastically connected concentric rings. The theory of the vibration of the beams is given in one paper [12] and the experiments for a two-beam system are described in another [13]. The complete study for the rings is given in a single paper [14]. The experimental apparatus for determining the kinematical characteristics of a two-beam system, that is the nodal patterns and the corresponding vibrational frequencies, is shown in Fig. 3(a). That for impact response is given in Fig. 3(b). In Fig. 4 may be seen a schematic arrangement for determining the nodal patterns and frequencies for a two-ring system. For anyone having a special interest in the vibrations of such systems, it is recommended that an examination be made of the references which we have listed. It is considered that the theoretical and experimental information found there adds substantially to the weight of evidence in favor of Newtonian model studies.

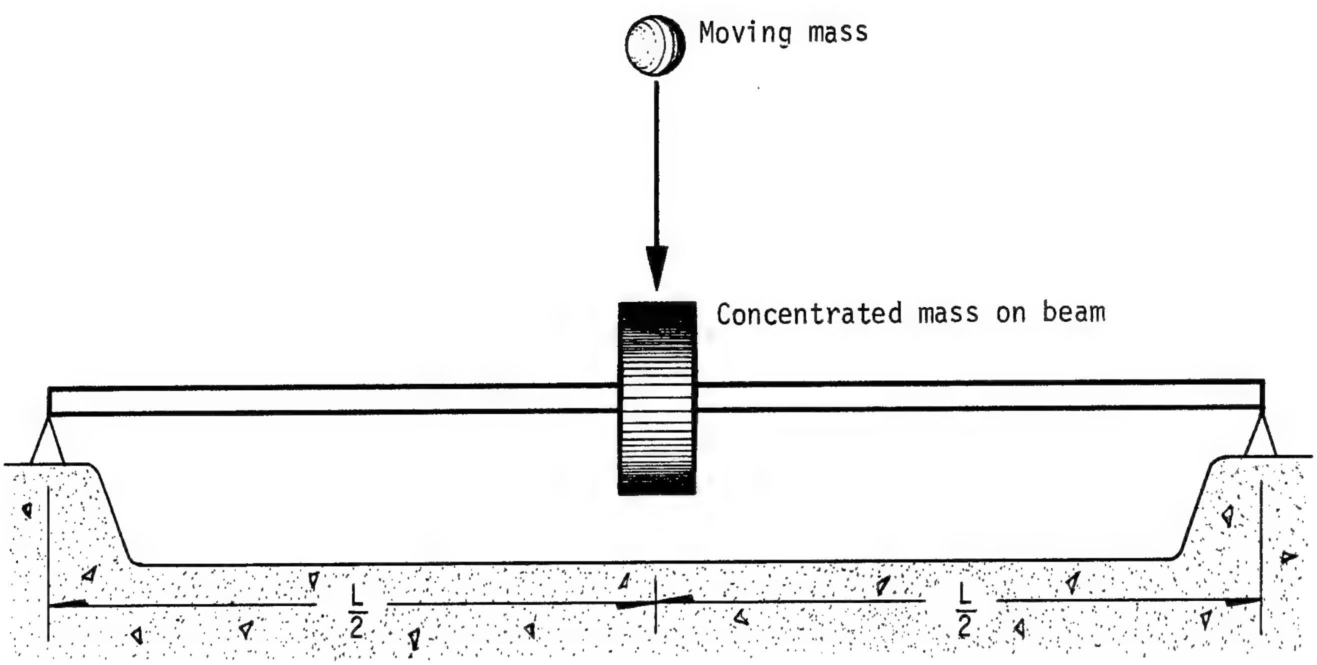
All of the models considered so far in the present chapter lend themselves to the study of essentially one-dimensional bodies. Such an assumption certainly carries restrictions but was quite satisfactory for our purposes. Now, however, it is proposed to pursue the question of the applicability of the Newtonian model in the field of two-dimensional solid bodies.

Deformation of Stiffened Two-Dimensional Elastic Surfaces

In the previous section we gave considerable attention to the dynamics of a one-dimensional structure as an example of Newtonian modeling. It is thought that an extension of the method to include two-dimensional structures may increase the understanding of our meaning of model. For that purpose we shall describe studies first of stiffened plates and then of stiffened shells. Finally, some model analysis of isotropic shells will be examined.

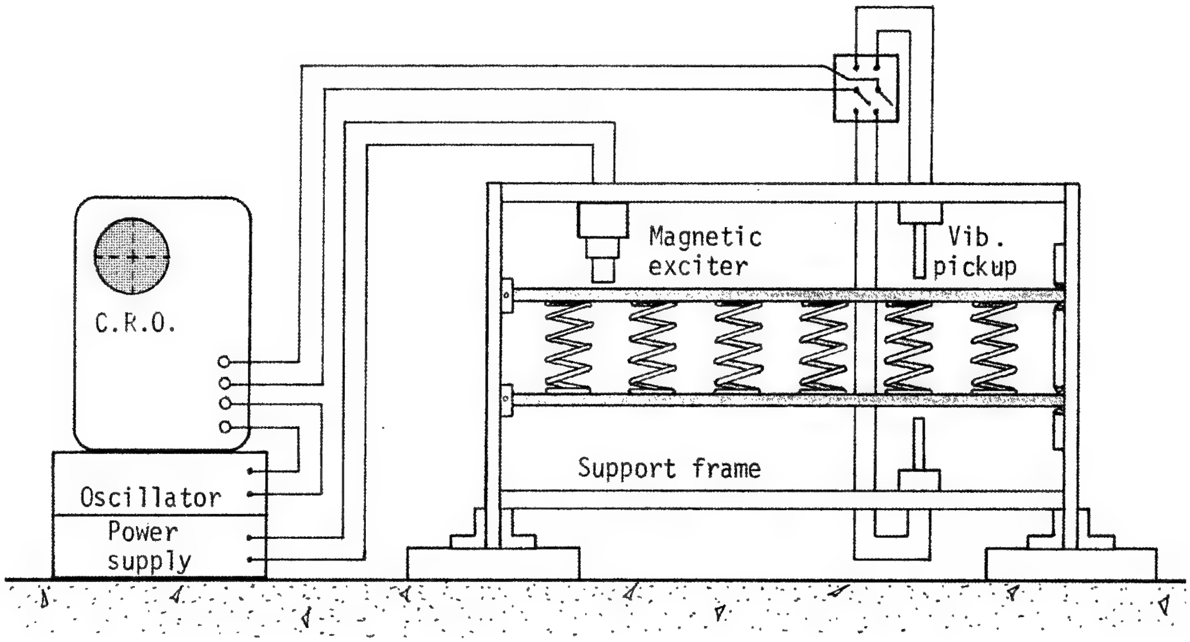


(c) Column with axial force F.

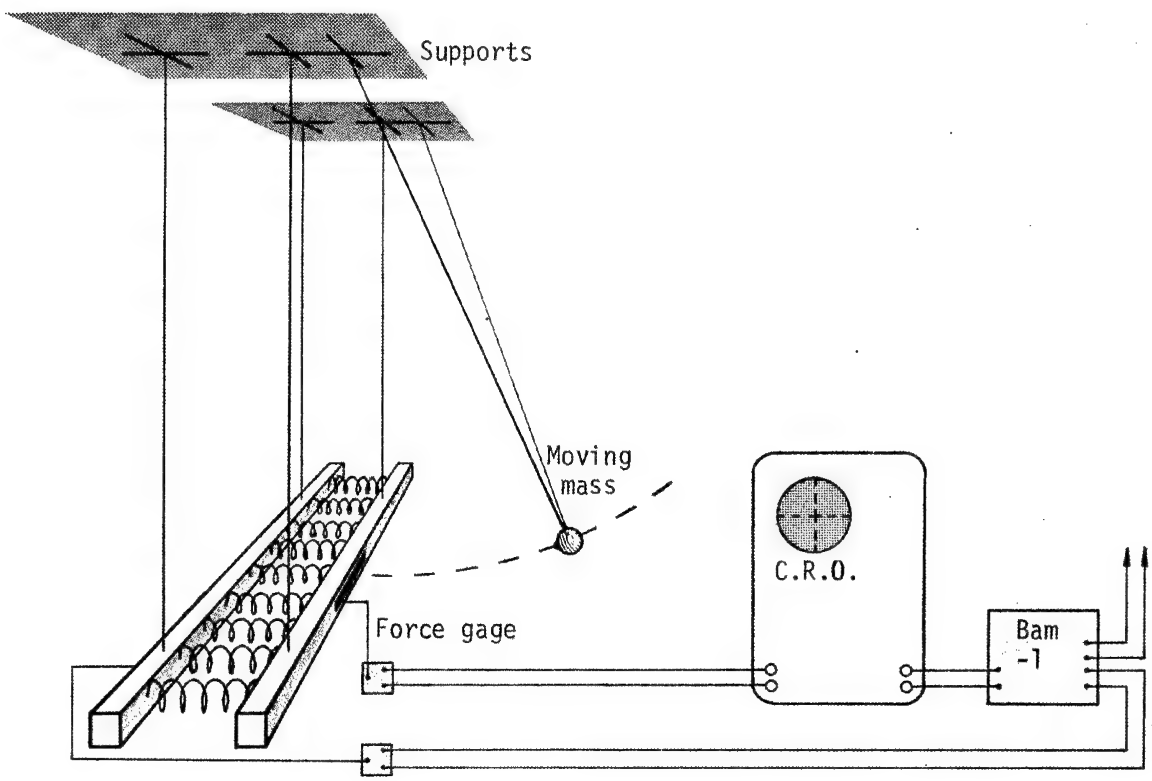


(d) Beam carrying a concentrated mass.

FIG. 2. (continued). Impact of a mass on a beam.



(a) Experimental apparatus for determination of vibrational characteristics of a two-beam system.



(b) Impact of a mass on a two-beam system.

FIG. 3. Elastically connected parallel beams.

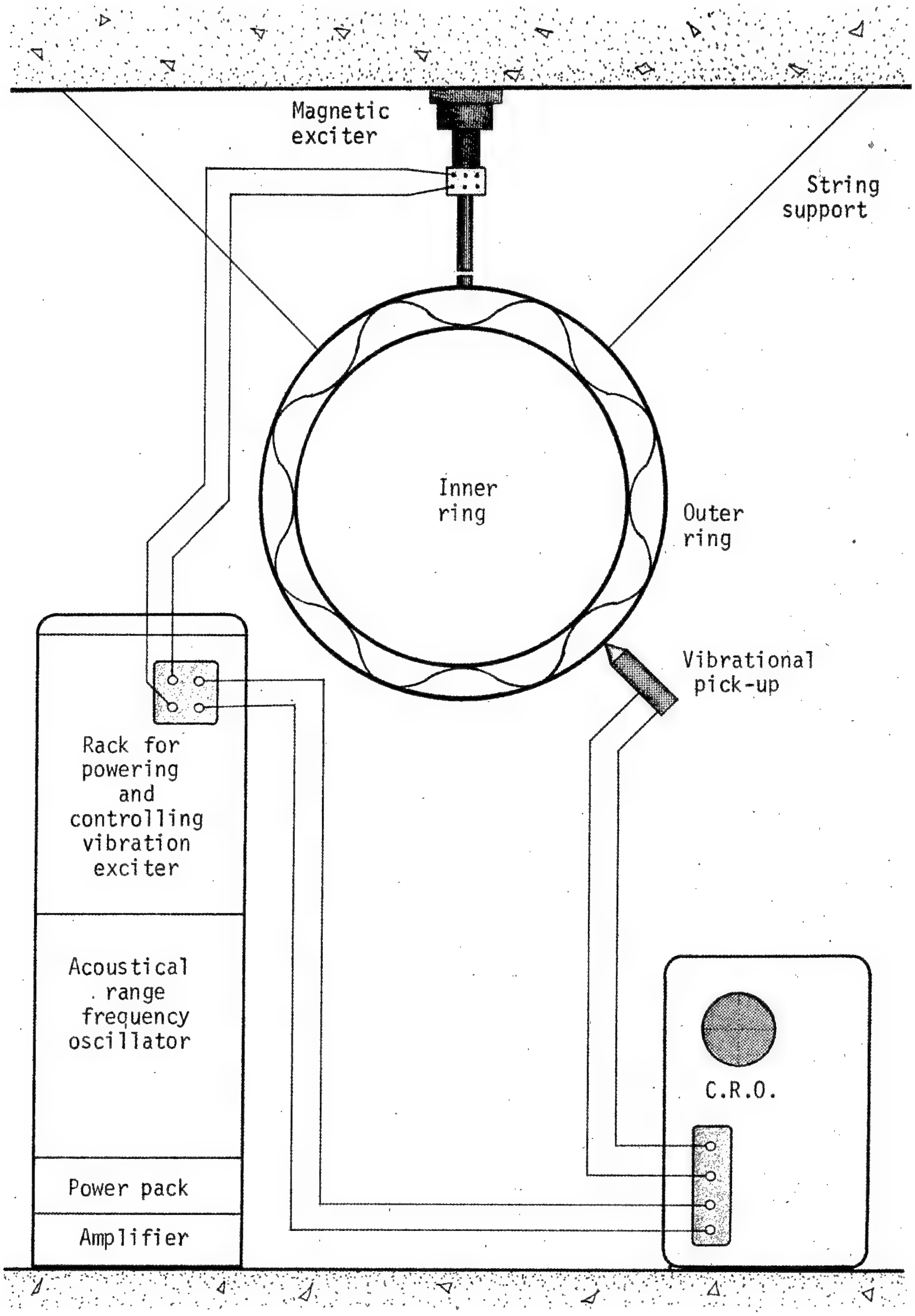


FIG. 4. Model of two-ring system.

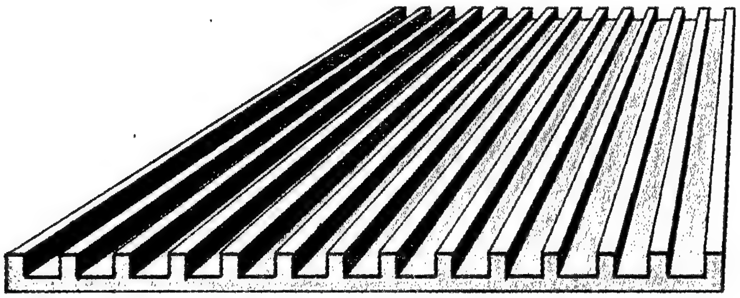
There are many structures composed of plates and shells to which are attached systems of closely spaced parallel stiffeners such as those shown in Fig. 5 and Fig. 6. In order to produce acceptable designs it is essential to be able to calculate the reaction of these structural elements to applied forces. During the last 75 years it became necessary to develop satisfactory methods for that purpose. Impetus was given to the effort to achieve such advances in structural design by the needs arising in shipbuilding and aircraft construction.

At the beginning of WWI the great need for more sophisticated structures in shipbuilding and the corresponding need for more advanced analysis was becoming apparent. As a consequence, efforts to solve increasingly more difficult structural problems were being made. By the advent of WWII a significant advance had been made in the design and construction of surface warships and submarines. More acceptable hulls for both were being constructed. Also, by this time the demands of the rapidly developing aircraft industry were lending weight to the effort to obtain strong lightweight structures. These latter demanded the use of stiffened plates and shells as well as the means for calculating their performance.

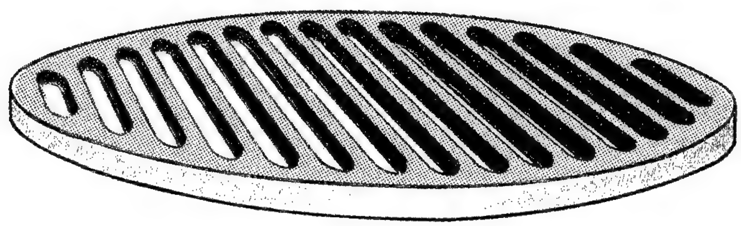
In 1914 M. T. Huber presented a theory of stiffened plates in which the plates were imagined to be equivalent in behavior to plates of constant thickness but composed of orthotropic material. Since then the theory has been applied by Seydel, Oldenbourg, Schade and others. A brief resume of the pertinent parts of this theory, with references to the literature, is given in a textbook by S. Timoshenko [15]. Our present interest in the subject is that it provides an excellent opportunity to illustrate further how the Newtonian model can contribute substantially to the solution of such technological problems. To do this we shall use a special method which we proposed in 1955.

It may be stressed that all of the previous investigators analytically determined the effective plate compliances for use with statical plate deformation problems using the elastic coefficients of the base material with which the structures were fabricated and the geometrical information about the stiffeners and the plates used in the given design. We proposed to determine the compliances by means of deformation experiments with physical models of stiffened plates. Also, we proposed to test the applicability of such an approach by vibrating the given plates and comparing the results with the corresponding theoretical predictions. The result was a rather extensive program of study which enabled us to discover how well the method enables one to determine the compliances. The vibration experiments provided checks on the statically predetermined compliances as well as knowledge about the vibrational characteristics of such structures.

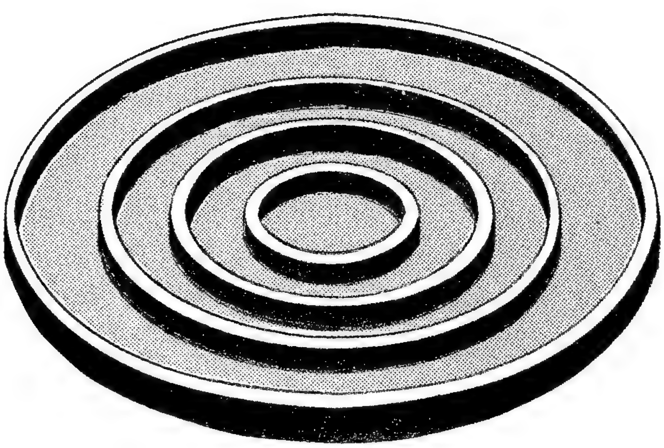
The method that we used to determine the compliances for the stiffened plates was based on one used by Bergsträsser in a study of



(a) Stiffened square plate



(b) Stiffened elliptical plate



(c) Circularly stiffened circular plate

FIG. 5. Models of stiffened plates.

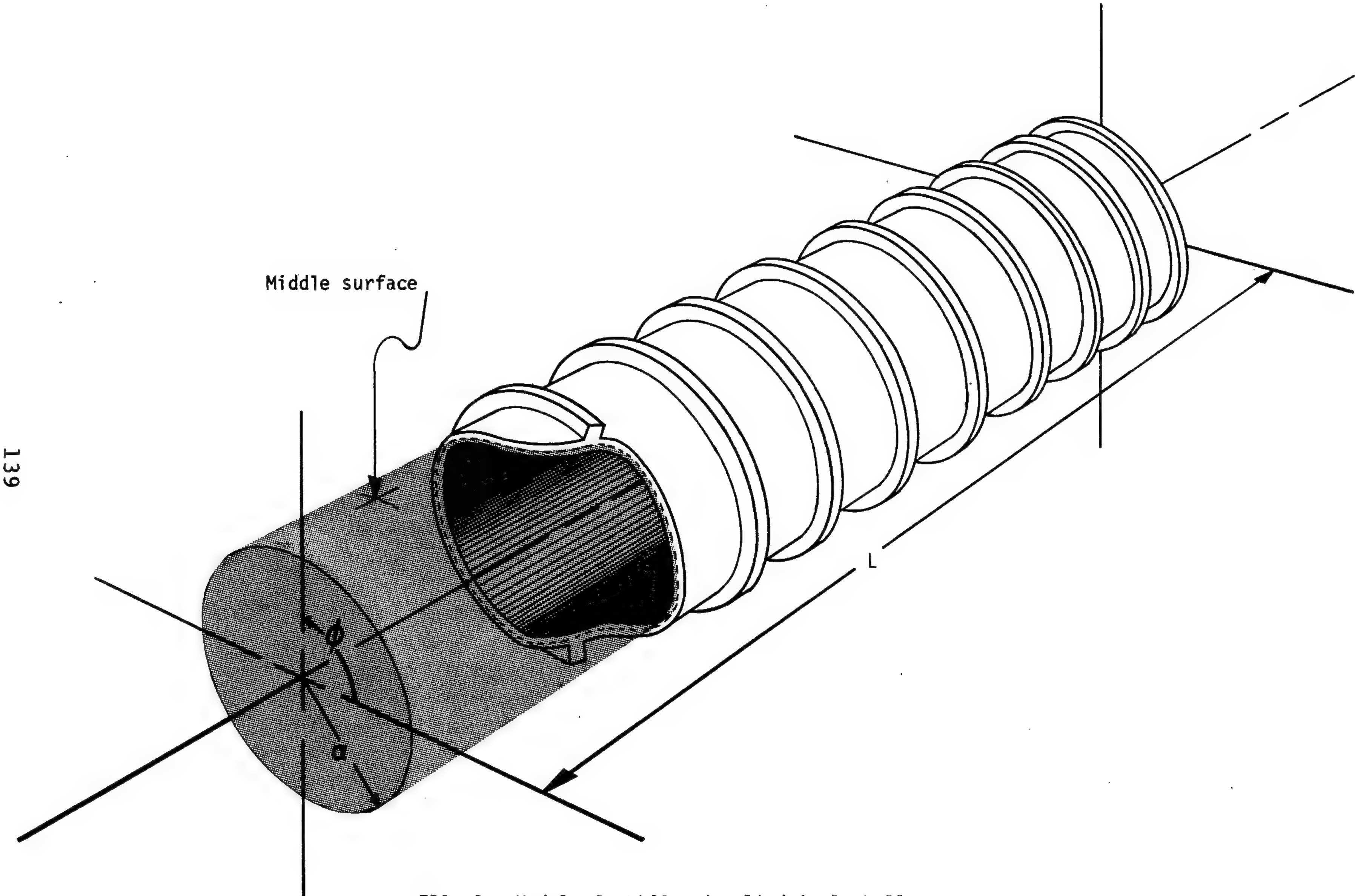


FIG. 6. Model of stiffened cylindrical shell.

isotropic plates. It requires the use of two rectangular plates and one square plate. One of the rectangular plates has the stiffeners parallel to the long edge; the other has the stiffeners parallel to the short edge. The rectangular plates are used in flexure experiments and the stiffened square plate is used in a twisting experiment. These experiments are described in a book on anisotropic elasticity by R. F. S. Hearmon [16]. It may be noted in passing that Hearmon's book is a very useful introduction to anyone concerned with all aspects of the application of anisotropic elasticity.

In order to make an extensive study of both stiffened plates and stiffened shells we began by carefully determining a complete set of elastic compliances for a plate with a given pattern of stiffeners. It may be stressed that in order to include shells in the program, additional types of compliances are required. Whereas in our first studies of flat plates it was necessary to obtain only flexure and twist compliances, we now needed the additional elastic coefficients associated with stretch and shear of the middle surface. All of the experiments and results are described in a published technical paper [17].

The twist and flexure associated with the following differential equation for the transverse deflection of a plate:

$$A_x \frac{\partial^4 w}{\partial x^4} + A_{xy} \frac{\partial^4 w}{\partial x^4 \partial y^2} + A_y \frac{\partial^4 w}{\partial y^4} = 0$$

where

w = transverse deflection

x,y = coordinates of point in 2-dimensional surface which represents the plate

A_x, A_y, A_{xy} = constants depending on physical compliances of the plate

The stretch and shear compliances were obtained from use of the usual theory of anisotropic elasticity in conjunction with experiments to determine the stretch and shear of the middle surface.

It turned out that the statical method used was an excellent on to obtain the required elastic constants. In order to use the above plate equation for dynamic experiments it is necessary to add the mass times acceleration term on the left-hand side as required by Newton's laws of motion. When this is done the equation may be solved in accord with given boundary conditions to provide the normal mode shapes and corresponding frequencies of vibration. Vibrational experiments amply substantiated the correctness of the compliance data we obtained in the statical experiments. The stiffened square plate used in the

investigation was similar to that shown in Fig. 5 (a). It provided the nodal patterns for free flexural vibrations as well as information concerning the combination of modes under certain conditions. The experiments have been described and results presented elsewhere [18]. In another publication results of vibrational experiments with stiffened elliptical plates like that in Fig. 5(b) are given in detail [19]. There also will be found the differential equation of motion of stiffened elliptical plates, based on Newton's laws of motion.

A somewhat different kind of stiffened plate is shown in Fig. 5(c). It is referred to as a cylindrically aeolotropic plate. The stiffeners are a set of concentric circular bars or beams. The differential equation of transverse deflections was developed and solved so that the compliances could be determined from experiments in which concentrated forces are appropriately applied to the model [20]. Vibrational experiments were conducted with these models and the results reported [21].

As pointed out in the paper on stiffened square plates there was a discrepancy between the calculated and measured frequencies of about 10 to 15 percent. Careful examination of the experimental techniques indicated that all of the experimental data were probably correct, showing that the physical models were reliable. That then left a question concerning the theoretical formulation based on Newton's equation of motion. Further study suggested that the omission of rotatory inertia might explain the discrepancy. Such indeed was the case and the new formulation of the problem, including the effect of rotatory inertia, provided calculated frequencies which agree very closely with measured values [22]. While the original findings had been somewhat disconcerting on account of discrepancy between theory and experiment, the improved theoretical method was satisfying, especially in that it demonstrated how the Newtonian model approach could lead to very useful findings.

The success of the program with the stiffened plates led ultimately to a rather extensive study with stiffened shells. As already mentioned, it is essential to have two sets of elastic compliances for use with shells; those that correspond to flexure and twist, as with the flat plates, and additional elastic constants that correspond to the stretch and the shear of the middle surface.

Our models were limited to the case of stiffened cylindrical shells such as shown in Fig. 6. In addition to studying shells with circular stiffeners, consideration was also given to shells having longitudinal stiffeners in the direction of the generators of the cylinder.

As might be expected both the mathematical analysis and the experimental techniques are more demanding than in the case of plates. Notwithstanding, it was possible to develop the theory satisfactorily and predict the normal mode shapes and corresponding frequencies [23].

Little suspected vibrational phenomena developed during the study with the models [24]. Again, it was apparent that the Newtonian methodology was invaluable.

Before leaving the problems of shell vibrations, two other experimental programs, which include examples of the Newtonian model may be mentioned. One was concerned with shallow spherical shells such as shown in Fig. 7 and the other with paraboloidal shells such as shown in Fig. 8. In one paper [25], the experimental method and results are presented for the spherical shell, and in another they are given for the paraboloidal shell [26]. It may be seen in the first reference that for the shallow spherical shell it was possible to solve the differential equation of motion and compare predicted results with those obtained from experiments. However, for the paraboloidal shells we were not able to obtain theoretical solutions, except in the case of shallow shells. On the other hand, it was possible with experiments on physical models to obtain many nodal patterns and corresponding frequencies.

While we consider that the studies with plates and shells clearly demonstrate the value of the Newtonian model in dealing with the deformation of solids, one more interesting example of its use in connection with the mechanics of deformation of deformable bodies will be presented. It concerns the role of couple stresses which conspicuously entered the field of solids in the twentieth century.

In order to render the concept of couple-stress clearer to engineers who based their analyses on classical elasticity, which originated with Navier and Cauchy, we introduced two experimental programs with models. It is our purpose to describe these now in the following and show how they relate to our concept of Newtonian modeling. However, it may be appropriate to describe first the advance of continuum theory which now includes the couple-stress and give a very brief historical sketch.

One of the present authors was influenced in his thinking on the problem of couple stresses mainly by the questions which may arise in connection with the nature of the general strain field. Specifically he was interested in the study of strain made by Leroux in 1911 [27]. It now seems somewhat odd that the older elasticians concentrated their thinking on the concepts of lineal strain and shear strain. When they finally introduced the relations of the strains to the stresses the constitutive equations naturally turned out to be the generalized Hooke's law which represents a linear relation between normal and shearing stress and lineal and shearing strain respectively. Leroux concentrated his attention on the gradients of the displacement field in a much more general manner than was the custom in his time.

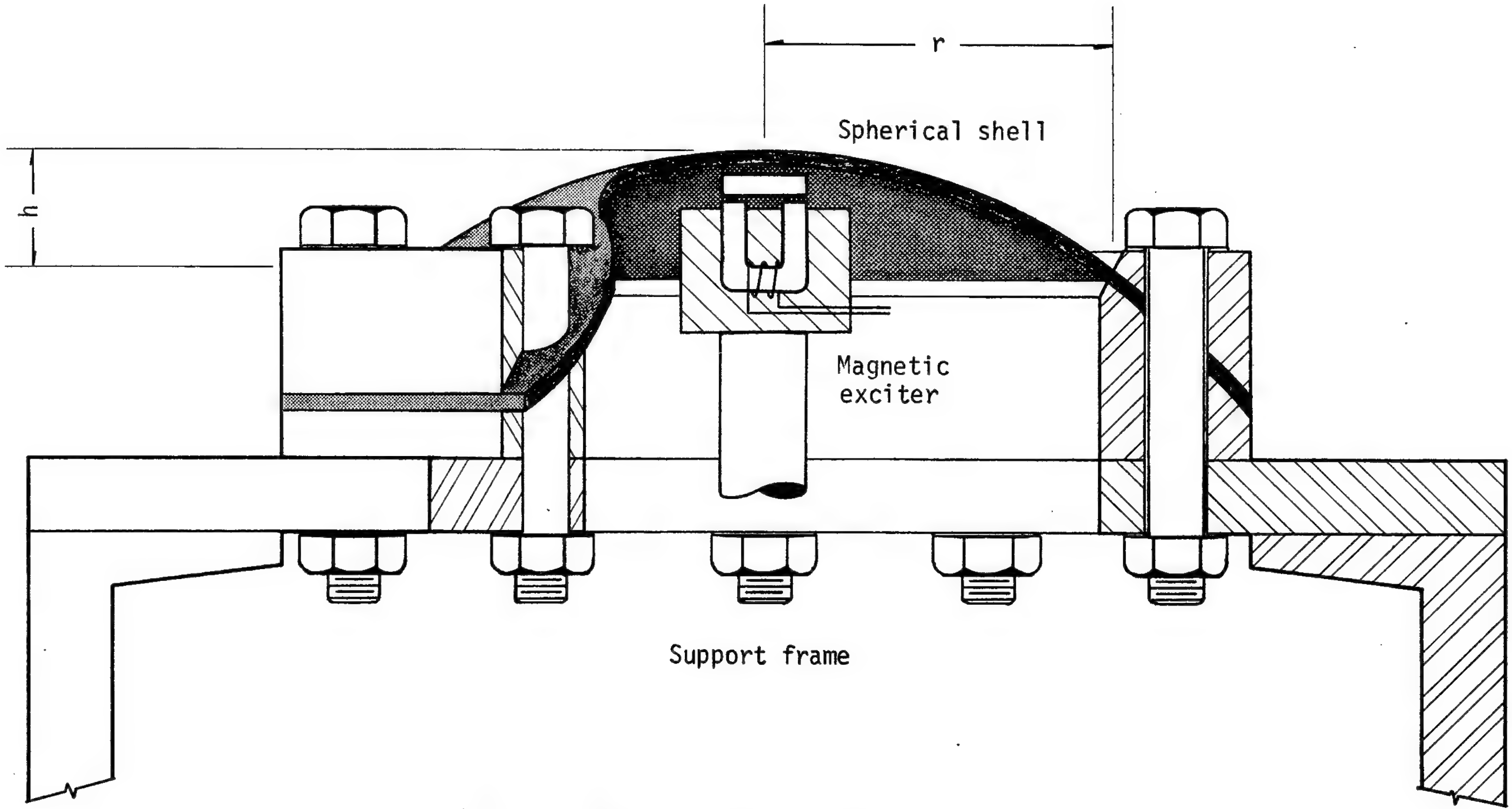


FIG. 7. Model of shallow spherical shell.

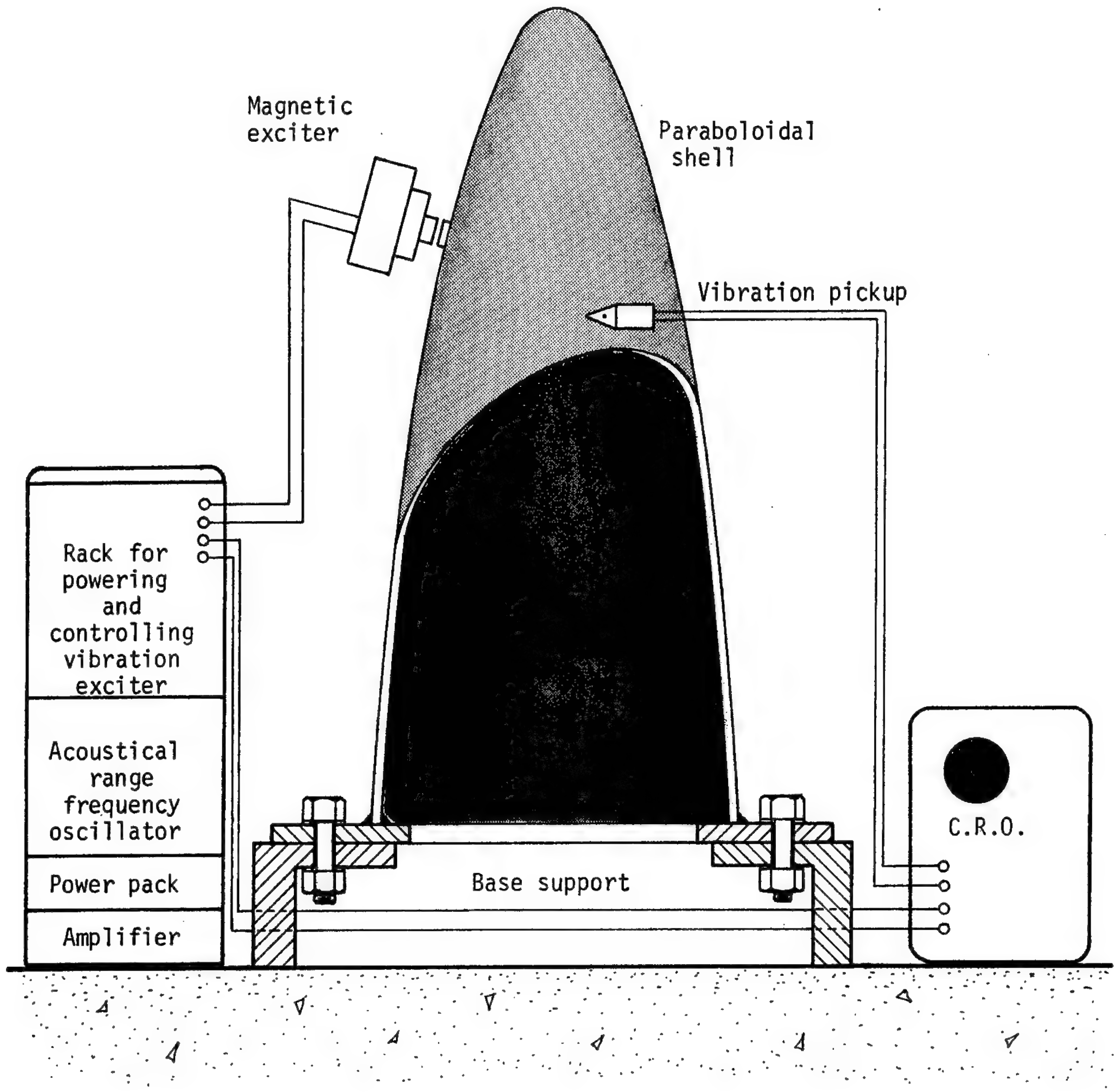


FIG. 8. Model of paraboloidal shell.

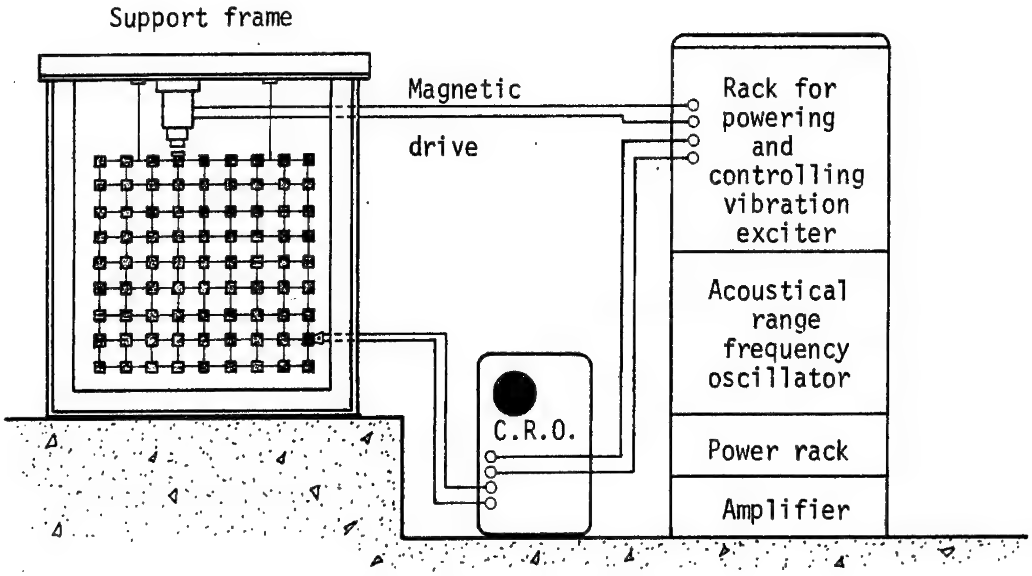
An important consequence of the more general treatment of strain is that some question is raised concerning the validity of the classical theory of elasticity in which the stress tensor is symmetric. Also, on a purely logical basis it would seem that the couple-stress could be postulated for the field theory with just as much reason as the classical stresses, which were originally postulated. The analogy of stresses or their force and moment resultants with force and couple in ridgid body mechanics is obvious. E. Cosserat and F. Cosserat in 1909 finally wrote a textbook on deformable bodies that included couple-stress [28].

On a purely physical basis, L. Boltmann questioned the universal validity of the principle of the symmetry of the stress tensor as related to the elastic continuum [29]. For various reasons Kelvin and Voigt also considered the same question. If the admissability of the symmetry of the classical stress tensor is denied, the door is opened immediately to the couple-stress.

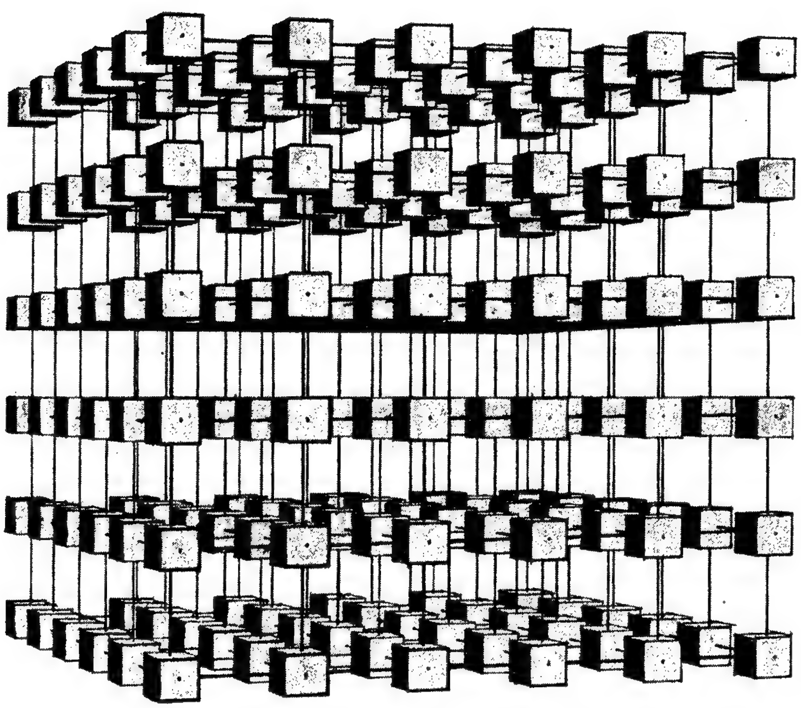
There was a considerable lapse of interest in the whole question until the last decade or so when a strong resurgence occurred. One probable reason for lack of concern in engineering circles was a lack of awareness of the seriousness of the question. Also, engineers would be reluctant to generalize classical design analysis to include couple-stress because of the rather considerable increase in complexity of the subject. On the other hand, attention was to be forced onto the problem by the rapidly growing knowledge of solid-state physics. For example it was inevitable that interest in dislocation theory would lead to a reinvestigation of the sufficiency of the classical theory of elasticity. Evidence of such a trend was demonstrated by the investigations of E. Kröner who introduced the concept of couple stresses in his studies of the deformation of metals [30]. A notable elastician, R. D. Mindlin, saw the need for greater generality in the formulation of the theory and wrote several important papers on the subject [31].

In order to render the concepts clearer to the engineer, who is concerned with structural design calculations, which may involve couple stresses, we decided to perform some experiments on models which could illustrate couple-stresses. For the purpose we designed and constructed some mechanical models of material which might develop couple-stresses. Solid metal blocks were held together in an array by either flat elastic strips or small rods. A 2-dimensional model is shown in Fig. 9(a) and a 3-dimensional one in Fig. 9(b). The couples were applied at predetermined points by means of parallel wires which pull in opposite directions.

Using the theory which includes couple stresses, a specific problem in elasticity was solved for a given body and loading. The latter were simulated with a physical model. Displacements at various points in



(a) Two-dimensional model.



(b) Three-dimensional model.

FIG. 9. Apparatus for vibrational experiments on model of a crystal.

the body were measured. In this manner the elastic constants for such a generalized continuum were determined. The models, experiments and calculations of elastic contants are described elsewhere [32].

Very interesting results were obtained with vibrational experiments on both the 2-dimensional and 3-dimensional models [33]. Normal mode shapes were determined with their corresponding frequencies of vibration. Some of the results seem to be at variance with calculations based on classical elasticity.

Again the didactic power of the Newtonian model is demonstrated. Furthermore, many examples of the Newtonian method from the work of other investigators in solid mechanics could be shown. However, we consider that sufficient material has been introduced for present purposes and now we shall turn to the field of fluid mechanics for more illustrations.

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CHAPTER 11

NEWTONIAN MODEL EXPERIMENTATION WITH FLUIDS

The Newtonian model is just as effective in the study of fluid flow as it is in the study of the deformation of solids. Again we wish to discuss specific applications of the model, but this time with the flow of liquids. Important problems which we have investigated include viscometric flows, the flow of dilute solutions of polymers, and tidal flows. The first of these involves questions concerning the mechanics of the cone-plate viscometer.

Flow in a Cone-Plate Viscometer

Among the various flows associated with the mechanics of fluids, one of particular interest for the determination of physical properties of liquids, is that which is generated by a rotor immersed in the liquid. The reason lies in the fact that relatively simple apparatus may be used to generate the flow and permit the ready measurement of some of the constitutive parameters. A well-known example is the cone-plate viscometer. The measured quantities are the torque imposed on the cone by the liquid and the angular velocity. From these data a so-called coefficient of viscosity is usually said to be determined. The determination requires some theory of flow on the basis of which the coefficient is obtained. Ordinarily a simple flow pattern is assumed to exist, and the Navier-Stokes equation for steady flow is used for the purpose of making the necessary calculations. The equation, which is given in textbooks on hydrodynamics, may be written as follows [1]:

$$\rho\frac{D\overline{V}}{Dt} = \rho\overline{F} - \overline{\nabla}p + \mu\nabla^2\overline{V}$$

where

ρ = density, $\overline{\nabla}$ = gradient operator, $\overline{V}$ = velocity, p = pressure

t = time, μ = coeff. of viscosity, $\overline{F}$ = body force, ∇^2 = Laplacian Operator

On the left is the inertia force in accord with Newton's second law and on the right are the various forces which are applied to an element of the fluid.

The solution of the equation was developed by others on the basis of some rather severe simplifying assumptions. The assumptions were thought to be justified by the fact that the angle between a generator of the cone and the flat plate is less than 4° and usually less than even 1°.

Over a decade ago we decided to investigate the flow lines and general performance of this type of viscometer. In particular we were very much interested in the assumption that the particles of the liquid

moved in perfect circles about the axis of rotation of the cone. The very small angle between cone and plate made it almost impossible for technologists to observe how the liquid actually flows. We knew that the particles attached to the surface of the cone did move in circular paths. The motion of other particles we questioned. As a consequence, we decided to use a Newtonian Model to fully explore the problem. The first need was a suitable physical model and so we designed and built a rotational fluid flow generator for the purpose. The machine and the first experimental results are described in the Transactions of the Society of Rheology [2].

The basic features of the flow generator are shown in Fig. 10. Several requirements were apparent from the beginning. We consider that the angle α between cone and plate should be varied from very small values, of the order of 1°, to very large values, of the order of 60°; the bearings in the machine should be of such quality as to practically eliminate anomalous motions; the container for the liquid should be transparent in order to permit direct observation and satisfactory photography; and finally, a sensitive torque meter, attached to the shaft of the rotor, should be provided. We were indeed fortunate to be able to meet all of these requirements.

In addition to making satisfactory measurements of torque at various angular velocities, observations were made of the flow lines in the liquid. For the latter purpose, flow visualization techniques were developed, using various dyes. The dyes were injected into the liquid with a hypodermic needle, which was quickly removed after a certain amount of dye was delivered at a pre-determined point. As the dye diffused throughout the moving liquid and the structure of the flow became apparent, pictures were taken both with still and movie cameras.

Information obtained on the shape of the lines of flow demonstrated that the major premise of the simple theory is at least to some degree false. The particles of the liquid do not rotate simply in concentric circular paths about the axis of the cone, but spiral around the central curve of a vortex as they also move around the axis of the cone. The particles actually move over toroidal surfaces which are clearly shown in the liquid by photographic means. Photographs are presented in the previously mentioned paper. Moving pictures of these dye etched flow patterns were also made. A sketch of the flow lines in a meridional section is shown in Fig. 11.

For a small cone-plate angle it is very difficult to observe the nature of the flow. However, with the aid of fine, bright particles we were able to demonstrate that radial flow exists. Using a transparent bottom to the container for the liquid, it was observed that particles move from the periphery toward the center and from the center toward the periphery. By using the complete adaptability of the physical model and varying the α from 1° to 60° in stages, it was clearly possible to shown that the circulation of the fluid about the vertical axis approached

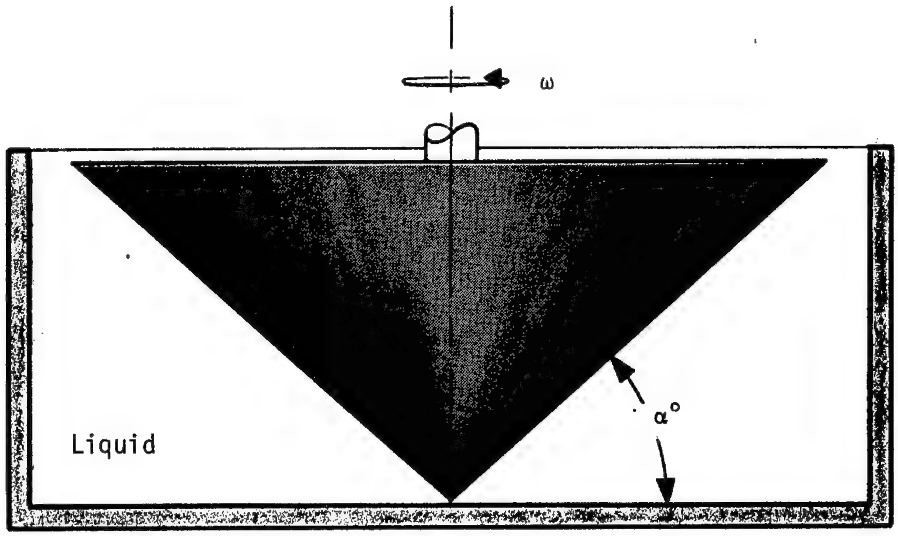

FIG. 10. Model of rotational fluid flow generator.

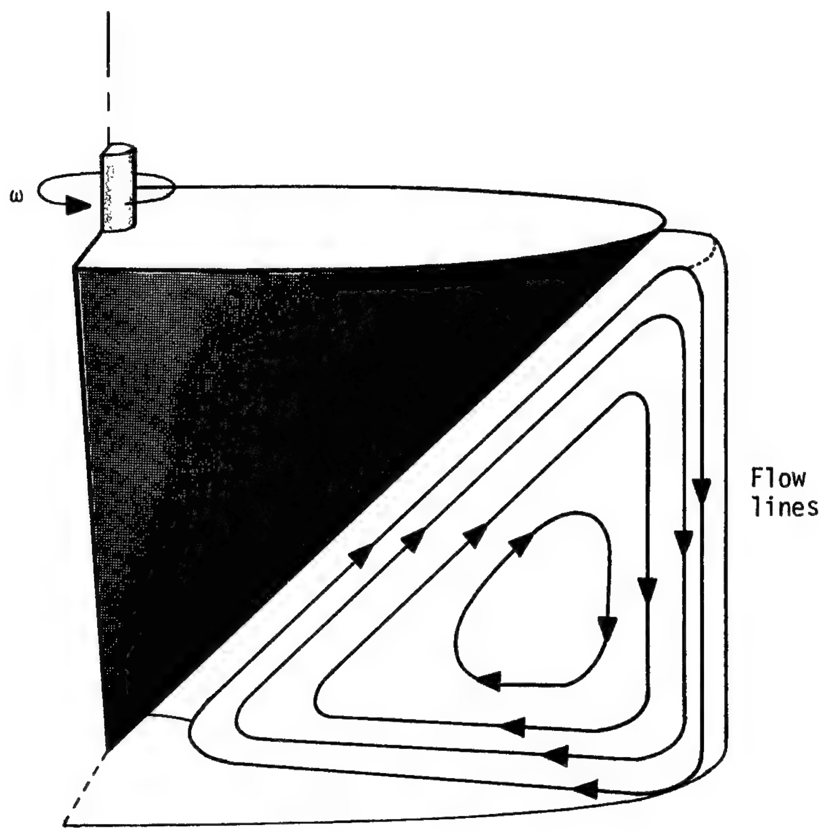

FIG. 11. Flow lines in meridional section of fluid flow generator.

zero as the angle α approached zero. It turned out to be possible to time the passage of a single particle as it moved around its own toroidal surface. Many observations on particles traveling around different toroidal surfaces led to the important conclusion that the circulation approaches zero. It is this fact that probably justifies the rheologists in making the assumptions they have concerning the behavior of liquids in the cone-plate viscometer, at least for Newtonian liquids. Also, it was determined with our fluid flow generator that the formula previously developed for the relation between the torque on the cone and the angular velocity of the cone is quite satisfactory for Newtonian liquids, if the angle α is small. For moderately large angles, however, using castor oil as the medium, it was shown that errors as large as 800 percent resulted.

The experience with the fluid flow generator clearly demonstrated again the value of the Newtonian model. It must be admitted, however, that the equation of motion for the general case has not yet been solved. Most of our studies were in connection with the physical, kinematical and kinetic models. A rather complete study along these lines was made and reported at the 1963 International Congress on Rheology in Providence, Rhode Island. The findings are published in the Proceedings of that Congress [3].

Flow of Non-Newtonian Liquids in Rotational Fluid Flow Generator

Knowing that rheologists used the cone-plate viscometer for studies of non-Newtonian, as well as Newtonian liquids, it seemed highly desirable to investigate the flow of both classes. By this time we had a clear idea of how Newtonian liquids behaved in our flow generator, but no idea of the nature of flow of any non-Newtonian types. Knowing of the peculiar normal stress effects of many of these liquids, especially those investigated by Weissenberg, we decided that a sensitive thrust meter for measuring axial forces on the rotating cone should be provided. Fortunately, we were able to design and build one that satisfactorily served the purpose.

The liquids which we studied were sweetened condensed milk, solutions of polyisobutylene in cetane, castor oil, tap water, and pure decalin. The last three substances were included so that a comparison could be made of the behavior of both classes of liquids. As we suspected, there was a radical difference in the character of the measured thrusts. A report on the experiments was published in Nature [4]. For the convenience of the reader, a reproduction of the thrust curves is shown here in Fig. 12. It can be seen that for the lower velocities, the resultant thrust for the non-Newtonian liquids is upward on the cone, in a direction opposite to that for the Newtonian liquids.

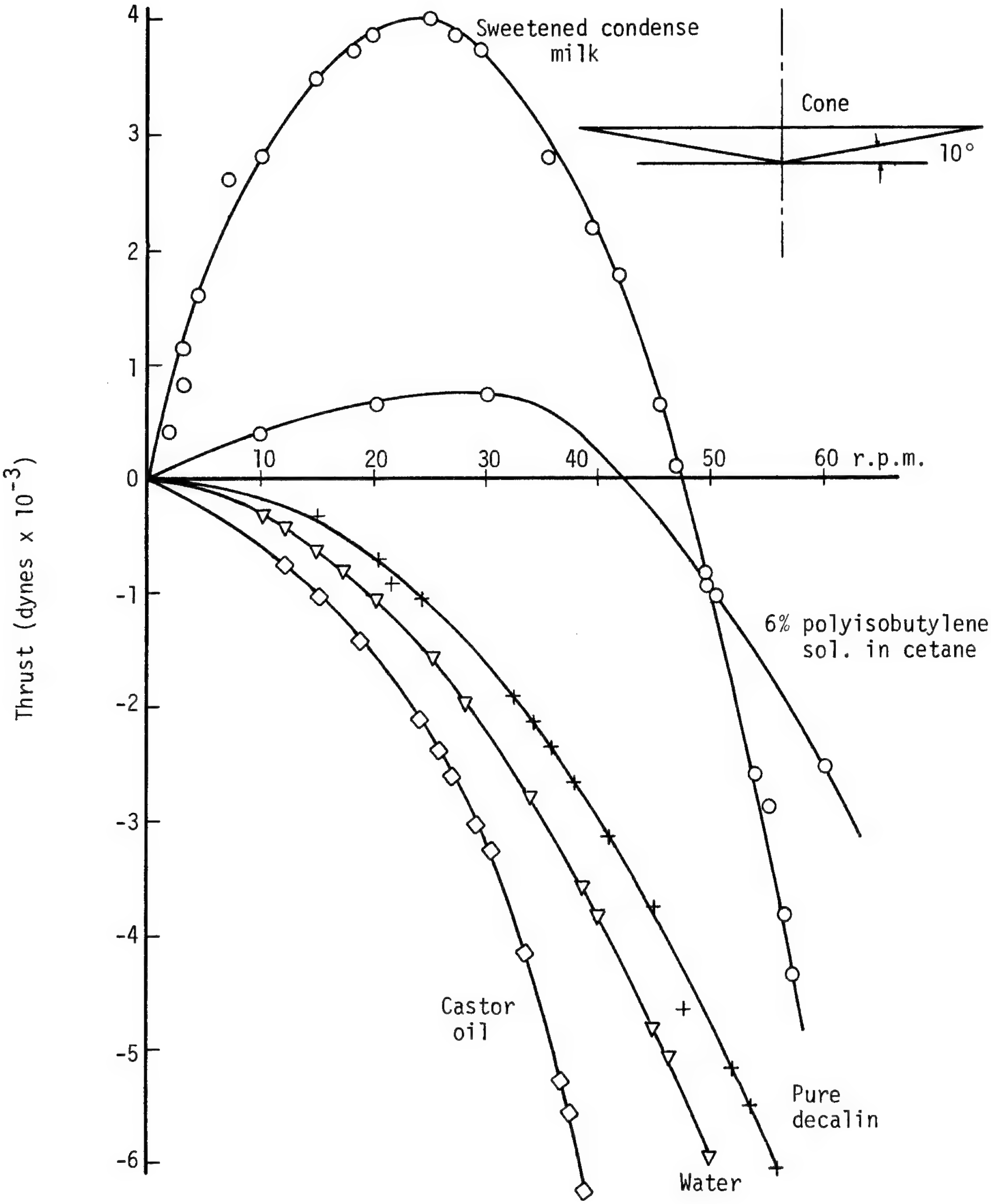

FIG. 12. Axial thrust on rotating cone as function of angular velocity.

Another very important finding, made with the model, is that instead of a single vortex, as for the Newtonian liquid, there are multiple vortices as shown in Fig. 13 and Fig. 14. This fact must be of crucial importance to any interpretation of torques and thrusts on the rotating cone. An investigator, unaware of the structure of the flow, would be in an untenable position trying to rate various liquids on the basis of their torque-angular velocity relations.

Complete data on the design of our thrust meter are given in the Journal of Applied Mechanics [5]. An extensive study, providing a substantial amount of torque and thrust data, is reported elsewhere [6]. For convenience, the torque data are reproduced here in Fig. 15. The peculiar vortical flows which we discovered with our flow generator suggested the importance of knowing the pressure distribution on the rotating cone. On the basis of the thrust data obtained it was suspected that the pressures for the Newtonian liquids would be relatively negative and those for the non-Newtonian liquids, conversely, positive. As can be seen in Fig. 16, such indeed turns out to the case. A complete discussion of these results is provided elsewhere [7]. These findings on pressure distributions as well as the multi-vortical flows, should give pause to those who may unthinkingly attempt to classify materials solely on the relationship of torque to angular velocity obtained with a cone-plate viscometer.

Some rheologists have referred to our research in connection with their own treatment of the subject. In particular, H. Giesekus made an intensive study of the cone-plate viscometer and presented the results in the Rheologica Acta [8]. He verified the multi-vortical character of the flow of non-Newtonian liquids which we had previously reported. He introduces the notion of secondary flow in his work. Besides the paper of Giesekus, there has been published a monograph on the subject by Coleman, Markovitz, and Noll [9]. They too referenced our work. All of these findings have been subsequently treated from a theoretical point of view by Truesdell and Noll in the Encyclopedia of Physics [10]. Since all of these authors use the term secondary flow in connection with the cone-plate viscometer, we would like to take this occasion to emphasize that the term may lead to some misconception. For example, G. I. Taylor in his famous study of flow in the cylindrical viscometer, discovered vortical flows at a critical value of the angular velocity. Theoretically he deduced the flow as a perturbation of the purely circular flow known to exist in such viscometers at sufficiently low angular velocities. Such a procedure is fully justified by experiment. The finding is really one which designates an instability of flow. Such is not the case for the rotational fluid flow generator using a cone as a rotor. For the Newtonian liquid the single vortex and for the non-Newtonian liquids the multi-vortices occurred at the

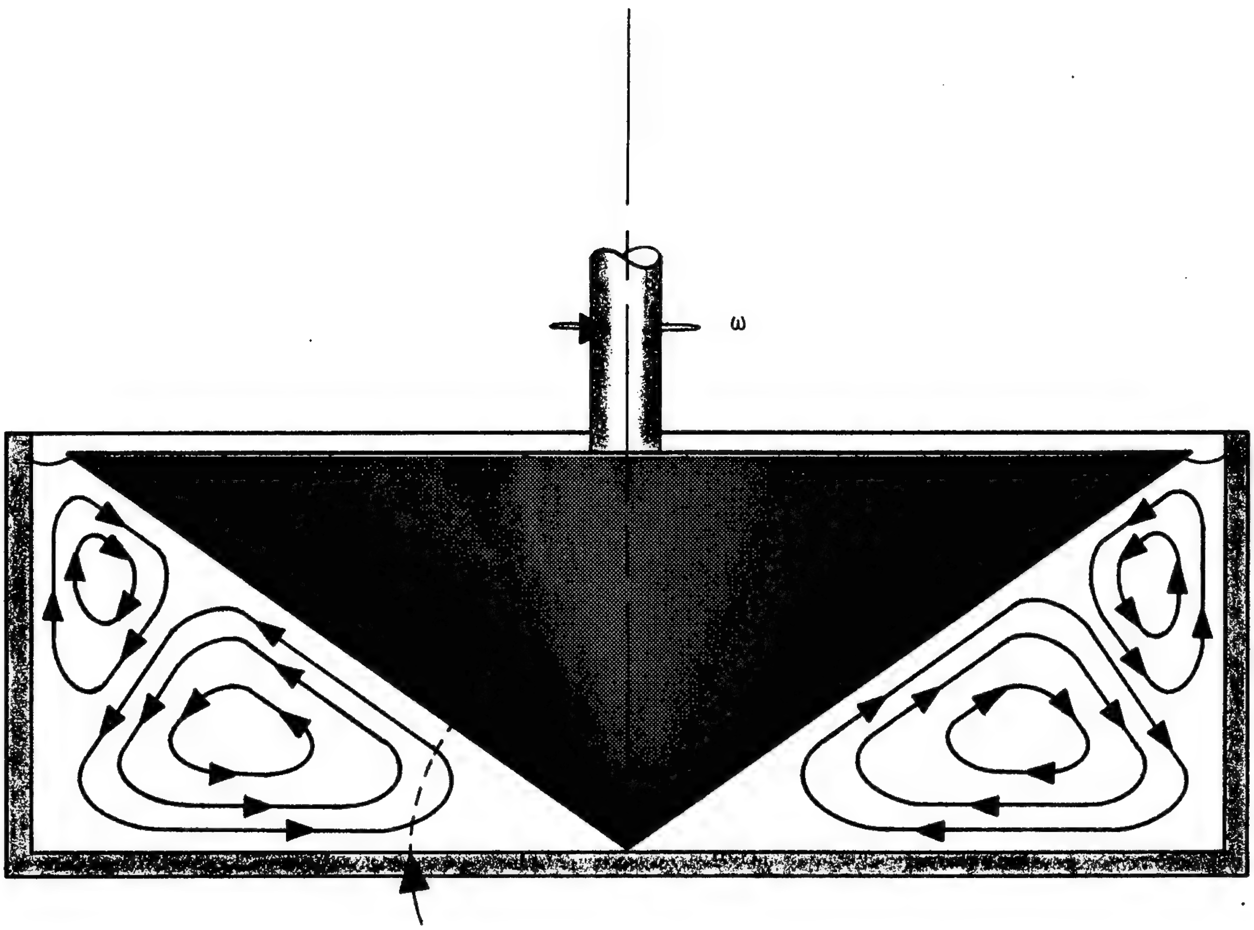

FIG. 13. Sectional view of flow generator showing flow lines.

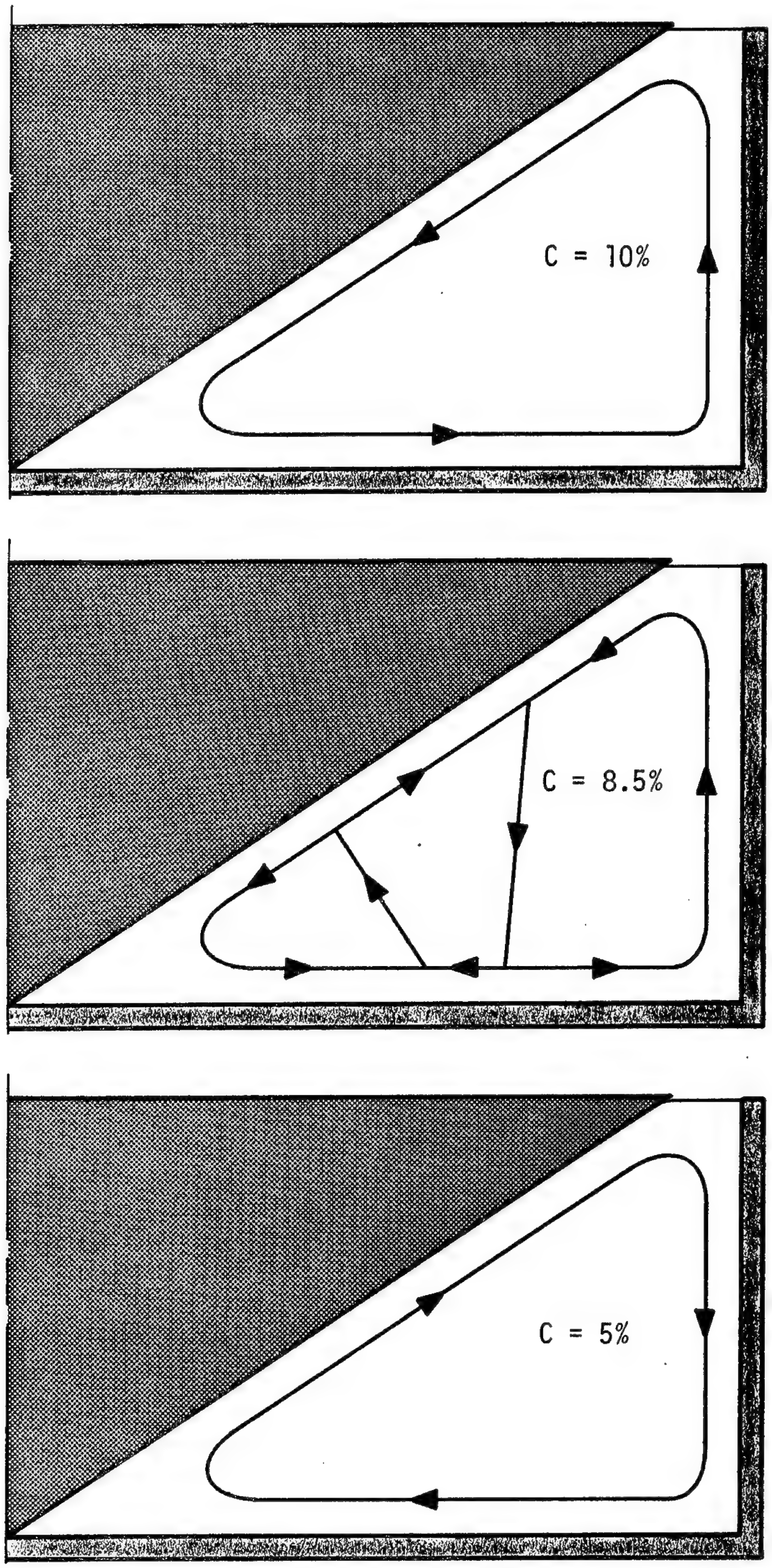

FIG. 14. Half-section of flow generator showing flow lines for several different concentrations of polyisobutylene solution.

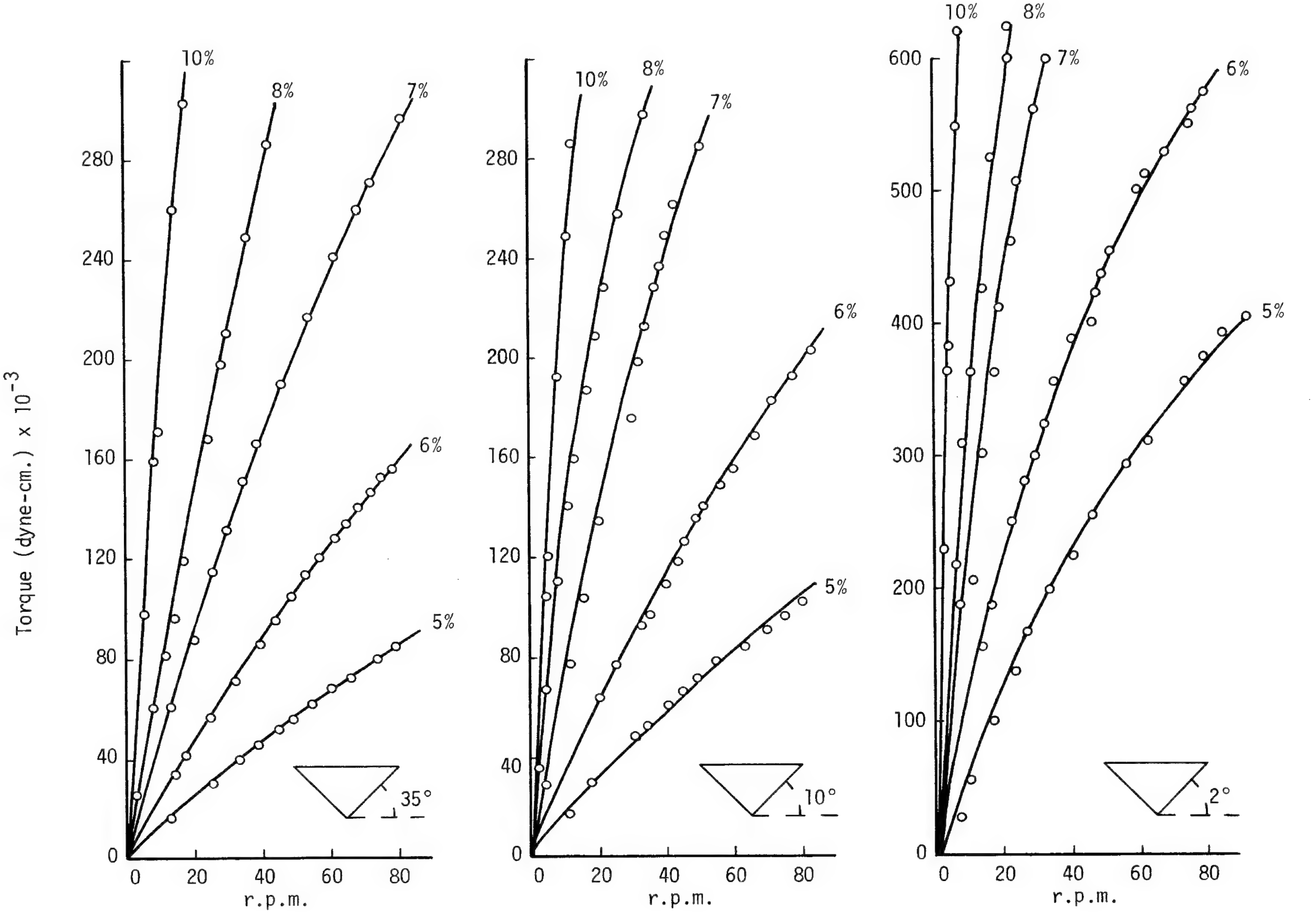

FIG. 15. Torque on cone.

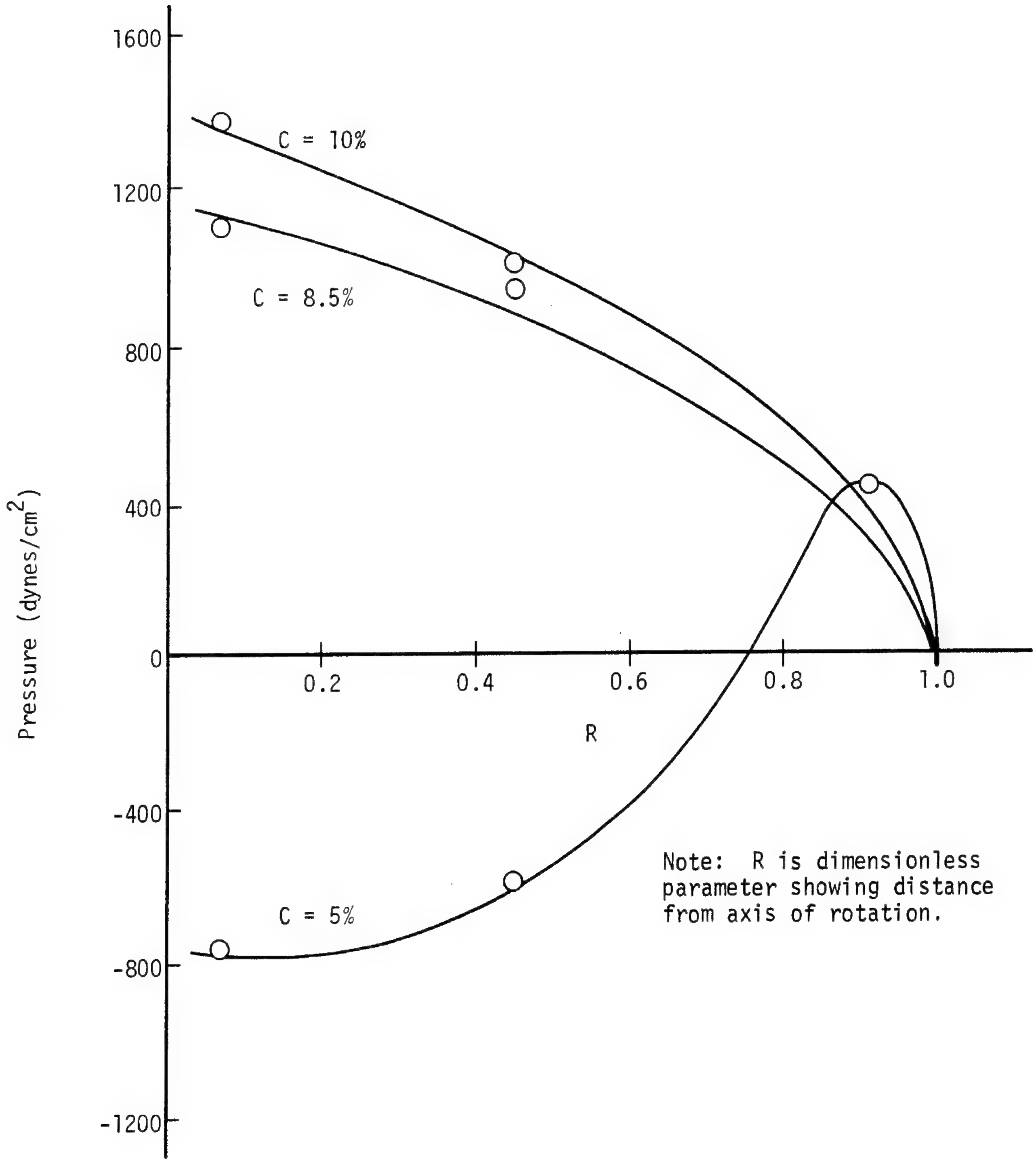

FIG. 16. Pressure distribution on surface of rotating cone for several concentrations of polyisobutylene solution in cetane.

lowest possible angular velocity. Hence we would prefer to speak simply of flow in our flow generator and not introduce the expression secondary flow. It is quite clear, as we have previously remarked, a particle of liquid moves both around the vortical center and around the axis of the cone. One may refer to the motion around the axis of the vortex as a secondary flow but we would like to object to any implication of instability of flow as evidenced by this fact.

Now we would like to discuss another type of fluid flow generator as an example of a Newtonian model.

Oscillating Rectilinear Fluid Flow Generator

One conclusion that may be drawn from the discussion in the previous section is that the science of viscometry needs more investigation The old practice of determining a single physical constant, usually called an equivalent coefficient of viscosity, should be re-examined. It has been amply demonstrated by our own study that serious considerations should be given to the structure of the flow field as well as to the relationship between total torque on the rotor and its angular velocity. Also, our findings with the thrust meter and the pressure transducers prove that additional kinetic variables should be measured if one is to develop sufficient background to realistically determine constitutive equations for any given liquid. The laws of internal resistance for the various media are much more complex than that assumed to involve only an equivalent coefficient of viscosity. It is true that the demands on the rheologist will be much greater, but the probable advance of technology certainly warrants it.

In the light of our experience with the rotational fluid flow generator, it seemed desirable to investigate the possibility of a radically different type viscometer, which would permit observation of other stresses and flows. In particular it was thought desirable to be able to examine microphotographically the texture of flow over a large region of easily accessible moving liquid. As a consequence we decided to pursue another Newtonian model investigation. After some consideration, it seemed that the most useful device for our purpose would be a rectilinear flow generator. Consequently, we set about designing and studying the performance of what we chose to call an oscillating rectilinear fluid flow generator. An oscillating type was developed for the purpose of studying the rheological properties and flow characteristics of both Newtonian and non-Newtonian liquids.

The generator consists essentially of two long horizontal concentric tubes, in which the annulus between them is filled with the liquid under study. The outer tube is mounted on elastic supports while the inner tube can be harmonically oscillated axially at a predetermined frequency and amplitude. The motion of the outer tube and the resultant force (liquid drag) acting on it are readily measurable at any time.

The principle of the apparatus depends on the fact that the outside tube motion is dynamically coupled to the inside tube motion by the liquid in the annulus which itself is caused to move by the controlled oscillations of the inner tube. It is assumed, at least in principle, that if the motion of the outer tube is know for a prescribed motion of the inner tube, the constitutive equations for the liquid can be dtermined. Or conversely, if the constitutive equations are known, the motion of the outer tube can be calculated for a given motion of the inner driving tube.

A sketch of our model is shown in Fig. 17. It requires a relatively small quantity of liquid for a given experiment. The driving mechanism, which is simple in design, consists of a speed controlled motor, an eccentric, a crank, and a guide. Arrangement was made to attach a variable mass to the spring supported outer tube in order to be able to vary its natural frequency. Properly sealed tube ends, which did not interfere with the motion, were installed in order to hold the liquid without leaks. The entire machine is mounted on a rigid base.

The only experience gained so far with the model has been with Newtonian liquids about which considerable is already known. For this case, it is necessary to obtain the solution of the Navier-Stokes equations subject to the boundary conditions in order to determine the value of the viscosity coefficient. We were able to obtain the solution for viscous liquid between two infinitely long tubes, for the case in which the fluid motion is generated by a harmonic axial motion of the inner tube, while the outer tube is assumed to be supported by springs and moving parallel to the longitudinal axis. The velocity field and shear stress for the liquid were obtained, as well as asymptotic solution for drag force, and tube motion. The coefficient of viscosity for a Newtonian liquid is then readily calculable. The apparatus and experimental data are fully described in the Journal of Applied Mechanics [11].

The coefficient of viscosity was determined for SAE oil 30, for SAE oil 10W, and for castor oil. The value of the latter checked the well-established value in the literature to within 1.3 percent. Before leaving this interesting model it should be pointed out that an unexpected result of the tedious calculations, which required the services of a computer, was the prediction of a sharp discontinuity in the drag-frequency curve. This prediction was later verified with the physical model and an example for SAE 10W oil is shown in Fig. 18.

While at present we do not have results from any study of non-Newtonian liquids with the model, it is known that it provides a clear and controllable field for microphotographic studies of the texture of the flow. Also, the velocity profile in a cross-section can readily be determined. Desirable features of the model are the small amounts

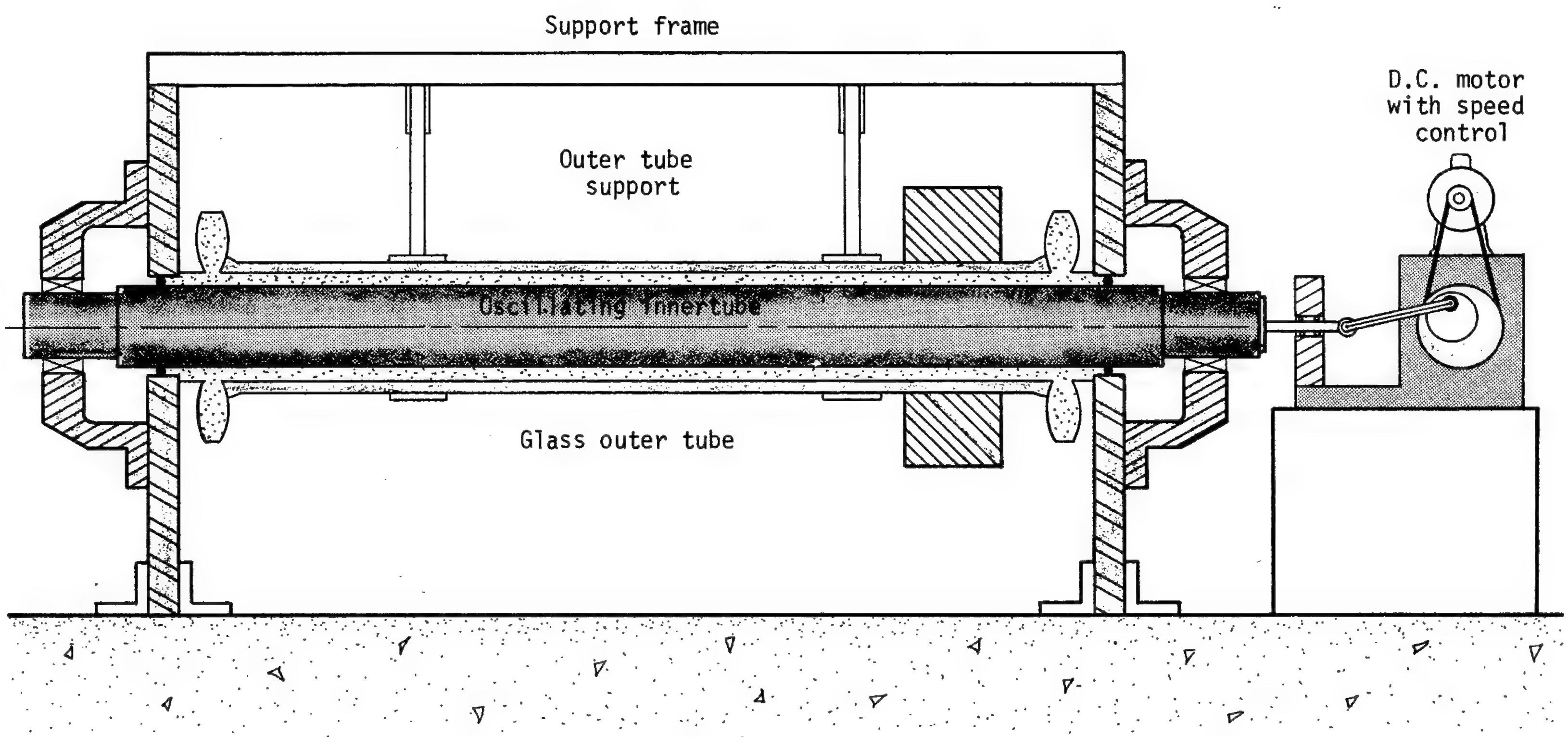

FIG. 17. Oscillating rectilinear fluid flow generator showing liquid between outer and inner tubes.

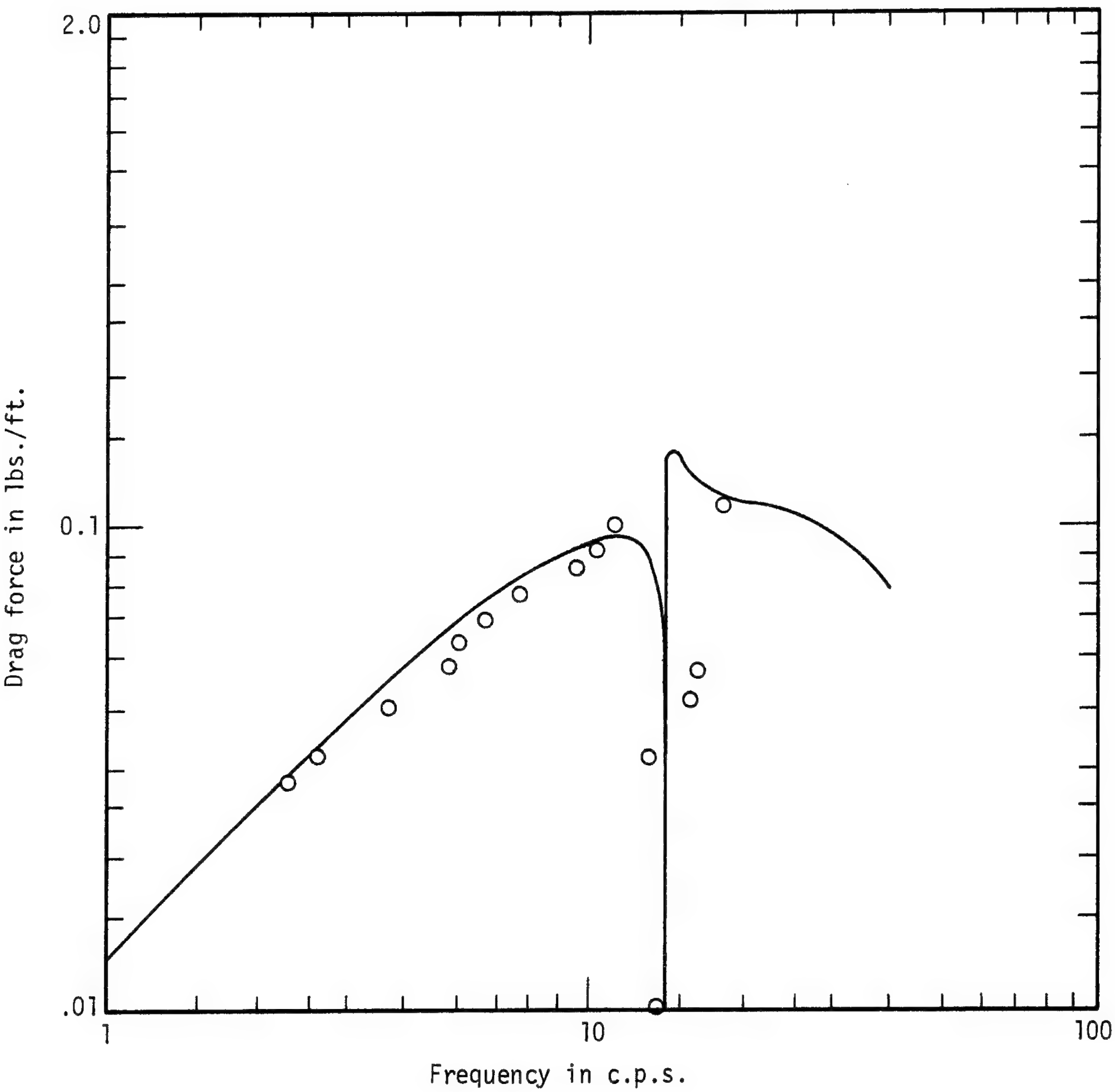

FIG. 18. Drag force on external tube for SAE 10W oil.

of liquid required for experiments, total drag force on the outer tube can be readily measured with strain gages, and the driving characteristics of the mechanism driving the inner tube can be fully determined. The solution of the equation of motion of the liquid subject to the known boundary conditions ties together the kinematic, kinetic, and physical models in the desired manner. The fact that the coefficient of viscosity for Newtonian liquids, obtainable from reliable sources, can be readily checked increases our confidence in the model.

We consider that the examination of the Newtonian model as a device to study the problem of viscometric flows has been carried far enough to show its value and so now we turn to another important flow problem for further study of the methodology.

Drag Reduction by High Polymer Solutions

In a 1948 paper, B. A. Toms reported a peculiar phenomenon of pressure loss reduction for flow in pipes as influenced by the introduction of small quantities of certain high polymers [12]. In a subsequent paper in the Proceedings of the Fourth International Congress on Rheology, A. G. Fabula referred to the paper by Toms and named the phenomenon after him, indicating his precedence in a very important discovery [13]. In his paper, Fabula also referred to similar findings by himself and others for the drag on rotating disk. His report, however, was solely devoted to the question of reduction in pressure loss for water flowing in pipes. Many subsequent papers on the subject by others were also concerned entirely with flow in pipes.

About a decade ago, because of the great technological value of the discovery, we decided to use a Newtonian model to study the phenomenon in connection with a model of a submarine or torpedo. We simply wished to determine any decrease in drag which might result from the injection of a weak solution of a high polymer into the boundary layer.

It was necessary to design and construct a small water tunnel in which the experiments might be conducted. A schematic arrangement of the model and a section of the water tunnel is shown in Fig. 19. For the purpose, the apparatus was made to provide a fully turbulent flow of water. It was also essential to develop a very sensitive dynamometer to properly measure the drag force. This was accomplished so that any drag across the model of the submarine could be readily measured with the required accuracy. The drag was measured both with the water alone and then with water into which a solution of the high polymer was injected. As a consequence it was reliably determined that the drag was really reduced by the low concentration solution to a remarkable degree.

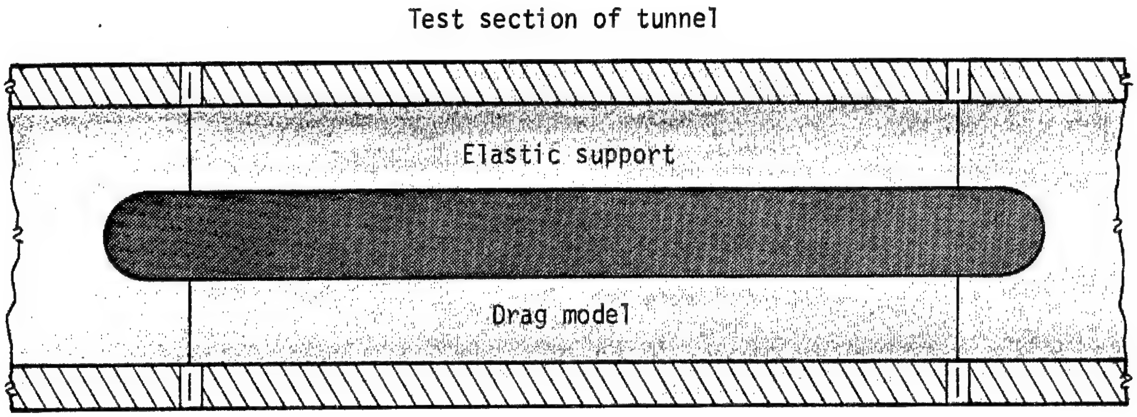

FIG. 19. Schematic arrangement of model and section of water tunnel.

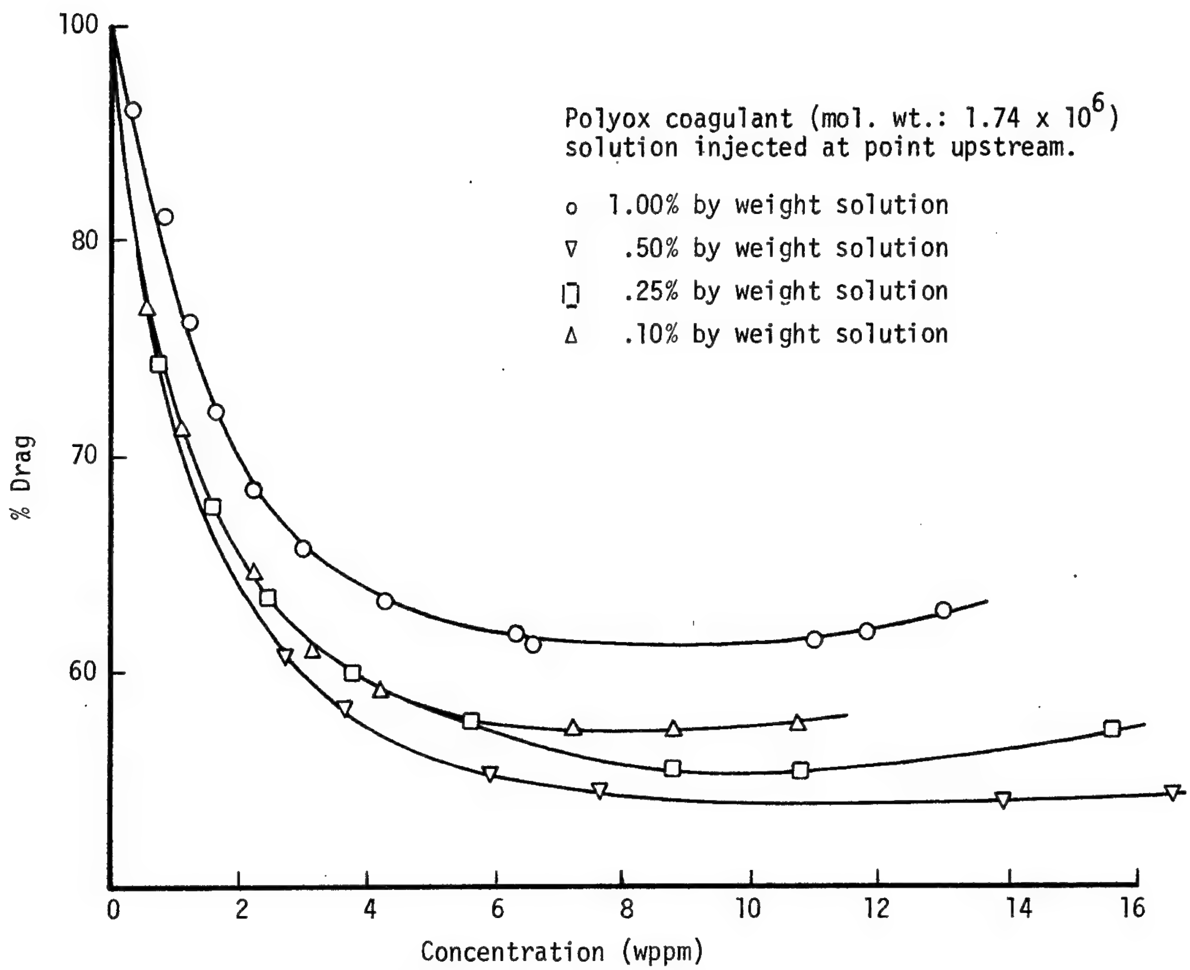

FIG. 20. Drag reduction for model as function of concentration of polymer solution in parts per million by weight.

Some of the experimental results are reproduced in Fig. 20. It is impressive how great the reduction is for a solution which contains only a few parts per million by weight of the solution.

In addition to measuring the reduction in drag, we also took the occasion to examine the reduction of pressure loss in the pipes composing the water tunnel, caused by the injection of a weak solution of the polymer. These results agreed with the findings which are published in the literature on the subject. We have published a complete report on our findings in a publication on engineering science [14].

An interesting aside on this matter are the remarks by S. Jakowska in her speculation that fish may use a mucus secretion for reducing the force which opposes their motion through the water. We have discussed the matter in some detail in the Annals of the New York Academy of Sciences [15].

It must be admitted that a satisfactory explanation of the constitutive laws which relate to the drag reduction have yet to be found. However, it must be admitted that our use of the kinetic and kinematical aspects of the Newtonian model have revealed results of great technological importance.

A final example of Newtonian model experimentation with fluids will now be discussed in connection with the phenomenon of tides.

Tidal Models

Although tidal models are related closely to similitudinous models and, in some sense, could be classified as disclosive, we considered that since they essentially involve the Navier Stokes type flow equations, at least in some sense, we would make them the last example of our Newtonian model experimentation with fluids. Although they are nothing new, having been of interest to hydraulic engineers since the time of the first tidal experiments by Osborne Reynolds, we became interested in both their theoretic aspects and current practical applications.

The first serious tidal model study was made by Reynolds in connection with some flow problems of the Mersey River near Liverpool, England. It was immediately clear to him that the scaling of such models would introduce difficulties that do not exist for those who conduct experiments with models of ships. In the latter, geometrical similarity can naturally be preserved. However, for modeling estuaries it turns out that greatly different scales must be used for the horizontal and the vertical directions. It is obvious that the estuarial stretches are many times greater than the maximum depth of the water. If geometrical similarity were insisted upon, and the horizontal dimensions reduced sufficiently to produce a reasonable model, the depths would be minuscule and hence lead to impractical flow and measurement problems.

In a review of the hydraulic scale models of Reynolds, by A. T. Ippen, some of the details of the original research are presented [16]. In his first model of the Mersey estuary, Reynolds chose a horizontal scale of 1:31,800 and a vertical scale of 1:960, a distortion factor of 33.12. Because of his success with the first model and because of some questions concerning the effect of scale distortion, he built a larger model with a horizontal scale of 1:10,560 and a vertical scale of 1:396. The second model is approximately three times as large as the first. It is not our present purpose to review the accomplishment of Reynolds and the subsequent success with such modeling, but rather to draw attention to its relation to Newtonian model experimentation. We also wish to outline a current project which we have initiated in order to investigate some theoretic questions and to attempt to develop some practical applications in connection with environmental problems.

One of our prime practical interests is a tidal model of very small size in order to study the movement of surface debris and such undesirable materials as oil slicks. If the very small models are reliable to any degree then the effect of tides on the movement of substances, especially on the surface, could effectively be studied with many tidal reversals in a relatively short time interval. The basic reason for this possibility rests on the fact that the tide period varies as the horizontal scale divided by the square root of the vertical scale. On the basis of such a formula, Reynolds estimated the period of his model tide to be about 80 seconds. It is therefore obvious that a very large number of tide reversals in the model could be accomplished in a single day. Such a feature is very valuable for the purpose of observing how the tides move refuse to the ocean.

Notwithstanding the success of Reynolds and of many others with the use of such models, we considered it necessary to first investigate the theoretic meaning of such large scale distortions. The experimental station of the U. S. Army Engineers at Vicksburg, Mississippi has been using scale models of harbors and rivers for many years. However, the sizes of their models are many times larger than what we contemplate. During a visit which we made to that establishment, a director of hydraulic research, H. Simmons, told us that he himself was planning a study on a family of models which are much smaller than those now in use. He wishes to investigate scale effect and, also, determine the usefulness of the mini-model in many present day studies which involve important environmental problems.

In our own program we chose to model a rather large body of water which is readily available for study. It is the system of waterways associated with Winyah Bay at Georgetown, South Carolina. For the restricted amount of laboratory space available to us, we chose two quite small models. The horizontal scale of the larger model is 1:17,000 and that for the smaller model is 1:34,000. Using the formula for tidal period, the period of the smaller model was set at about 50 seconds and that for the larger model about 70 seconds.

It was necessary to develop tidal generators for both models. They are of a plunger and tank type and are driven by motors with thyratron speed controls. A sketch of one of the models is shown in Fig. 21. With the apparatus it is possible to develop tides with the predetermined periods. Depth measurement, as function of time, with improvised transducers enabled us to measure the rise and fall at any point in the models.

The flow in the estuary portion of the model is very interesting, especially in the creeks. It merits a great deal more study. Preliminary studies, also, demonstrated the feasibility of determining the mobility of surface refuse. It is quite possible to measure the surface motion and relate it to that which might be predicted by suitable equations of motion.

Many problems still exist for such a program with tidal models, but our considered judgment is that the Newtonian model methodology again finds useful application in an important field.

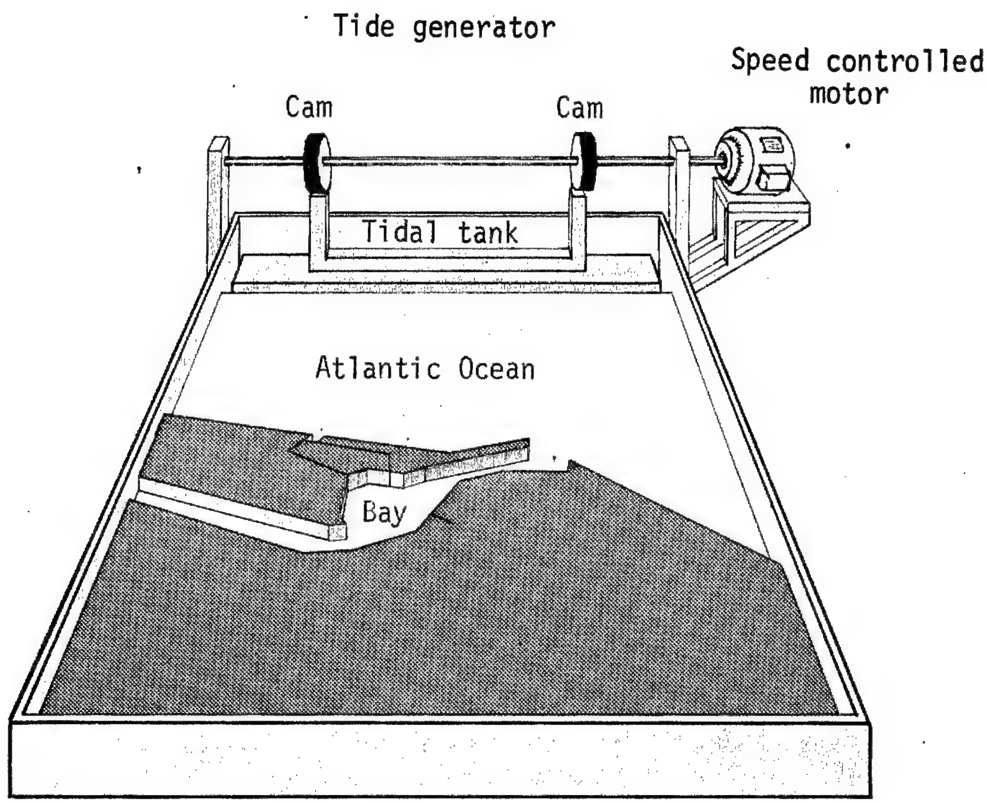

FIG. 21. Small scale model of Winyah Bay at Georgetown, South Carolina.

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CHAPTER 12

ENLARGEMENT OF CONCEPT OF MODEL

It seems to us that the concept of Newtonian model should be enlarged to include other fields of science besides mechanics. In the present chapter we will provide examples of such from the literature. However, before doing so we consider it appropriate to cite at least several examples of what we have been calling Newtonian model, by authors other than ourselves. These are chosen at random and the choice is based on conformity to definition and not necessarily on special merit of content.

The first example is from a paper on pulse propagation in tubes by Goldsmith, et. al., [1]. It contains the use of equations which are based on Newton's laws of motion as well as discussion of impact experiments based on those equations. Specifically, longitudinal impact experiments were conducted on aluminum tubes as models, using the so-called Hopkinson bar method. Impulses were impressed longitudinally on the ends of the tubes by collision with steel spheres. Dynamic strains were measured on the surfaces of the tubes. Comparisons of theoretically and experimentally determined strains are presented as functions of time for several points along the models. One of the authors, W. Goldsmith, previously published a textbook on impact, which contains a bibliography of over four hundred references on the subject [2]. We consider that that work provides a large number of illustrations of what we have called the Newtonian model. Furthermore, the contributions to technology of such a model approach is amply demonstrated.

A similar impact study was made by Mortimer et al [3]. They investigated the longitudinal impact of cylindrical shells which have cross-sectional discontinuities. Their theoretical work is based on the Newtonian equations of motion. Here again are presented calculated results for strains at given locations as functions of time. Impact experiments were conducted on two models which were dynamically loaded in the longitudinal direction by projectiles mounted on a pendulum. Comparisons between theory and experiment are provided. Again, as for the previous paper, interesting and technologically useful results were obtained.

In the field of plate vibrations, a study by Kung and Pao illustrates the use of the Newtonian model [4]. Circular plates with clamped edges were oscillated transversely with an electromagnetic drive to various finite amplitudes and at predetermined frequencies. The non-linear response curves showing amplitude to thickness ratio as function of drive frequency to linear fundamental frequency are given.

A final example chosen from the field of the mechanics of solids is described in a paper on the resonance of stiffened plates by Duffield and Williams [5]. They developed a theoretical analysis based on the Lagrangian form of the equations of motion. The principal interest of the authors was centered on the parametric instability of simply supported stiffened rectangular plates subject to in-plane sinusoidal dynamic forces. A study was made of the effect of stiffeners on the instability of the plate. Experimental and theoretical results are compared and as a consequence the authors claim that excellent predictions for onset of parametric instability can be obtained.

The general theory of vibrations, with applications to plates, can be found in the well-known reference by S. Timoshenko [6]. Here we have a very useful exposition of that subject, accompanied by many references to the technical literature. Much of the work of this justly famous man is closely allied to the concept of the Newtonian model.

A similar example can be taken from the field of statics of materials. For this purpose consider the collected experimental papers of P. W. Bridgman, the distinguished physicist [7]. The accomplishments of the lifetime effort of this great man will again reveal the incisive value of the Newtonian model as we have defined it. The six volume work of Bridgman contains reproductions of nearly two hundred of his original papers.

We have been describing several cases of our subject as treated by others working in the field of solids. We will now cite some pertinent references from the field of fluids. For the first case we select a paper which provides a very interesting study by Vanyo and Likins on liquid-filled precessing spherical cavities [8]. Their research is based on the Navier-Stokes equations for a viscuous, incompressible fluid in a spinning and precessing spherical container. To obtain the motion experimentally, spin- and precession-motors were used. The support frame was considered to be a Newtonian or inertial frame. Many conditions of operation were investigated both with water and with silicone products as the fluid. The findings were considered to be of particular interest to the space industry.

Another paper which may be cited is one by Novotny and Eckert on viscoelastic liquids flowing between parallel plates [9]. A simple experiment was devised in order to study the constitutive equations for certain non-Newtonian liquids. Various polymer solutions were of interest. The physical model for studying the flow consisted of two parallel plates arranged close together in such a manner as to form a channel through which the liquid was constrained to flow in an approximately steady-state condition. Several transducers were used to measure normal forces. During the investigation attempts were made to measure both shear and normal stresses. As is well known by rheologists, normal stresses play a significant role in the motion of such liquids. The authors emphasize the fact that constitutive equations for these liquids are complex. They do

consider, however, that their model represents progress in a most difficult area of research. Again we see a Newtonian model used effectively to study the flow of non-Newtonian liquids. It exhibits the theoretical and experimental features which are always associated with the type of model. The constitutive equations, which are of interest to the authors are grounded in the Newtonian mechanics.

The general theory of fluid mechanics, with application in many fields of technology, can readily be examined in the encyclopedia work which was produced under the editorship of V. L. Streeter [10]. Most phases of the subject of fluids are treated in that work in such a manner that the reader can readily see its relationship to what we have called the Newtonian model.

A very penetrating analysis of the fluid problem as related to the broad subject of rheology is presented in an extensive work by Truesdell and Noll [11]. In addition to providing a clear analysis of the subject, stressing the role of the constitutive equations, the authors provide a truly extraordinary bibliography. With appropriate insight one can readily discern the modelistic aspects of the subject.

Although we have treated a number of problems related to the vibrations of solids, we have not specifically mentioned the technologically important subject of acoustics. As is well known, that subject deals with oscillatory phenomena in gases, liquids, and solids. Therefore, inasmuch as Newton's laws of motion find direct application in that field and physical models are used extensively, we shall include it, along with mechanics, as an obvious exemplification of the Newtonian model. While we will not present here the usual equations associated with acoustics in order to demonstrate their relationship to the Newtonian equations of motion, we can refer the reader to an excellent expository paper on the subject of acoustics by Morse and Ingard [12]. Another long paper which considers some important applications of the subject is by Truell and Elbaum [13]. Both of these papers are contained in a two volume treatement of the subject, in the <u>Encyclopedia of Physics</u>, which may be considered definitive even if not exhaustive. Futhermore, as is characteristic of that great encyclopedia, there is a large number of references to important technical papers in the extensive literature on the subject.

In addition to the encyclopedic material, we would like to cite some of the research by a pioneer in the application of the principles to the acoustical design of auditoriums. For many years V. O. Knudsen has devoted his effort to the development of the field of architectural acoustics His life's work is exemplified by one among several papers in a recent symposium on the subject in the <u>Journal of the American Acoustical Society</u> [14]. In the article, Knudsen clearly demonstrates the forceful use of the Newtonian model concept in the study of buildings requiring very careful specifications of acoustic characteristics. We concede that the bulk of the interest shown in that work seems to concentrate on such

matters as reverberation time, sound sources, echos and the like. However, it should be emphasized that all of this technology is based on the wave phenomena associated with the Newtonian equations of motion.

Having presented examples of the Newtonian model used by various authors in the fields of solids and fluids, as well as demonstrating that acoustics may rationally be classified with such applications of mechanics, we will now outline a case for the enlargement of the concept of Newtonian model to include all of the subjects arising in the field of physics as well as in those subfields which depend on physics. It seems appropriate to call such cases extended Newtonian models, acknowledging that such a designation is purely technical and limited to problem areas that have to do only with physics. At this point it may be clarifying to state that further extension of the model concept is contemplated, but in areas so radically removed from physics that we shall reserve for these the classification disclosive model. In the meantime it behooves us to review briefly the ground from which we contemplate present day physics. In order to accomplish this, it is at least convenient, if not essential, to use as a reference a truly astonishing work produced recently under the aegis of the National Research Council of the National Academy of Sciences. The results of that arduous task, undertaken by many prominent physicists, are contained in an impressive three volume study which has been recently published [15]. The first volume puts in perspective the field of physics as it is conceived at present by leaders in that field. The second volume, called volume II, Part A, deals with what are called the core subfields of the general subject. Finally, the third volume, called volume II, Part B, treats the interfaces between physics and other important scientific areas which fundamentally rely on the basic physical principles.

It may be useful for our purpose to review briefly the domain encompassed by the subject of physics. It is instructive to see such a classification in terms of the entities composing it and the scale of dimensions involved. All will agree that the largest entity in the list is the metagalaxy. It has a diameter which is said to be 10^{25} centimeters. Following in order in magnitude is the diameter of our galaxy, the distance to the nearest star, the distance to our sun, the radius of the earth, man, the cell, the atom, and the atomic nucleus. The order of magnitude of the smallest item in the list is 10^{-10} centimeter. As is easily seen, one may conveive of the magnitude of anything, such as that of a large mountain, fitting properly at some point in the list.

Some of the nineteenth century textbooks showed that physics of that period consisted of mechanics, acoustics, heat, light, optics, electricity, and magnetism. The beginnings of serious studies of atoms and molecules came in the latter part of that century.

One could take as the basic defining characteristics of physics the force and force fields. The physicists say that the basic forces are four in number: gravitational, electromagnetic, weak nuclear, and strong nuclear. The field of the gravitational and the electromagnetic forces extends to infinity. The weak nuclear force exists deep within the particle, which has not yet been discovered. Finally, the strong nuclear forces operate within the small radius of one fermi.

The gravitational force surrounds all matter in all directions to infinity. It controls the stars and the galaxies. The much stronger electromagnetic force cancels out at long range, since there are equal numbers of positive and negative charges in the universe. It controls the world of atoms and molecules. The weak nuclear force is known to exist, but its carrier has not yet been detected. The strongest force, the strong nuclear force, controls most effects in the compacted nuclear and subnuclear world. Our basic reference, the NAS-NRC Report, gives credit for the form of the foregoing classification of forces to the World Book Science Annual of 1968.

The sciences, besides physics, that use this notion of force and force field may be grouped broadly in four divisions: a science of physical substances, including chemistry; life sciences; generalized astronomy, including earth sciences; and engineering science. The NAS-NRC Report further states that chemistry may be regarded as a study of complex systems dominated by electrical forces. It also says that quantum mechanics and electromagnetic theory, as now formulated, provided a complete theoretical foundation for the understanding of the interactions between atoms and molecules that characterize chemistry.

The extension of classical mechanics to include relativistic considerations and quantum dynamics, provides the fundamental basis for research in modern physics.

Our principal interest in the present diversion is simply to make definite the basis for our introduction of the concept extended Newtonian model. We will give some specific examples of such models taken from the engineering literature, but first it may be useful to provide a description of present day physics and its applications taken from the NAS-NRC Report. It is as follows:

From Vol. II, Part A, The Core Subfields of Physics

1. Elementary - Particle Physics

2. Nuclear Physics

3. Atomic, Electronic, and Molecular Physics

4. Physics of Condensed Matter

5. Optics

6. Acoustics

7. Plasma Physics and Physics of Fluids

From Vol. II, Part B, The Interfaces

8. Astrophysics and Relativity

9. Earth and Planetary Physics

10. Physics in Chemistry

11. Physics in Biology

12. Instrumentation

It may be well to make a distinction between two aspects of our problem of models as related to the field of physics. First there is the ever expanding forefront of fundamental knowledge and then there is the field of physical systems to which the principles apply. An example of the former is the Bohr atom and of the latter the turbulent flow of matter in a galaxy. An interesting study could be made of the evolution of physical concepts from antiquity to the present, but that would be outside the scope of our present development of the concept of model. An interesting reference, however, can be made to reflections on the matter by a scientist who was truly one of the great ones. He is Henri Poincaré who has provided a profound study of the nature of the physical sciences [16]. Peculiarly he distinguishes the Latin mind from that of the Anglo-Saxon. He says the former finds it possible to express truth only by mathematical equations while the latter uses physical models which are crude and constructed of commonplace materials. Unfortunately we cannot pursue such a psychological thesis but we can say that Poincaré sees a close relationship between the two viewpoints. Our own position is that the complete model includes both the equations and the physical paradigm.

It may be noted that heat is not explicitly mentioned in the NAS-NRC compartmentation of physics. Notwithstanding it is true that the theory of heat and thermodynamics are of paramount importance to the physicist and he considers them in connection with all of the subfields. A noteworthy one is that which he designates "Physics of Condensed Matter," which we will now examine. For the purpose we will give some examples of research in heat transfer conducted by engineers in recent times. They exemplify aspects of physics which lie outside that which is susceptible to treatment by the strict Newtonian model. They are logically included under the extended Newtonian model.

The major topics treated in the classical theory of heat are conduction, free and forced convection, and radiation. Our examples cover all of these phases. One of special importance concerns the flow of heat across the areas of contact of two bodies. Such cases are complicated by the indeterminableness at the area of contact. Experimental models are increasingly required for the generation of knowledge concerning such phenomena.

The first thermal problem we wish to cite is a contact problem treated in a paper by Mikic and Carmasciali [17]. It deals with the thermal contact resistance between two bodies pressing against one another. For the purpose they used solid cylinders as physical models. One of the cylinders, of a given material, was pressed longitudinally against the other cylinder, of a different material, with various platings at the interface to reduce thermal contact resistance. The interface forces were measured and the hardness of the plating material determined. The field equations for the theoretical analysis are the classical heat conduction equations and are obviously different from the Newtonian equations which we considered with our strictly Newtonian models. Nonetheless they play a similar role in the theory of heat conduction to that played by the Newtonian equations of motion in classical mechanics. The authors used physical models for the study and provide a comparison of experimental results with those obtained from solutions of the equations. Results of such investigations are known to be of value for applications in technologically important problems.

A significant free convection problem which represents another of the modes of heat transfer was studied by J. W. Yang, et. al., [18]. They investigated the heat transfer to a laminar fluid from a plate which was subjected to sinusoidally time varying temperatures. For such an example the equations of motion of the fluid are of the Newtonian type but the heat transfer equations are not. Hence we refer to this type as an extended Newtonian model. The authors provided a physical model with which velocity profiles and temperature profiles were determined. As a consequence a considerable amount of useful data has been provided for an important heat transfer problem, again demonstrating the effectiveness of model studies.

Another technologically important study of free convection may be found in a paper by J. J. Noble, et. al., [19]. It concerns the circulation patterns in molten glass which occur during the production process. The field equations for temperature and velocity are used in the investigation of a small scale physical model using glycerine as a modeling fluid. Thermocouples were used for the temperature measurements and streak photography for the velocity field determinations. Comparisons between theoretical and experimental results are presented. Again, the model, which we are now calling the extended Newtonian model, proves its value for developing the art of an important industrial process.

Another interesting and important study of a free convection problem was made by Gentry and Wollersheim [20]. They were concerned with the determination of the heat transfer rates from isothermal cylinders to non-Newtonian fluids. Momentum and energy integrals were explicitly used. With the physical model free convection data were obtained with five different liquids which consisted of water and four aqueous solutions of carboxypolymethylene. Temperature measurements were made with copper-constantan thermocouples in the fluid and teflon insulated copper-constantan thermocouples in the aluminum walls of the cylinder. The Fourier conduction equation was used to determine surface temperatures. Velocity profiles for the fluids were determined with motion picture photography using a dye streak technique. Integral solutions are compared with experimentally determined results.

A third important mode of heat transfer is radiation and we wish to offer as an example the work reported in a paper by Chan and Tien [21]. They studied the radiative transfer of heat through a packed bed of microspheres. They claim that a new model is provided by them for studying such thermal problems. As for all of the problems we are considering, mathematical equations as well as a physical model are used. The physical model consisted of a slab of identical metalcoated spheres packed in a simple cubical arrangement. A considerable amount of analysis and data are presented by the authors. An important part of their work is the determination of the scattering diagram of a unit cell, the optical properties of a series of thin microsphere layers, and the solution of the appropriate two-flux equations. An important finding is the strong dependence of the radiative properties on the particle diameter and the emissivity. Reasonable agreement was established between calculations and experimental data.

A serious engineering problem associated with heat transfer was studied by E. J. Thorgerson, et. al. They used a model, which involved both conduction and convection to predict so-called convective subcooled critical heat flux. The results of the research, given in a brief paper in the technical literature [22], are based on a lengthy doctoral dissertation by Thorgerson. The authors define the problem as one of prediction of burnout or critical heat flux in forced convection boiling In more general language we may say that the principal objective is to obtain reliable criteria of damage to metal tubes which are used in heat transfer from nuclear sources. The effective use of the extended Newtonian model analysis is clearly demonstrated, especially in the more lengthy dissertation.

For example of a very complex flow problem, which involves considerations of heat transfer and thermodynamics, we refer to a recent paper by Pletcher [23]. While he limits his treatment to the sphere of mathematical analysis, he does rely heavily on experimental results obtained by others. His use of the word model applied to the mathematical aspects alone. As the title indicates, the investigation is concerned with what the author calls prediction of transpired turbulent boundary layers. In

looser general language, we may say that we are interested in the turbulent boundary layer between a fluid and a solid surface through which fluid may be blown or sucked. The author deals explicitly with the momentum, energy, continuity and state equations, along with the appropriate specified boundary conditions. It is not our purpose to go into much detail in the papers we are using as examples, but it does behoove us to urge the reader to peruse the present paper to get a strong impression of the use of the extended Newtonian model in the analysis of a serious turbulent flow problem which involves heat transfer.

Before leaving our project of providing examples of what we call extended Newtonian models we wish to consider a bioengineering study which lies in the transition region between the extended Newtonian model and the much more general field of what we will call disclosive models. The study is reported in a paper by Birkebak et al on Thermal Modeling Applied to Animal Systems [24]. To master the formidable task of estimating the exchange of heat energy between animal and environment, the authors correctly imply that the problem must be studied with the basic heat transfer equations, using know material properties. Their heat transfer studies were made both with birds and small mammals. The bird chosen for study was the cardinal and the mammal was the shrew. The authors readily admit that their analysis is a great oversimplification of the problem. Notwithstanding that, we are of the opinion that the study represents a valuable start on the problem of heat transfer as related to animals and their environments. The reader is urged to examine the paper for some insight into the possibilities of gaining biological knowledge by using physical models.

By now we hope that we have developed a sufficiently strong argument in favor of our proposed concept of extended Newtonian model. It should be clear that the five categories of models, which we have so far proposed, cover a large area of human knowledge. Again we repeat, those models are the iconic, the analogic, the similitudinous, the Newtonian. and now the extended Newtonian. Despite the use of this fairly large number of categories, it is not sufficient for all purposes. There are other types which do not fit readily or do not fit at all in our present scheme. As we pointed out in the beginning, the generalized model should include all regions of human interest. We therefore find it necessary to anticipate a more generalized concept than what we have used so far. The additional category we shall call disclosive model. Undoubtedly as time passes and one becomes more familiar with models, the disclosive model itself will divide into more specific forms. Our further study will be concerned with such generalizations.

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CONCEPT AND USE OF MODELS

PART II

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CHAPTER 13

DISCLOSIVE MODEL AND THE DEVELOPMENT OF KNOWLEDGE

In chapter three on the critique and classification of models we introduced the disclosive model as the last of six classes. The first five, the iconic, the analogic, the similitudinous, the Newtonian, and the extended Newtonian we consider to be well-defined and thoroughly understood. The disclosive type will now be examined in considerable detail, especially because of the great importance which we attach to it. As the reader may recall, the term disclosive was introduced from Models and Mystery by Ramsey [1].

The five classes which we consider sufficiently well understood cover such things as city maps, membrane analogy, towing tank models, dynamic structural models, and free convection thermal models. A very large class of models is outside the scope of such models. We require a specific designation for the purpose of classification so we have decided to use the expression disclosive model. We fully realize that while Ramsey did not consider systematically the five types with which we begin our taxonomy he did appreciate the need for a special term to cover the models in his own discipline, which was theology. As we have mentioned before, it is also apparent that because of his particular background he did not see the need for a more extensive classification. One conversant with many diverse disciplines knows that a large number of distinct classes is required.

In the present chapter we wish to consider some of the reasons for requiring a sixth category. Not only have we not considered problems peculiar to theology in our own treatment so far, we also have not considered proper classification for models in modern physics, chemistry, biology, and many other important current fields. All of these we plan to treat under the general designation of disclosive models. It can be appreciated, with some reflection, that the need arises from the fact that mankind is still on the frontier of discovery and knowledge with regard to all such fields of intellectual activity. Knowledge associated with these problems is evolutionary. Nascent knowledge is characteristic of man's striving to progress. We wish to exemplify the present state of learning with specific examples from many different disciplines. This we will do both in the present as well as in future chapters. However, we first wish to consider some of the most prominent features of the disclosive model. In doing this we do not intend to repeat our general treatment in chapter three.

As Ramsey so cogently points out, a model should provide a means for dialogue and develop knowledge. While this remark applies also to models in our first five classes, it is peculiarly important for models

in the sixth class. It may be useful to review briefly some of the things that Ramsey said with respect to his field of theology so that we may gain some appreciation of the more general problem.

While we propose the term disclosive, Ramsey actually used the term disclosure. He said, "Now it is with these models which Black calls analogue models and I call disclosure models that contemporary science is most concerned; ...." Ramsey further says that disclosure is at the heart of every model. Disclosure embraces phenomena and their associated model. He also sees that a model is better the more prolific it is in producing deductions which may be experimentally verifiable. We may add that Ramsey considers that there is a far-reaching parallel between models in science and models in theology.

Little reflection is needed to arrive at the conclusion that the disclosive model in physics is mathematical while in theology it is non-mathematical. Such a distinction will be manifest in our further development of the subject. We may emphasize that our first five classifications are mathematical with the possible exception of some aspects of the iconic model. We may recall that the derivation of the word iconic is associated with the term image which at times may have a religious connotation.

If we agree that disclosive implies a basis for dialogue and the development of knowledge, one can readily see that our first five classifications have these characteristics also. The meaning of this obviously resides in the fact that the expression disclosive model is probably generic and includes all sub-classes of models. Notwithstanding this fact we shall reserve the term, at least for the present, to distinguish models of the first five classifications, which are well-defined, from any other kinds. The specificity of each distinct type of model arises from special constraints placed on that type. For example, the analogic model, as we defined it, is such that identical mathematical equations describe the performance of two radically different physical systems under consideration. This is a severe constraint or requirement placed upon such models.

A very special aspect of modeling arises from the science of mathematics. In the next chapter we shall attempt to clarify the use of the concept of model within the subject of mathematics itself and also, in those disciplines which allegedly use mathematical modeling.

We wish now to discuss the aspect of model which Ramsey associated with mystery. He claims that the model enables one to reduce mystery in religious matters. The same can obviously be said also of physical science, at least that part which is at the forefront of knowledge. We assume that the reduction of mystery is the same as the increase of knowledge. In contrast with this aspect of model is the fact that models of our first five classes generate information rather than knowledge

which is the reduction of mystery in the religious sense. For example, the similitudinous model, which is exemplified by the towing tank or wind tunnel, enables us to calculate the performance of the prototype from measurements on the model in a very systematic and unerring manner. It is true that for a radically new design of ship or airplane there is a great mystery about its performance until experiments with models are made, however we are assured in advance that when a well-known plan of model testing is conducted all of the mystery will be removed. Such assurance does not exist on the forefront of physics or in the area of the most profound problems of theology. In process theology, for example, the notion of a god who apparently develops in an evolutionary sense given rise to some tantalizing paradoxes. Besides modern physics and theology there are other disciplines that utilize models in a systematic manner. During World War II the advent of the electronic computer and operations research led to the rapid development of many activities that hitherto made little use of the power of modern mathematics. Systems analysis and decision making began to apply advanced mathematical methods and the term model became commonplace. The root of their method is mathematics. To illustrate some of the thinking associated with the newer use of models we refer to a book by R. L. Ackoff and his co-workers [2]. In particular we call attention to their concept of model as expounded in chapter four of their book. It will be found that they classify models as the iconic model, the analogue, and the symbolic model. It may be remarked that their use of the term iconic is the same as ours. Also, their analogue is quite similar to our analogic model, but without the clear-cut stipulation concerning the role of the defining mathematical equations for the physical systems which are said to correspond. They do not introduce the similitudinous model at all and have no apparent reason to recognize what we have called the Newtonian and the extended Newtonian models. The symbolic model plays a fundamental and exhaustive role in their brief taxonomy and it is this model which we wish to consider now in some detail.

The Ackoff school states that in symbolic models the properties of the thing represented are expressed symbolically. They further state that the mathematical equation is a symbolic model. Also they definitely enunciate the proposition that models in which the symbols employed represent quantities are usually called mathematical models.

In order to emphasize their thinking about symbolic models we wish to quote directly from their book. They say, "Models of problem situations, as we have discussed them, should always take the following special form:

$$V = f(X_i, Y_j)$$

where V = the measure of the value of the decision that is made

X_i = the variables which are subject to control by the decision; the decision variables define the alternative courses of action

Y_j = the factors which affect performance but which are not subject to control by the decision maker

F = the functional relationship between the independent variables X_i, Y_j and the dependent variables V."

The authors also refer to this symbolical model as a decision model. Simulation is an important term for their methodology. Simulating seems to be a systematic method which they use. They go further and say that a simulation in which decision making is performed by one or more real decision makers is called gaming. They also point out that gaming has come into increasing use, particularly in the study of complex military and industrial operations. Furthermore, it is now beginning to be used in the study of governmental problems at the municipal, national, and international levels.

We have reviewed enough of the work of Ackoff and his co-workers to indicate some of the directions for the development of the concept of model at the present time. Other authors and researchers could equally well have been used as references for our present purpose. The technical journals proliferate with analysis and discussion of subjects which belong in this category of what has been called symbolic models. In future chapters we will examine these matters further.

Nothing said so far in the present chapter leads us to add to our taxonomy of six classes of models. In subsequent chapters we will deal specifically with what has been called a symbolic or mathematical model. Mathematics plays a very special role with respect to all classes of models to which it applies. It is conceivable that there are models to which it may not apply. It probably does not apply to such fields as theology, cosmological evolution in the large, ethical systems, world history, the theory of psychological types, and so forth.

Disclosive models, in addition to being essential to physics and theology, are also important to the economical, industrial, military, medical, legal, governmental, religious, esthetical, psychological, educational, literary, and biological sciences. There are probably many more fields which should be included in this list. In recent years such fields as biophysics, biochemistry, bioengineering, ecology, industrial management, urban planning, aircraft handling, and dozens of others have developed at surprising rates and demonstrated the important role of the model and the science of modeling. All of such models fall within the category which we at present term disclosive model.

Having now added the class of disclosive model to our original set of five classes to bring the total to six, we wish to discuss a very important aspect of model taxonomy. It may be recalled that in chapter three on a critique and classification of models we discussed the taxonomy suggested by Mihram [3]. The columns of his matrical representation we criticized adversely, although the rows we accepted as legitimate. His six columns and four rows provide twenty-four categories of models. It might seem that our own formulation in terms of six classes and our acceptance of Mihram's four rows gives us also twenty-four types of models. However, as we said before and repeat now, Mihram's classification seems vastly over formalized for the present state of the science of modeling. Also, for us to insist on his four rows along with our six columns seems to involve a great deal of awkwardness and possible ambiguity. Instead of assuming a rigid matrix for the specification of our model classification we will go along with our six categories and consider them as affected by the four items which constitute Mihram's matrical rows. The latter are designated as statical, dynamic, deterministic, and stochastic. These important four concepts we will treat at length in later chapters. We agree that they apply in some sense to all models. However, their full consideration with respect to some kinds of models, say for example the iconic, may be meaningless.

As we have already emphasized, mathematics is essential to the concept of model and the science of modeling. Nevertheless we find that a considerable number of ambiguities arise and these almost exclusively with regard to the disclosive model. In the next chapter we wish to examine the use of the model within the science of mathematics itself and, also, the concept of mathematical modeling. After we study the relationship of mathematics to the concept of model, we then wish to devote the remainder of the book to the explicit analysis of many kinds of applications of the disclosive model. It is hoped that on the basis of such a study we can effectively recommend a course for the future use of the model in attempts to solve the many problems which crowd in upon us at the present time.

We wish to end the present chapter with a discussion of what we may call a disclosive model which at first sight appears to be simply iconic. It concerns organizational charts, but those which are evolutionary. They are used primarily in groping towards important operational processes. An example of what we have in mind can be found in the Novemeber 1974 issue of <u>Physics Today</u>. In that issue there is a very brief but important article entitled <u>Approaches to a National Science Policy</u> [4]. Its main thrust is contained in an evolutionary set of organizational charts of the governmental departments which have had concern with research and development programs for a long time. The first chart is the organization of the various departments such as defense, education, agriculture, health, and so forth, under the Cabinet which in turn operates under the direction of the President subject to the controls of the Congress. This chart the authors refer to as a plural-

istic model. It is the old arrangement of governmental departments and in recent years it has come under criticism from many quarters because it seems to be rather ineffective for the conduct of vigorous research and developmental programs in science and engineering. It is what we would call an iconic model because its purpose seems to be simply a guide to the arrangement of departments under the cabinet.

In order to quickly outline the evolution of the organizational chart toward optimal functioning, which by its nature falls into our category of disclosive model, we will refer directly to the article. It is pointed out that in the "pluralistic" system, common in many countries until the 1950's, finanical resources were assigned to each autonomous government R and D sector which then made its own independent decisions. Now universally rejected, such a plan produces a wasteful competition among the branches. The authors state that alternatives include the "coordination model", in which an advisory agency links the individual science policies together somewhat weakly, and the "centralized model", in which a strong centralized agency is supposed to exert complete control over all government scientific activities. They suggest that a so-called concerted-action model would combine the best features of the two extremes, with individual agencies developing specific science policies, complemented by some kind of strong and effective central organization.

A similar discussion is presented for the United Kingdom and for France. In addition to charts which correspond to the four models mentioned above, there are three additional charts which show the national organization of science and engineering, both within and without the governmental departments of the United States, the United Kingdom, and France. As is well-known, all organizations in the United States come under the President; in the United Kingdom and in France they come under the Prime Minister.

The authors of the article state that the information which they provide is derived from "A Science Policy for Canada", a report of the Canadian Senate Special Committee on Science Policy, "A Government Organization for the Seventies". It can readily be seen that such an organizational study and its implications relates to models which are far removed from the simple iconic type. There is an obvious dynamism present in the whole activity which is directed toward the optimization of scientific and engineering R and D programs. In the United States the Council on Economic Policy and the National Science Foundation have a decided impact on the structure of research and development funding.

For the present we have completed our definition and discussion of the disclosive model. In subsequent chapters we will return to its use in many different applications.

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CHAPTER 14

BRIEF HISTORY OF THE MODEL IN MATHEMATICS

As in all intellectual activities, the notion of model has been present in mathematics since its early inception during the long course of man's evolution. During the development of the various cultures in the world the idea of model grew stronger and clearer. It was used more and more consciously with the passage of time. It seems, as we have remarked earlier, that modeling is essential for thinking. Now in the latter half of the twentieth century the scholar becomes increasingly aware of the essential role of the model in the development of knowledge. The fact is particularly clear in current mathematical thought as in all other branches of science.

As with history in general so with the history of mathematics in particular, one may follow the development of the concept of model since the time that man began to develop his consciousness. We go back to prehistoric times in order to obtain the first glimmer of the idea. In the present chapter, we wish to outline briefly some of the history of mathematics in order to obtain a clear view of the long and rather slow development of the concept of model in that discipline. As we have said on previous occasions, we are not historians and hence not competent to provide a complete and fully appropriate history of mathematics as a whole or of its particular branches. However, we do consider it essential, for our purpose, to again examine the historical side of the subject of models. This time it is in the context of mathematics. We leave to others, more competent than ourselves, the intricate story of the complete development of the history of the idea of model from the earliest times until the present. In the meanwhile it is most important for us to sketch as effectively as we can the history of the idea. It is clear to us that an awareness of its growth is essential to the proper understanding and use of the concept of model in the last quarter of the twentieth century.

As we have previously suggested, the notions of analogy, similarity, homology, and one-to-one correspondence are vital to the understanding of the concept of model and its conscious use in the development of knowledge. Anyone at all familiar with mathematics knows that these terms are part and parcel of the stuff of which that subject is made.

Although a useful insight into the history and development of mathematics can be obtained from such works as the Encyclopedia Britannica, we shall depend for our study on texts by Carl B. Boyer [1], M. Kline [2], Florian Cajori [3], and David Eugene Smith [4]. The first two volumes are recent treatments of the history of mathematics from the earliest times up to the most complex recent developments.

The latter two are well-known old references on the subject from the beginnings up to the nineteenth century. We have no intention of dealing in critical comment of an historical nature or of trying to assess priority of claims with regard to the various races of man. It is well established that there were important developments in India, China, Egypt, Mesopotamia, Babylonia, Greece, Rome, and the Arabic countries long ago. In order to emphasize the concept of model in the total growth of mathematics we will freely use the histories which are at our disposal. It may be well to note at this point that the two more recent references above contain excellent and lengthy lists of references on mathematics, including such gems as the contributions of Professor van der Waerden. It was our privilege to know van der Waerden when he was a visiting professor at the John Hopkins University years ago. There one could observe his dedicated and competent treatment of the history of ancient parts of the subject. It might be added for those who do not know that van der Waerden was, also, a superb mathematician.

Two things were characteristics of very eary man - his propensity to count and to draw. It is known that he used pebbles to count and lines on a surface to represent figures. He made simple calculation and produced primitive drawings. According to our taxonomy of models, both the pile of pebbles and the configuration of lines are iconic models. They are imagery and representations. They are, also, very useful in various activities. In the sense that they permit dialogue and help develop knowledge we may also classify them as disclosive models.

It is well-known and significant that the early number-system was based on the scale of ten. A little thought makes the reason for this fact obvious. The man of prehistoric times undoubtedly counted on his fingers and maybe, also on his toes. The child of today, as well as certain aborigines, does the same. Evidence attests to the use of the fingers for counting among the ancient Egyptians, Babylonians, Greeks, and Romans. Aristotle, in his day, commented that the widespread use of the decimal system is but a result of the anatomical accident that most of us are born with ten fingers and ten toes.

The use of the counting process demonstrates the awareness of the meaning of that all important mathematical concept — one-to-one correspondence. It finally led, in the twentieth century, to the definition of cardinal number as the class of all classes equivalent to the given class. Here the use of model is apparent because the process involves comparison, analogy, and one-to-one correspondence. Two classes are compared to determine whether they do or do not have the same cardinal number.

Besides the cave drawings of prehistoric times, there is another peculiar example of geometrical configuration. It arises during the era of primitive man. David Eugene Smith says, "A further prehistoric

stage of mathematical development is seen in the use of such simple geometric forms as were suggested by the plaiting of rushes, the first step in the textile art." He further says, "The geometric ornament, however, became in due time a favorite one among nearly all early people." Finally he says with respect to pottery, "The early pottery of Egypt and Cyprus shows very clearly the progressive stages of geometric ornament, from rude figures involving parallels to more carefully drawn figures in which geometric design plays an important part and in which the mystic symbols as the swastika are found. Art was preparing the way for geometry."

As we suggested at length in our introductory chapter, religion and mysticism stimulated the early beginnings of science and mathematics. Man was filled with wonder. He was awed by the heavens, by life itself, and especially by death. Everything was a great mystery. The peculiarities of geometrical form and simple numbers electrified his imagination. The mysteries of form and number he deeply associated with the mystery of the Cosmos. It is our conclusion that in these early observations the concept and use of models were being born. We hope to have fully established our thesis by the end of the next chapter in which we exhibit the conscious use of model at a high level of mathematical abstraction. It is our position that even though early historic man had no word for model he was actually using the concept and that he continuously developed it until the present time.

It is instructive to examine the growth of mathematics in the various cultures of mankind. Such a project is truly enormous and well beyond the scope of our book, however we will very briefly sketch the history of the development of the subject in such a country as China. A recent book by Frank Swetz treats the subject in some interesting detail and we will rely upon it for our factual statements [5].

In his historical perspective, Swetz observes that the origins of Chinese mathematics are obscured by legend and mysticism. He says that the legendary emperor Huang-ti emerged as the first patron of mathematics. This early history is very suggestive of the manner in which the iconic model was used in the development of the subject. We will stress this in passing. Educational institutions are first mentioned in the history of China during the Shang dynasty (1523 B.C. - 1028 B.C.). Despite its ups and downs, China has a fairly continuous development of mathematical education. Mathematics, as such, is first mentioned in the history of the study of mathematics in the schools of China about the time of the Chou dynasty (1027 B.C. - 256 B.C.). It is clear that the subject was developed in the context of utilitarian pursuits. As for all other peoples, except the ancient Greeks, there was little or no abstract treatment of fundamental concepts. Mathematics was a body of inductively conceived knowledge resulting from concrete experimentation. It was a collection of computational techniques formulated through an extensive reliance on mechanical computing aids designed to solve concrete

problems. It can readily be speculated that it was rich in the application of the nascent concept of model. The historical period of indigenous mathematical achievement in China came to end with the arrival of Matteo Ricci a Jesuit scholar, at Peking in 1601. He translated into Chinese two mathematical works of his former teacher, Christopher Clavius. These were on the geometry of Euclid and on the principles of arithmetic. Jesuit predecessors of Ricci had introduced logarithms and refined the native theories of trigonometry. It seems that this infusion of European knowledge was necessary to activate the growth of abstract mathematics in modern China. There can be no doubt that during its development, Chinese mathematical thought retained and cultivated its propensities for empiricism and utilitarianism. We can see that the Chinese must have used the iconic and disclosive type models in their development of mathematics. However, the subtle use of model in the axiomatic formulation of the subject which we shall discuss later was totally unknown. It is well-known that the axiomatic approach was introduced by the Greeks and developed systematically for the subject of geometry. We wish now to consider the history of geometry in the Western World, from the time of the ancient Greeks, in order to see the full development and use of model in modern mathematics.

In order to proceed with our consideration of the model in geometry, from ancient times to the present, we first draw attention to the developability of iconic and disclosive models. Such models can readily be made of threads, cardboard, plaster and other materials in 3-space as shown on page 72 of volume 15 of the 14th edition of the Encyclopedia Britannica. Also, a relatively recent book on the subject under the title Mathematical Models has been published by Cundy and Rollett [6]. A more recent treatment using snapshots has been provided by Steinhaus [7]. We simply wish to emphasize that such physical models can be readily constructed. The fact was not lost on the early Greek mathematicians and it can be appreciated how they studied the conics as sections of solid cones. In some sense it was a help and hindrance. For an elucidation of this remark we refer to page 123 of the text by Boyer [1]. He says, "To guarantee that a locus was really a curve, the ancients felt it incumbent upon them to exhibit it stereometrically as a section of a solid or to describe a kinematic mode of construction." One should be willing to concede that such a use of iconic models is also disclosive in the sense that it permits important dialogue and the development of knowledge. However, the severe requirement can be a possible hindrance. Boyer, in developing his thought along these lines, says, "That Apollonius, the greatest geometer of antiquity, failed to develop analytical geometry, was probably the result of a poverty of curves rather than of thought. General methods are not necessary when problems concern always a limited number of particular cases. Moreover, the early modern inventors of analytic geometry had all Renaissance algebra at their disposal, whereas Apollonius necessarily worked with the rigorous but far more awkward tool of geometrical algebra." While we think that Boyer makes an important point, it must

be conceded that the great Greek geometer who wrote so effectively on the conics (a very limited class of plane curves, to be sure) was a man of his time. He and others of his time did exceedingly well with the models which were available to them. For example, Strabo (63 B.C. - 25 A.D.), who was an eminent geographer, considered the earth to be a sphere (model) and introduced geographical coordinates which foreshadowed analytical geometry. We also wish to emphasize that Apollonius, who incidentally was born at Perga about 225 B.C. and was a student in the Euclid school at Alexandria, demonstrated decisive properties of cones. He showed that a plane section of a circular cone, parallel to a generator, yielded a parabola. Also, other appropriate sections yielded ellipses or hyperbolas. The axiomatic development of geometry by the great Greek mathematician Euclid we will examine later in this chapter.

If we wish to follow the road laid down by Apollonius for geometry, it is a startling fact that to reach the next revolutionary advance it is necessary to go over a millenium and a half to the time of the Renaissance. It is this era which we now wish to examine more closely, but again we cannot go into the many details which relate to the invention of analytical geometry. For example, both Pierre de Fermat (1601 - 1665) and Rene Descartes (1596 - 1650) were responsible for the introduction of this very important geometry. Despite differences in approach to coordinate geometry, Descartes and Fermat became involved in controversy as to priority of discovery. Such misunderstandings existed in mathematics at that time and probably throughout its long history. Both the geometry and the algebra were at hand and both men used such knowledge to introduce a separate and extensive treatment of coordinate geometry. Although it is said that Fermat discovered the basic ideas of coordinate geometry in 1629, La Geometrie of Descartes was published in 1637. Descartes understanding of coordinate geometry is said to have originated in 1619. Be that as it may, we wish to stress the implicit use of model in the establishment of the subject. We may say that a physical model exists in the line system which is drawn to represent the geometrical objects and that a theory of analysis of such representation lies in the associated algebra. It should be emphasized that the coordinate geometry enabled one to study a much vaster system of curves than the conics, which were the objects of geometrical study by the ancient Greeks. Descartes was well aware of the greater power of his method. As we know now, the development of two-dimensional coordinate geometry led to the geometry of 3-dimensions and finally to that of N-dimensions. As plane curves could readily be studied in the 2-dimensional geometry so surfaces could be in the 3-dimensional geometry. Having observed the powerful impetus given to geometry in the seventeenth century by Fermat and Descartes we would like now to review in somewhat more detail extensions of the geometry to higher dimensions than three. An excellent simple treatment of the subject, which we can relate to the model viewpoint, is given by Constance Reid in her book entitled

A Long Way from Euclid [8]. For our purpose we will now give a brief summary of some of her work in order to illustrate again our position concerning the role of the concept of model.

As Descartes observed that his method gave him greater power than the Greeks had to study curves, we will now examine how the extension of analytic geometry to dimensions greater than three can be accomplished and used. In order to do this we observe that a point tracing out a line is a one-dimensional geometry; a line tracing out a plane is a two-dimensional geometry; and a plane tracing out a higher space is a three-dimensional geometry.

We ask now whether a three-space (solid) can trace out a four-dimensional space. One can really obtain a visual idea of a four-dimensional space by a logical extension from three dimensions. Take the simplest figures in each of the dimensions:

a) a line is bounded by two points
b) a triangle is bounded by three lines
and c) a tetrahedron is bounded by four triangles.

Should there not be, in a 4-dimensional space, a figure bounded by five tetrahedra? Obviously we are dealing with the principle of analogy, which is essential to the concept of model. As we have stressed in the past, the pure analogy suggests possibilities. The next step is to see whether proofs can be obtained. In the present analysis such is the case. The logical extension of the three figures above we call a pentahedroid. When the five tetrahedra are regular, the pentahedroid (it can be proved) is one of the six regular bodies possible in a 4-dimensional space. To our above set of three statements a, b, c, we may add a fourth as follows:

d) a pentahedroid is a figure bounded by five tetrahedra.

We can "see" a hypersolid in a manner similar to that in which we are accustomed to seeing 3-dimensional solids in 2-dimensional space. We are all familiar with the real appearance of these objects in photographs and paintings (iconic models). Of course, it is our actual experience with objects in three dimensions which gives a reality to their representation in two, and this actual experience is not possible with four dimensional objects. Nevertheless, we can construct a perspective model in three dimensions of a never-seen and never-to-be-seen but logically thought out figure in four dimensions.

In the 3-dimensional representation of a 4-dimensional pentahedroid we have before us a tetrahedron, which is one of the faces of the pentahedroid. We can make 3-dimensional patterns of 4-dimensional hypersolids almost as easily as we can make 2-dimensional patterns of 3-dimensional solids.

As an example consider a regular tetrahedron, which is obviously composed of four equilateral triangles, developed out on to the plane which contains its base. The result is an equilateral triangle divided into four smaller equilateral triangles.

Now an analogous but 3-dimensional pattern for a pentahedron involves spreading out the hypersolid into three space. The resulting pattern would be a tetrahedron erected upon each face. One can imagine this 3-dimensional configuration folded back into a 4-dimensional pentahedroid.

We can algebraically conduct investigations of the above type not by actually sculpturing the forms but by resorting to the corresponding N-dimensional coordinate geometry. Recalling that:

$$ax + by + c = 0$$

represents a line in 2-dimensional space and:

$$ax + by + cz + d = 0$$

represents a plane in 3-dimensional space.

The extension is simple and we see that:

$$ax + by + cz + dw + e = 0$$

represents a hyperplane in 4-dimensions.

Furthermore, if (x,y) and (x',y') are two points in a plane, the distance between these points can be written:

$$\delta^2 = (x' - x)^2 + (y' - y)^2$$

where δ is the distance.

Similarly for 3-dimensional space:

$$\delta^2 = (x' - x)^2 + (y' - y)^2 + (z' - z)^2$$

The extension to 4-dimensional is obviously:

$$\delta^2 = (x' - x)^2 + (y' - y)^2 + (z' - z)^2 + (w' - w)^2$$

In a similar manner the equations of a circle, sphere, and hypersphere can be written:

$$x^2 + y^2 = R^2$$

$$x^2 + y^2 + z^2 = R^2$$

and

$$x^2 + y^2 + z^2 + w^2 = R^2$$

It turns out that the area of the circle is:

$$A = \Pi R^2$$

the volume of the sphere is:

$$V = \frac{4}{3}\Pi R^3$$

and the hypervolume of the hypersphere is:

$$H = \frac{1}{2}\Pi^2 R^4$$

The equation can be generalized to higher dimensions as follows:

If the number of dimensions is even, say n equals 2k, we have:

$$\frac{\Pi^K}{K!} R^{2K}$$

and if the number of dimensions is odd, say n equals 2k plus one, the general expression is:

$$\frac{2^{2K+1}\ K!\ \Pi^K\ R^{2K+1}}{(2K + 1)!}$$

As one goes further into n-dimensions he never knows at just what n the extension becomes suddenly very difficult. At present there is no one, for example, who can tell how to pack 9-dimensional spheres in 9-dimensional space.

The geometry of n dimensions might just as well be called the algebra of n variables. However it cannot be lost on the reader that the extensions flow naturally by analogy and the motivation is stimulated by the original iconic models in physical 3-space. However, proofs must follow the underlying principles of logic and algebra.

We see in these few examples the mirrored evolutionary nature of all knowledge and the intrinsic lack of FINALITY. Development progresses indefinitely. The model and the analogue play key roles in the process.

Another important example of the model in mathematics relates to the subject of non-Euclidian geometry. It serves an essential

purpose in the proof of consistency of the axiomatic foundations.

As we have been doing so far in the present chapter, we will sketch briefly the history of the subject and use appropriate references. A classical reference is the small monograph by Roberto Bonola [9]. More recent references are by Carruccio [10], Coxeter [11], and Sommerville [12]. A book of considerable interest to us and a fine example of historical writing in mathematics is Mathematical Thought from Ancient to Modern Times by Morris Kline [2]. Its publication in 1972 has provided us with an expert exposition, especially in the field of non-Euclidean geometry.

As anyone who is at all acquainted with the subject knows, the parallel postulate of the ancient Greek mathematician Euclid was questioned from the beginning, even by Euclid himself. However, the first modern mathematician to publish a work devoted exclusively to the theory of parallels was P. A. Cataldi in the seventeenth century (see Bonola, page 13). The really serious work on the subject does not come until a much later date. However, it is our opinion that the serious student should examine several sources for the forerunner of modern geometry, which includes the non-Euclidean as an essential part.

The Renaissance provided the work of Fermat and Descartes on coordinate geometry as we have previously reviewed. No doubt their studies were revolutionary and provide a strong contrast with the program of Euclid. Another influence of considerable importance to geometry came during the Renaissance and from a different source. The story we wish to present briefly was emphasized by both Boyer [1] and Kline [2]. Discussing Leonardo da Vinci, Boyer says, "Leonardo frequently is thought of as a mathematician, but his restless mind did not dwell on arithmetic or algebra or geometry long enough to make a significant contribution. In his notebooks we find quadratures of lunes, constructions of regular polygons, and thoughts on centers of gravity and on curves of double curvature; but he is best known for his applications of mathematics to science and the theory of perspective." We stress here the word PERSPECTIVE. Kline dwells at great length on the influence of the theory of perspective in the development of mathematics. We will quote him freely and note especially what he has to say about the developments of the Renaissance. At one point Kline says, "In the Renaissance the depiction of the real world became the goal. Hence the artists undertook to study nature in order to reproduce it faithfully on their canvases and were confronted with the mathematical problem of representing the three-dimensional world on a two-dimensional canvas." Further he says that Filippo Brunelleschi (1377 - 1446) was the first artist to study and employ mathematics intensively. Georgio Vasori (1511 - 1574), the Italian artist and biographer, says that Brunelleschi's interest in mathematics led him to study perspective and that he

undertook painting just to apply geometry. In the fifteenth century the theoretical genius in mathematical perspective was Leone Battista Alberti (1404 - 1472). Alberti conceived the principle that became the basis for the mathematical system of perspective adopted and perfected by his artist successors. By use of a model consisting of two glass screens interposed between the eyes of the beholder and a specific scene, he observed that the sections would be different. His preoccupation with this problem really is the starting point for the development of projective geometry. Another painter concerned with the mathematical principles of perspective was Piero della Francesca (c. 1410 - 1492). His main work, De prospettiva pigendi, made advances on Alberti's idea of projection and section. Of all the Renaissance artists, the best mathematician was the German Albrecht Dürer (1471 - 1528). His book on geometry in 1525 was intended to pass on to the Germans knowledge he had acquired in Italy and in particular, to help the artists with perspective. In the period from 1500 to 1600 artists and subsequently, mathematicians put the subject on a satisfactory deductive basis. Finally, definitive works on perspective were written much later by the eighteenth century mathematicians Brook Taylor and J. H. Lambert.

We have dwelt rather at length on the introduction and growth of the concept of perspective because it clearly demonstrates the power of the model and, also, interdisciplinary influence. The artists orginally were greatly concerned with the idea of perspective and projection which later were to loom so large in mathematics as a whole. One most important mathematical aspect of this artistic development was that such thinking provided a distinctive advance over the Euclidean geometry of the Greeks. As important as is the theory of perspective for the development of geometry, there was also the theory of coordinate representation which Kline says changed the face of mathematics. For over one hundred years after the introduction of analytic geometry by Fermat and Descartes, algebraic and analytic methods dominated geometry almost to the exclusion of synthetic methods.

Finally, the stimulus to revive synthetic geometry came primarily from Gaspard Monge. In 1799 in his Traite de geometrie descriptiva he showed how to project orthogonally a three-dimensional object on two planes (a horizontal and a vertical) so that from the representation one can deduce mathematical properties of an object. The major effort of Monge and his pupils was finally devoted to what is now called projective geometry. This subject enjoyed a somewhat vigorous but short-lived burst of activity, with the creative work of Pascal and Desargues, in the seventeenth century, but was submerged by the rise of analytic geometry and the calculus. Projective geometry was revived by Lazare Carnot (1753 - 1823), the father of Sadi Carnot, but received its main impetus from Poncelet (1788 - 1867) an officer in the army of Napolean and a student of Monge. Poncelet's work centered around

three ideas. The first is that of homologous figures. Two figures are homologous if one can be derived from the other by one projection and a section, which is called a perspectivity, or by a series of projections and sections, which are called a projectivity. The second idea was the principle of continuity, and the third the notion of pole and polar with respect to a conic.

A modern presentation of projective geometry which may be strongly recommended is a text on the subject by H. S. M. Coxeter [13]. We will quote the last paragraph of his historical introduction to his book. He says, "Besides being a thing of beauty in its own right, projective geometry is useful as supplying a fresh approach to Euclidean geometry. This is especially evident in the theory of conics, where a single projective theorem may yield several Euclidean theorems by different choices of the line at infinity; e.g. if the line at infinity is a tangent or a secant, the conic is a parabola or an hyperbola respectively. Arthur Cayley (1821 - 1895) and Felix Klein (1849 - 1895) noticed that projective geometry is equally powerful in its applications to non-Euclidean geometries."

We now end our brief historical treatment of mathematics and the model which has run from prehistoric times to the twentieth century. However, we will find it necessary to provide some additional history of non-Euclidean geometry in the next chapter, where we consider special aspects of the model as related to modern mathematics. Other topics which we have chosen to review in the next chapter include logic, metamathematics, and the foundations of mathematics. It turns out that all of these subjects reveal very technical and essential connections with the concept of model.

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CHAPTER 15

TYPE OF MODELS IN MODERN MATHEMATICS

There are two areas in modern mathematics which we wish to use in order to exemplify the conscious and effective use of the model in the body of mathematics per se. The first is associated with geometry and is the one with the longer conscious use of model. The second concerns abstract algebra and its relation to logic and the foundations of mathematics. We will consider first the model as used in the development of non-Euclidean geometry.

Our association of model with non-Euclidean geometry is very technical from a mathematical viewpoint and we will hasten to establish the relationship. However, to provide a sound basis for our exposition of the matter it is necessary to provide some historic perspective.

For a very good review of the history one may refer to chapters 36, 37, and 38 of Mathematical Thought from the Ancient to Modern Times by Morris Kline [1]. Concerning our discussion of projective geometry as a variant on both the mathematics of the ancient Greeks and the coordinate geometry of Fermat and Descartes we wish to refer to Kline's chapter 38 which is entitled, Projective and Metric Geometry. He says, "with the non-Euclidean geometries now at hand the possibility arose that these, too, at least the ones dealing with spaces of constant curvature, might be some specialization of projective geometry. Hence the relationship of projective geometry to the non-Euclidean geometries, which are metric geometries because distance is employed as a fundamental concept, became a subject of research. The clarification of the relationship of projective geometry to Euclidean and the non-Euclidean geometries is the great achievement of the work we are about to examine. Equally vital was the establishment of the consistency of the basic non-Euclidean geometries." It is this last sentence which we will examine carefully as it relates to the concept of model in a special manner. However, we first wish to view briefly the history of the subject as it relates to the major investigators.

Anyone at all familiar with mathematics knows that it is the famous fifth postulate or axiom of Euclid on parallels that is at the root of our inquiries. From the time of Euclid it was suspect. Is it really an axiom or a theorem derivable from the other axioms? We have provided several references on non-Euclidean geometry, but for consistency in our present objective we shall adhere closely to the story presented by D. M. Y. Sommerville [2].

Among the first axiomatic investigators stands Gerolamo Saccheri (1667 - 1733). He devised an entirely different mode of attack on the problem of the fifth postulate of Euclid in an attempt to institute

a reductio ad absurdum. However, if Saccheri had a bit more imagination he would have anticipated by a century the discovery of the two non-Euclidean geometries which follow from his hypotheses of the obtuse and the acture angle.

Another scholar, J. H. Lambert (1728 - 1777), fifty years later, also fell just short of this discovery. By 1799 the great mathematician K. F. Gauss (1777 - 1855) began to study the subject. He was probably the first to obtain a grasp of the possibility of a non-Euclidean geometry, and we owe the very name to him. By 1829 he was in possession of many theorems of non-Euclidean geometry. We know that at this time Nikolai Ivanovich Lobachevsky (1793 - 1856) was studying the implications of the geometry of Euclid. By 1826 he explained the fundamentals of his "Imaginary Geometry", which is more general than that of Euclid. He had developed for himself the strong spirit of non-Euclidean geometry. This produced in 1840 his important book Geometrische Untersuchungen Zur Theorie der Parallellinien.

Both Wolfgang Bolyai and his son John were interested in the theory of parallels. By 1823, John had worked out the main ideas of non-Euclidean geometry. Gauss never published on the subject so that the honor of discovery goes alone to Lobachevsky and Bolyai. Their ideas, however, did not gain much recognition for many years.

It remained for the great Bernhard Riemann (1826 - 1866) to admit the hypothesis that the sum of the angles of a triangle may be greater than two right angles and that a straight line may be unbounded but yet of finite length. We owe these important concepts to Riemann who provided them in his famous Dissertation of 1854.

The conception of a geometry in which the straight line is finite, and is, without exception, uniquely determined by two points, is due to that accomplished mathematician, Felix Klein. Klein provided the names now associated with the three geometries. He called the geometry of Lobachevsky Hyberbolic, that of Riemann Elliptic, and that of Euclid Parabolic.

We wish now to stress the fact that although non-Euclidean axiomatic systems could reasonably be discussed along with Euclidean it was, in our opinion,ultimately essential to provide models ofthe various geometries in the form of surfaces, especially in order to examine the important question of consistency. On this point, we recall that years ago Charles Proteus Steinmetz wrote a small book on relativity and in a pocket attached to the inside of the back cover he had a set of paper surfaces which could stereoscopically illustrate the geometries we now wish to discuss in some detail.

It may be recalled that the non-Euclidean geometries that were most significant after the work of Riemann were those for spaces of constant curvature. It was Beltrami who recognized that such surfaces

are non-Euclidean spaces. He gave a limited representation of hyperbolic geometry on a surface which showed that the geometry of a restricted portion of the hyperbolic plane holds on a surface of constant negative curvature if the geodesics are taken to be straight lines. The lengths and angles on the surface are the lengths and angles of the ordinary Euclidean geometry on the surface. One such is known as the pseudosphere. It is generated by revolving a curve called the tractrix about its asymptote. It was one of the surfaces which Steinmetz provided in the set we mentioned above. Double images were provided so that the surface could be viewed in a stereoscope and provide three-dimensional effects. The pseudosphere is a model for a limited portion of the plane of Gauss, Lobachevsky, and Bolyai. On the pseudosphere a figure may be shifted about and just bending will make it conform to the surface, as a plane figure by bending can be fitted to the surface of a circular cylinder.

Heinrich Liebmann (1874 - 1939) proved that the sphere is the only closed analytic surface, free of singularities, of constant positive curvature and so the only one that can be used as a Euclidean model for double elliptic geometry.

The development of these models helped the mathematician to understand and see meaning in the basic non-Euclidean geometries. One must always keep in mind that these geometries, in the two-dimensional case, are fundamentally geometries of the plane in which the lines and angles are the usual lines and angles of Euclidean geometry. Hyberbolic geometry was developed in this fashion but the conclusions deduced from such models seemed strange and it took some time before they were accepted into the body of mathematics. The double elliptic geometry did not even have an axiomatic development as a geometry of the plane. Hence the only meaning attached to it was provided by the geometry of the sphere. A far better understanding of these geometries was obtained from a development that sought to relate Euclidean and projective geometry. Cayley, seeking to demonstrate that metrical concepts can be formulated in projective terms, devoted himself to the relationship between Euclidean and projective geometry.

Felix Klein showed that a hemisphere, including its boundary, is a model of single elliptic geometry. However, it is necessary to identify any two points on the boundary which are diametrically opposite. The great circular arcs are the geodesics of this geometry and the ordinary angles on the surface are the angles of the geometry.

We can see why Klein introduced the terminology hyperbolic for Lobachevsky's geometry, elliptic for the case of Riemann's geometry on a surface of constant positive curvature, and parabolic for Euclidean geometry. It was suggested by the fact that the ordinary hyperbola meets the line at infinity in two points and correspondingly in hyberbolic geometry each line meets the absolute in two real points. The ordinary

ellipse has no real points in common with the line at infinity and in elliptic geometry, likewise, each line has no real points in common with the absolute. The ordinary parabola has only one real point in common with the line at infinity and in Euclidean geometry, as generalized in projective geometry, each line has one real point in common with the absolute.

Shortly after 1870 several non-Euclidean geometries, the hyperbolic and the two elliptic ones, had been introduced. The FUNDAMENTAL question which had to be answered was whether they were CONSISTENT. All of the work produced by Gauss, Lobachevsky, Bolyai, Reimann, Cayley, and Klein might still have been nonsense if contradictions were inherent to the geometries.

Beltrami stressed the fact that Riemann's two-dimensional geometry of constant positive curvature is expressible on a sphere. Such a model makes possible the proof of the consistency of two-dimensional double elliptic geometry. The axioms and theorems of this geometry are all applicable to the geometry of the surface of the sphere provided the line in the double elliptic geometry is interpreted as great circle on the sphere. If there should be contradictory theorems in this double elliptic geometry then there would be contradictory theorems about the geometry of the surface of a sphere. Now the sphere is included in Euclidean geometry, which is considered to be consistent. Hence, the double elliptic geometry is consistent.

Beltrami had given the pseudospherical interpretation of hyperbolic geometry but this serves as a model for only a limited region of hyperbolic geometry and so could not be used to establish consistency for the entire geometry. The consistency of hyperbolic and single elliptic geometries was established by new models. The model for hyperbolic geometry was determined by Beltrami. However, the distance function used in the model was provided by Klein. The consistency of the hyperbolic geometry is made to depend on the consistency of Euclidean geometry.

We now consider that we have sufficiently stressed the use of the model concept in non-Euclidean geometry. We have pictures of geometric entities in a flat plane for Euclidean geometry and different kinds of pictures for the corresponding entities on curved surfaces for non-Euclidean geometry. With regard to such a synthetic treatment of geometries we complete this phase of our subject with a reference to the work of David Hilbert . In 1898 he proposed an axiomatic treatment of geometries. In his Foundations of Geometry he sought a set of axioms for Euclidean geometry which were both complete and independent [3]. In other words, all the theorems of Euclidean geometry can be deduced from basic axioms and no axiom can be deduced from the remaining ones once that axiom is deleted. His complete presentation consisted of five groups of axioms which consisted in the aggregate of seven on connection, five on order, one on parallels, six on congruence, and one on continuity.

With the set of axioms, Hilbert specified the Euclidean geometry, in the sense that one can state which of the Euclidean theorems require which axioms in their proof. Also, by dropping one axiom, replacing it by the negation of that axiom, and finding a model for the new set of axioms, he proved the independence of each axiom. Also, in finding such models for independence proofs, one is not restricted to models from geometry. Any set of known true propositions having the same logical forms as the revised axiomatic set will prove the consistency of the revised set, and hence the independence of the changed axiom. In this last sentence we anticipate the second part of our present investigation, mathematical foundations and logic, which we will treat presently. First, however, we wish to make one further examination of the nature of geometry. It consists of a brief review of an important program instituted by Felix Klein.

Powerful stimulus was given to the subject of geometry by the now famous Erlanger Program. The program was initiated by a talk given in 1872 by Klein on the occasion of his admission to the faculty of the University of Erlangen. Its English title is, A Comparative Review of Recent Researches in Geometry. His central idea is that each geometry can be defined by a group of transformations and the invariants under the group.

Projective geometry of two-dimensions is the study of invariants under the transformations from the points of one plane to those of another or to points of the same plane. In terms of homogeneous coordinates the transformation is:

$$x_j' = a_{ij}x_i \quad i = 1, 2, 3$$

where a_{ij} are real numbers and determinant $|a_{ij}|$ does not vanish. The invariants are linearity, collinearity, cross-ratio, harmonic sets, and the property of being a conic.

One subgroup under the projective group is the set of affine transformations. They are the same as above except a_{31} and a_{32} vanish. Under affine transformations, straight lines transform into straight lines, and parallel straight lines into parallel straight lines. However, length and angle are not invariant.

In a recent book on algebra and projective geometry, Reinhold Baer notes that every linear manifold defines a projective geometry and an affine geometry [4]. He calls a partially ordered set V(F, A) an algebraical MODEL of affine geometry. On his page 304, he constructs a projective model of an affine geometry.

The group of any metric geometry is the same as the affine group except that the determinant of coefficients $|a_{ij}|$ is equal to plus or minus one. The first of the metric geometries is Euclidean geometry.

Its transformations in two-dimensions are those of analytical geometry. One may recall these as:

$$x' = \rho(\chi \cos\theta - y \sin\theta + \alpha)$$

$$y' = \rho(\chi \sin\theta + y \cos\theta + \beta)$$

where $$\rho = \pm 1$$

The invariants are length, size of angle, and hence shape of figure.

One may classify a subgroup of the affine as parabolic metric group. Also, hyperbolic metric geometry is a subgroup under projective geometry which leaves invariant an arbitrary real non-degenerate conic in the projective plane. Single elliptic geometry corresponds to the subgroup of projective transformations which leaves a definite imaginary ellipse invariant. Another geometry that can be classified from the transformation viewpoint is double elliptic geometry.

For four metric geometries. Euclidean, hyperbolic, and the two elliptic geometries, the transformations are such that the motions are rigid body.

Klein considered more general geometries than projective but we shall go no further here because we consider that we have sketched out a sufficient number of cases to reveal the nature of geometry from the analytic viewpoint. Also, the model nature of the subject with which we are dealing should be clear. While we are not now dealing explicitly with the pictorial models, like the pseudosphere, sphere, and hemisphere of the previous synthetic approach, nevertheless it is clear that we have now a whole set of possible models in geometry which are analytically defined. The great power and generality of the method is obvious. Its triumphs are reminiscent of those of Fermat and Descartes in the field of elementary analytical geometry. Just as their accomplishment greatly extended the power of the ancient Greek geometers, so the Erlanger Program increased that of the geometers going into the twentieth century.

We have come finally to the second topic which we propose to treat under the title of the chapter, and one which projects itself into the second half of the twentieth century and probably beyond. We wish now to consider in some detail the model as related to metamathematics, logic, and the foundations of mathematics.

The latter part of the nineteenth century and the first half of the twentieth witnessed developments in mathematics that had scarcely been dreamed of before this time. Forerunners of the new era were the supermathematicians Gauss (1777 - 1855), Cauchy (1789 - 1857), and Weierstrasse (1815 - 1897). These men supplied the rigorous treatment of analysis in their day. However, the rigorization of

analysis did not prove to be the end of the investigation into the foundations of mathematics. All of the studies of those three eminent mathematicians assumed the existence and the acceptability of the real number system. However, as we now know, the subject was anything but satisfactory from the logical viewpoint. Although most of the mathematicians of the time did not think it necessary to investigate the logical foundations of the number system, at least Weierstrasse did begin his study of irrational numbers. It remained for Cantor, who introduced the theory of sets, and Dedekind to provide the required stimulus to lay the foundations, provide acceptable definitions and treatment in the field of real numbers, and ultimately to recognize the meaning of number itself. Although it is our principal intention to consider the relation of model to mathematics, we do think it necessary to examine, at least briefly, the history of logic itself. It is this subject which so profoundly relates to the mathematics of the twentieth century. It is essential to the proper understanding of logic, metalogic, mathematics, and metamathematics.

In order to accomplish our purpose we shall rely heavily on a single book. It is the History of Logic by I. M. Bochenski [5]. We do this for an important reason. Bochenski has both the interest and the competence to examine the ancient logic and the logic of modern times, especially as it relates to mathematics. As in the brief history of mathematics which we have perused and in that of the concept of model which we sketched earlier, we see, also, that the ideas associated with logic go far into the past of man. Bochenski on page four of his history quotes Petrus Ramus who lived in the sixteenth century. Ramus may be the first historian of logic, so one can see that it was rather late in general history when man began to tell a connected story of logic. Bochenski in a probably humorous vein says that Ramus's imagination far outran his logic. Ramus spoke of a Logica Patrum in which Noah and Prometheus figure as the first logicians. We ourselves are rather amused by Bochenski, and others, who seem to overlook the fact that the glimmer of both logic and model must have appeared far back in our evolutionary beginnings. No subject, with which we are familiar, jumps full blown from the head of Zeus. Having said this, however, we recognize that the first normal treatment of logic began with the Greeks of the Golden Era and with Aristotle in particular. Aristotle provided for the first time in history a systematic treatment of formal logic in his Topics. He developed doctrines of the so-called predicables and of the categories. Furthermore, he provided studies of the principle of contradiction, the law of the excluded middle, the syllogism, and the beginnings of a metalogical system.

Bochenski answers, and we see ground for agreement, that logic remainded more or less the same until the time of the scholastics in the twelfth and thirteenth centuries. Amongst others are Peter Abelard (1079 - 1142), Albert the Great (1193 - 1280), William of Ockham, and

John Buridan. Looming large in this period is Thomas Aquinas who basically was a theologian. Bochenski goes to great pains to show that these men not only revived the work of the Greeks but made substantial contributions of their own. He says that they introduced a metalogical method of treatment. In his history he states, "Generally speaking, whatever the scholastics discuss, even the problems of antiquity, is approached from a new direction, and by new means..... There is firstly the metalogical method of treatment. Metalogical items are indeed to be found in Aristotle, but in Scholasticism, at least in the later period, there is nothing but metalogic, i.e. formulae are not exhibited but described, e.g. in the De puritate artis logicae of Burleigh not a single variable of the object language is to be found." In addition scholastic logic, even at the end of the thirteenth century, is very rich, very formalistic and exact in statement. Some treatises undoubtedly rank higher than the Organon and perhaps than the Megarian-Stoic, too."

We have emphasized that there were two great schools of logic up until the Renaissance. These were the Aristotelian and the Scholastic. It is our opinion that Bochenski makes a strong claim for such a position. Furthermore we have to jump almost to the twentieth century for further significant advances and we wish to quote Bochenski directly on the matter. He says on page 258 of the history, "Formed by this logic and its prejudices modern philosophers such as Spinoza, the British empiricists, Wolff, Kant, Hegel, etcetera could have no interest for the historian of formal logic. When compared with the logicians of the 4th century B.C., the 13th and the 20th centuries A.D. they are simply ignorant of what pertains to logic and for the most part only knew what they found in the Port Royal Logic." Continuing in this manner, he points out that there was one exception, Leibniz (1646 - 1716), whom he calls one of the greatest logicians of all times. On such a note we wish to move rapidly to recent times so that we can examine the use of the concept of model in these logical areas. From Leibniz we go rapidly to the subject of the general foundations of mathematical logic. Even at the present time, its development is not complete and there are still significant discussions about its name and scope. Various names are used such as "mathematical logic", "symbolic logic", and "logistic." There is no universal agreement about the characteristics which distinguish it from other forms of logic. Mathematical logic is distinguished by its having a calculus and a constructional process. All other logics known to us make use of the abstractive method; the logical theorems are obtained by abstractions from ordinary language. Mathematical logicians proceed in just the opposite way, first constructing purely formal systems, and later looking for an interpretation from everyday speech.

A brief resume of names, dates, and works may be of value in orienting the reader in this last important subject which we are

discussing. G. W. Leibniz generally ranks as the original mathematical logician. Several important scholars followed him but the next important name we wish to cite is that of George Boole, whose pioneer work, The Mathematical Analysis of Logic, appeared in 1847. In the same year Augustus de Morgan published his Formal Logic. Several investigators took Boole's ideas in various directions but because of limited space we move on to outstanding figures for the newer developments. These are C. S. Pierce, Gottlob Frege, G. Peano, Bertrand Russell, A. N. Whitehead, David Hilbert, L. E. J. Brouwer, J. Lukasiewicz, St. Lesniewski, A. Tarski, R. Carnap, A. Heyting, and K. Gödel. These men presented important papers on the subject from about 1867 to 1930. The subjects which were treated are logical calculus, theory of proofs, metalogic, the concept of logic, Boolean calculus,propositional logic, propositional function, truth-values, predicate logic, the logic of classes, antinomies, and types. The topics are all discussed at length in the history of Bochenski.

We can briefly summarize the major achievements of this last period in the history of logic which takes us into the twentieth century. It is the era from the latter part of the nineteenth century up to and including the Principia of Russell and Whitehead. Mathematical logic at the end of that period is seen as a highly original variety of logic. It proceeds constructively in that it studies logical laws in a peculiarly characteristic language which it has devised. It has developed elementary relations of syntax and semantics. It uses formalism extensively and greatly surpasses such attempts by the Stoics of the early period of logic and by the Scholastics of the Middle Ages. Using the new methods of construction and formalism the previous intuitionistic ideas were reconceived and developed. Some of the results consist of the new logical forms, the distinction between language and metalanguage, the distinction between propositional and term logic, and the investigation of antinomies. There has been a long series of logical discoveries. Of paramount importance is the concept of complete proof, which was effectively developed. There are new concepts such as functor, argument, and quantifier. With these notions came the ideas of many-place functors and multiple quantification, which are strikingly new ideas in the long history of logic. Also, the sharp contrast of the logic of predicates and classes are new. The logic of relations, the theory of description, and the logical antinomies of this era are highly original. The large number of logical formulas which were enunciated and studied is characteristic of mathematical logic. The period contrasts clearly with that of the classical Greeks and of the Scholastics of the Middle Ages and represents a new development in the long history of logic. Such are the observations of Bochenski who represents a high order of scholarship in the field of logic.

The matter which we have been reviewing relates seriously to what we have to say about the place of model in mathematics but we will

now deal directly with some recent authors who have presented mathematical studies which explicitly use the concept of model.

A good reliable introduction to the phases of the subject which we now wish to consider is provided by Lucienne Felix in her book entitled The Modern Aspect of Mathematics [6]. She says in her introduction, "More and more our search into the unknown is guided by mathematical models." Felix sketches the new trends in mathematics and then discusses some aspects of metamathematics which concern logic and method.

Model of an abstract theory is essential to our thinking and so we wish to quote Felix directly on this subject. On page 82 she says, "If two theories J and J' are such that every element in the domain of each is associated to an element in the domain of the other (one-to-one correspondence), and if every operational or relational sign of one is similarly associated with a sign of the other, and if every true relation in one is translated by a true relation in the other — if all these conditions obtain, we say that the two theories define in their domains an isomorphic correspondence. For the mathematician the two theories are two translations of a unique theory — for he is concerned only with the relations between objects and not the objects themselves. If we refer the theories, by translation, to the same domain we obtain equivalent theories. They differ only in the method of exposition adopted. Isomorphic domains constitute models of the same abstract theory." In this statement we see clearly that we have associated with the concept of model in the past. There is a comparison of an A and a B. There is a one-to-one correspondence and there is an isomorphism. Felix goes on to exemplify these remarks by a consideration of geometry just as we did previously when we considered non-Euclidean geometry. An important statement is made on page 87 of her book. It is as follows, "These few examples illustrate well one of the essential characteristics of modern mathematics: the fact that its form is susceptible of many models therefore provides a great economy of thought." Again we have the case of a scholar justifying model by an extraneous remark. Apparently for her, the meaning of the model is that it provides economy of thought. It seems it is not apparent to her that not only is the economy of thought involved but even thought itself. As we stressed earlier in our book, concerning the role of model in physics, as for example in the Bohr-Rutherford model of the atom, the model is not dispensable in our thinking. It is an essential part of the model-theory dyad, or the model-theory-experiment triad. It seems to us that one cannot think theory in the absence of the basic model. It may at times be in the background but nonetheless it is there, everpresent. In her Appendix II, Felix says that the analogy of formalism in two areas of research permits the consideration of the two areas as models of a single structure, so that the experience acquired in one of the areas serves in the exploration of the other. She, also, refers to models of organic molecules and Bohr's atom. With respect to the concept of model she says further, "To prove that a property P is independent of the set A of the axioms of a theory T,

i.e., that P does not belong to T — we must construct a model, a theory in which all the axioms of T (and other axioms ultimately) are true and in which P is not true." It may be recalled that we illustrated this principle in our discussion of non-Euclidean geometry. In her book, Felix cites a few well-known examples which illustrate the use of models in the sense of analogy and ismorphism. These may be listed as follows:

1. The set of real numbers and the set of positive integers.

2. Points on an axis and real numbers.

3. The three symmetries S_1, S_2, and S_3, with respect to three axes concurrent at angles of 60° and the permutations on the triple of three letters: a, b, c.

4. Translating an analytic theory by means of an algebraic model: The Laplace Transformation in which differentiation corresponds to multiplication and integration to division.

The growth of a conceptual model may be likened to the extension of theories by the creation of new elements. It is the very process by which mathematics grows. The entire history of numbers is a classic example of growth by extension. The evolutionary process carries us from natural integers, to fractional numbers, to signed numbers, and finally to algebraic numbers. The passage to complex numbers is not such a straightforward extension unless one disregards the order relation as a requirement. After the complete growth of the number system from integers to real numbers we observe that the real line is defined as a set consisting of its points and it serves as a model for the set of real numbers.

To appreciate how recent is the use of the term model in mathematics one need only recall that J. W. Young in his Fundamental Concepts of Algebra and Geometry never once uses the term even though he extensively presents the development of the complete number system, the abstract foundations of mathematics, and geometric theory. On the other hand, Mal'cev, the Soviet mathematician, in his Metamathematics of Algebraic Systems does [7]. His book is said to be pervaded by the theory of models, that broad region on the boundary of logic and algebra, and to lean more toward metamathematics than toward universal algebra. The title of his book suggests the breadth of his study which includes general models (relational structures), algebras (algebraic structures), and partial algebras. The term model is used extensively by Mal'cev as reflected by thirty-four of the references in his book. We will now quote several instances of the use of the term model by Mal'cev. At various places in his book he makes the following statements:

1. Every assignment of truth-values to the elementary propositions

is called a model. Hence, for every segment S^{λ} there is at least one model M^{λ} for which S^{λ} is true.

2. If B is a domain for S, and R is a complete consistent configuration on B satisfying S, then we shall say B, R are themselves a model satisfying S_1 or simply, S.

3. A base set A with a sequence of predicates defined on it will be called a model. In the usual manner, instead of operations one can take the corresponding predicates and consider algebras to be models. Let us agree to say that a class K of algebras is locally definable (or just local) if from the fact that every finite submodel of an arbitrary algebra M is isomorphic to a submodel of some K-algebra it follows that A itself belongs to K.

4. A nonempty set M supplied with a finite sequence of predicates $p_1, \ldots, P_s$ is called a model.

5. The algebra whose base is the set of natural numbers and whose single basic operation is the addition of numbers can be denoted by [(0, 1, 2, ...), +]. Similarly [(0, 1, 2, ...), $\leq$, 1] denotes the model with the same base and with the order relation $\leq$ and the divisibility relation 1 as basic predicates.

6. An arbitrary system of similar models is called a class of models.

We wish to end our comments on the work of Mal'cev by a reference to model in connection with the problem on the border between algebra and logic. He says, "In this report I want to survey some results and problems in a mathematical discipline which has arisen in the last decade on the boundary between mathematical logic and classical abstract algebra, and which to date has no generally accepted name. It is most frequently called model theory or universal algebra, or sometimes general algebra."

For our purpose, the study of models, a very fine exposition of the situation concerning the foundations of mathematics and its relationship with logic is presented by Beth, Wilder, Stabler, Kac, Ulam, Luxemburg, Keisler, and Robinson. They demonstrate clearly the connection between the concept of model and mathematics. For convenience in presentation and in order to outline some of the substance of the various pertinent subjects we will touch upon briefly the treatment by these scholars. We by no means imply that we are referring to all the expounders of the modern doctrines, or even to the best, but we do maintain that their studies reveal the heart of the matter.

In his introduction Beth comments on the various domains which

are being treated in the study of the foundations of mathematics [8]. He says, "Research into the foundations of mathematics shows an increasing tendency to split up into various separate domains, each of which is more or less related to some branch of contemporary mathematics: set theory, abstract algebra, analytic topology, and so on." He recognizes Tarski as one of the outstanding mathematical logicians of the contemporary scene. He comments that Tarski's semantics provide precise definitions for such notions as fulfillment, denoting, definition, model, logical consequence, logical identity, and truth. These, together with other fundamental terms of metamathematics, come very close to current usage by leaders in the field.

Beth further tells us that the calculus of systems and semantical method relate metamathematics to the study of certain algebraic systems which belong to the field of the theory of partial order. Such a development gives importance to logical algebra and makes metamathematics one of the most characteristic features of contemporary mathematics. It is pointed out that one can characterize a deductive science or theory T as being the set of all statements — usually called the theorems of T — which can be derived, starting from a certain set of fundamental statements — usually called axioms, postulates, or hypotheses underlying the deductive science T — by means of logical inference. The study of logical inference is of a general nature and can be made for several deductive theories at the same time. Such is currently the task of logic.

Beth specifically outlines the postulational position on page 118 and relates it directly to the concept of model. In order to see how he does this we will now sketch his exposition. Let us assume a set N of elements a, b, c, ..., an operation F which transforms certain elements a into elements F(a) and a single element e, which together satisfy the following postulates:

1. e belongs to N;

2. If a belongs to N, then F(a) is defined and belong to N;

3. If a and b belong to N, and if F(a) equals F(b), then a equals b.

4. If a belongs to N, then F(a) is not equal to e.

5. Let a be any element of N; then by starting from e and applying again and again the operation F, we finally arrive at a.

In order to explain the purpose of the different postulates, let us try to establish a model of them that is, an assembly { N, F, e } of a set N, an operation F, and an element e which satisfy the five postulates. Two models of a given set of postulates which have a

simularity of structure are usually called ismorphic. Any two models of the five postulates are ismorphic.

Dedekind's theory of number system relates to the above postulates but the fifth postulate is written differently as follows:

If a belongs to N, then a belongs to every set K which has the following properties:

(i) e belongs to K

(ii) if b belongs to K, then F(b) also belongs to K.

Dedekind's postulates constitute and axiomatization for the theory of natural numbers. Consider the system $\{N_o, F_o, e_o\}$, N_o being the set of all natural numbers, $F_o(n) = n + 1$ for every natural number n, and e_o the natural number one.

Algebras have been subjected to elaborate analysis by the method of formalized axiomatics. Such research was inaugurated independently by A. Robinson and A. Tarski. We also know that geometries have been so axiomatized. As a result of modern research (after 1880), a number of axiom systems for elementary plane geometry have been constructed, all of which are completed in the sense that they constitute a sufficient basis for a rigorous derivation of classical theorems. We recall that David Hilbert was foremost amongst such investigators.

It is known that topology can also be developed as an autonomous discipline, called abstract topology, with an axiomatic basis of its own. An important topological system was developed by F. Hausdorff in 1914.

A Boolean algebra may be defined as a system $\{B, +, .\}$ with axiomatic base as follows:

1. $x + y = y + x$

2. $xy = yx$

3. $x + (yz) = (x + y)(x + z)$

4. $x(y + z) = (xy) + (xz)$

5. there is in B an element u such that, for every v in B:

   $v + xu = v$, and

   $v(x + y) = v$ (See Boole's Laws of Thought.)

Any field of sets can obviously be considered as a Boolean algebra.

Under the heading Completeness Theorems for Logical Systems, Beth discusses the use of models. He also discusses the calculus of systems and models. He further discusses elementary logic and higher order logic. He systematically uses the concept of model and even speaks of semi-model, non-standard model, and complete model. He obviously finds an essential use for model in his study.

Wilder presents the subject of axiomatics in much the same vein as Beth [9]. In his study of axiomatic systems he introduces the term model. He uses it to denote the result of the assignment of meanings to the undefined terms in Σ which is an axiom system. This is also called an interpretation of Σ. Wilder says further that it is not uncommon practice to obtain a model of an axiom system Σ in another branch of mathematics - even in a branch of mathematics that is, in its turn, based on an axiom system Σ'. Stabler speaks of the great variety possible in the models of a Boolean algebra, when he interprets H, a set of elements a_i, as a set of classes [10]. Any model of this type can be referred to as an algebra of classes. Furthermore, he suggests a model of the postulates which could be called an algebra of propositions. Like many others, he also exhibits a model for Riemannian geometry. Kac and Ulam give their views on the axiomatization of geometry and, also, treat at length mathematical algebra which, they say, has become today largely a study of such abstract systems as groups, rings, and fields [11].

Some remarks in a book by Luxemburg concerning the rather recent developments are pertinent for us [12]. He says in his introduction that in a 1934 paper in volume 23 of the Fundamenta Mathematicae Thoralf Skolem provides critical existence theorems in the subject of mathematical structures, which led to great interest in the subject by other mathematicians during the next fifteen years. In fact, an intensive study of these and similar structures began and became known as the study of nonstandard models of arithmetic. The concept of model was by this time being consciously and extensively used in mathematics. In the Symposium whose Proceedings were edited by Luxemburg there were analyses of Boolean-valued models for set theory, ultra-products in the theory of models, saturated models, homogeneous universal models, model theory as related to the metamathematics of algebra, and continuous model theory. In the Journal of Symbolic Logic 25, 1960, H. J. Keisler discusses theory of models with generalized atomic formulas. He also provides a fairly large bibliography which contains numerous references to logic which uses the concept of model. We will terminate our discussion of this rather recent development of mathematics by use of models with a discussion of some of the work by Abraham Robinson who is one of the most talented and respected scholars in the field.

Robinson's work is highlighted in a 1974 book which is entitled Introduction to Model Theory and to the Metamathematics of Algebra [13]. This book grew out of an earlier one on the metamathematics of algebra, which was published in 1951. The earlier work was concerned with the logical analysis of the methods of abstract algebra. It was a contribution

to algebra using the methods of symbolic logic. What is important to our thesis is that it developed certain topics in what is now known as "Model Theory." The 1974 revision contains many of the most recent developments in the subject. Robinson, for example, emphasizes the fact that numerous important concepts of algebra possess natural generalizations within the framework of the theory of models. The author says that these developments are interesting and important. Specifically he stresses that the theory of algebraic ideals and varieties, the notion of an algebraically closed extension, and the notion of a system of resultants to a given set of equations can be discussed profitably in a metamathematical setting. Importantly, he notes that apart from providing a unity of outlook, this approach also produces new algebraic results, e.g., in the case of a differentially closed field. In the last part of his book, Robinson provides an introduction to what is called non-standard analysis, a new application of model theory. It provides an effective calculus of infinitesimals and appears to have considerable potentialities. Specifically concerning the concept of model we quote from page 10 of his book. He says, "If all sentences of a set K hold in a structure M under a correspondence C, then we say that M is a model of K (under C). If K contains only a single sentence Y then we shall say also that M is a model of Y." In a certain theorem it is stated that if a set of sentences K and a sentence Y are such that Y is defined and holds in any structure which is a model of K then Y is deducible from K. Hence, Y is deducible from a subset of K. Robinson defines isomorphism in the following manner, "A one-to-one correspondence C between the individuals and relations of a structure M and the individuals and relations of a structure M' will be called an <u>isomorphism</u>; if the relations of any given order corresponds to one another, and if, whenever a relation holds between certain individuals of M the corresponding relations hold between the corresponding individuals of M , and vice versa. If M and M' are similar it will be taken for granted that C maps every relation on itself. If M equals M' then the isomorphism is said to be an <u>automorphism</u>. Structures between which there exists an isomorphism are called <u>isomorphic</u>.

Robinson points significantly to the concept of extension of model. He says that a fundamental property common to all the usual algebraic concepts is, roughly speaking, that the intersection of two models, or indeed of any number of models is a model. Furthermore, Robinson uses the concept of model systematically for the generalization of algebraic concepts. There are various concepts in algebra which are based on certain fundamental ideas whose scope, though vague, would appear to be more general than appears from the concrete definition. Among these in particular the concept of a polynomial ring of n variables adjoined to a given commutative ring; the concept of a free group; the concept of an algebraic number; the concept of an ideal.

We could proceed much further with exemplary uses of models in these mathematical and logical subjects but we consider that we have sketched sufficiently the situation vis a vis the model which has developed in the

twentieth century. We have shown how non-Euclidean geometrical development has depended on the concept of model and furthermore how later the same thing was done in logic and algebra. Before finally ending these considerations we wish to mention a most remarkable contrast between the study of non-Euclidean geometries and non-Aristotelian algebras.

In an extraordinary book on logic and methodology which was edited by Anna-Teresa Tymieniecka there is a cogent chapter on the contributions of the logician Vasilev by George L. Kline [14]. We will now sketch the position on multivalued logics, by Vasilev as reported by Kline. In his article Kline recalls, what we have previously discussed, that Lobachevsky himself had called his geometry "imaginary'; the term non-Euclidean became current somewhat later. Vasilev calls his logic imaginary and by analogy with non-Euclidean geometry, "non-Aristotelian". A given logic, Vasilev points out, rests on several independent axioms, not just one, so that, by omitting different axioms, we get different logics. In non-Euclidean geometry -- whether Lobachevskian or Riemannian -- Euclid's fifth (parallel) postulate is omitted; in non-Aristotelian logic Aristotle's Law of Noncontradiction is omitted, or at least the scope of its application is drastically limited. According to Vasilev Euclidean and non-Euclidean geometries have in common those Euclidean theorems which do not depend on the parallel postulate; similarly, Aristotelian and non-Aristotelian logics have in common those propositions which do not depend on the Law of Noncontradiction. Just as in Lobachevskian geometry straight lines may be either:

1. interesecting

2. non-intersecting

3. parallel

so in non-Aristotelian logic propositions may be either:

1. Affirmative

2. Negative

3. "Indifferent"

Vasilev may have clarified his position by a distinction between object-language and meta-language. He does not make such a distinction, but then, neither did his more celebrated contemporaries in the period around 1912.

Vasilev concludes that the "true-false" dichotomy of standard logic, like the "intersecting-parallel" dichotomy of standard geometry gives way to a trichotomy in the "imaginary" disciplines. Vasilev admits that Aristotelian logic and Euclidean geometry are the "simplest" systems; but he insists that non-Aristotelian logic has just as much right to be called logic as non-Euclidean geometry has to be called geometry.

We consider that in this chapter we have clearly demonstrated the conscious and effective use of both the term and the concept model in modern mathematics. Also we have shown that instead of declining in use and value since its conscious inception it is being more intensively cultivated in the current literature.

In the beginning of our present study we were convinced that it was important to emphasize the story of the development of and the use of the concept model in the discipline of mathematics itself as contrasted with the subject that is frequently referred to as mathematical modeling. It is clear to us that the latter use of model is distinctly different from the use of model in mathematics itself. We do contend however that the two separate disciplines depend fundamentally on the concept of model as we conceive it.

In the next chapter we will begin to investigate the subject of mathematical modeling which really is the act of applying the established principles of mathematics in a host of different kinds of human activity.

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CHAPTER 16

MATHEMATICAL MODELING

We know that there is a long history of mathematics that led finally to its fully axiomatic formulation in the twentieth century. We can readily see something of this development by tracing the history of geometry.

As we stated in chapter 14, in connection with the entire body of mathematics in China, the subject was developed in the context of utilitarian pursuits. It was inductively conceived knowledge resulting in computational techniques and dealing with concrete problems. Empiricism and utilitarianism were the hallmarks of mathematics from the very earliest beginnings in China about the sixteenth century B. C., emerging from legend and mysticism. Mathematics had a similar course of development amongst other peoples, such as the Babylonians, the Egyptians, the Romans, and the Arabs. The possible exception among the nations was Greece, where the abstract nature of mathematics was recognized and developed.

In Greece, the earliest abstract mathematician was probably Thales of Miletus (640 - 546 B.C.). He introduced some definite theorems and the first abstract geometry of lines. The name geometry, as we know, is derived from the Greek, and originally denoted earth measuring. It is probably true that Thales learned from the Egyptians who had a utilitarian approach which involved such things as measuring portions of the surface of the earth, after flooding of the Nile, in order to reestablish land boundaries. It was Euclid, a great Greek mathematician of the third century B.C., who established a set of postulates or axioms as the basis of geometry. As we know, his fifth postulate, on parallels, is celebrated in the history of mathematics. After Euclid, the first really axiomatic investigator was Gerolamo Saccheri, who lived in the seventeenth century A.D. It is easily verified that his studies arose out of an attempt to deal with problems of the 5th Euclidean postulate.

By the middle of the twentieth century, not only had the axiomatic foundations of geometry been established by such mathematicians as David Hilbert, but the very foundations of all of mathematics were under study and being planted on a fully axiomatic base. We have already reviewed the axiomatic and modelistic nature of the basis of all mathematics in chapter 15 in which we examined non-Euclidean geometry, the foundations of mathematics, metamathematics, and logic.

The reason we have referred again to these matters is to stress three facts:

1. Mathematics finally, in the twentieth century, has attained a definite axiomatic formulation, has its relations to logic clearly drawn, and systematically uses the concept of model.

2. Mathematics required a very long time -- about four millenia -- of intensive development to accomplish the solid foundation for itself in the twentieth century, and

3. Mathematics is a science of relations and logic and does not concern itself with things per se. Here we may recall Bertrand Russell's remark that mathematics is a class of propositions p that imply q. Also in a semi-humorous vein he remarked that mathematics is a subject in which one never knows what he is talking about or whether what he says is true. There is a certain biting truth in this last sentence. Mathematics is never concerned with the fundamental nature of things which are physical, psychological, biological, etcetera.

The mathematics program of the twentieth century is certainly an ideal. It suggests to anyone the importance of axiomatization of any discipline. Such attempts have been made by scholars in various fields during the present century. This is especially true in mechanics, which is a branch of physics. A great deal of progress has been made along these lines for subjects which are based on the particle dynamics of Newton and on the electrodynamic field theory of Maxwell.

In an excellent article by Patrick Suppes, a clear call is made for axiomatization in all disciplines and a proper use of what he considers to be the concept of the model [1]. We now wish to take some space to present our viewpoint on the speculation of Suppes on the axiomatic bases and on the model. It is our opinion that he has not properly allowed for the complete potentiality of the concept of model.

In recent years there has been an increasing number of seminars and colloquia devoted to the concept of model and the process of modeling. Two of them in particular are representative. We refer to a colloquium in Holland in 1960 [1] and a seminar in Venice in 1971 [2]. The latter consists of a rather broad coverage of application and the former is somewhat more concerned with a theoretical treatment of model. In the 1960 colloguium we wish to make particular reference to the paper of Patrick Suppes and, also, to that of Leo Apostel. It is the paper by Suppes which enables us to significantly illustrate one side of the modern approach to the understanding of the concept of model and its applications. We agree with a fundamental premise of Suppes, but take issue with what seems to be his lack of appreciation of the use of the model in its most general sense, especially as it applies to disciplines which have not as yet or cannot in principle utilize mathematics.

It is interesting to note that Suppes begins his paper with a set of examples of the use of model, cited by direct quotation and significantly he starts with Tarski's definition of model which states that a possible realization in which all valid sentences of a theory T are satisfied is called a model of T. We stress that such a view as that of Tarski is very close to the aspect of model which relates intimately with pure

mathematics and logic. We have previously discussed this aspect of model. Additional examples are cited by Suppes from the field of spectroscopy, the statistical model of Gibbes, the theory of games, the study of group opinions (Delphi method), the theory of learning, and stochastic processes. Of these subjects Suppes says that the first is taken from a book on mathematical logic, the next two from books on physics, the following three are from works on the social sciences, and the last one from an article on mathematical statistics. He further stresses that additional uses of the concept model could easily be collected in another batch of quotations. At the end of his brief survey of various fields he points out that he has omitted one of the more prominent senses of the word. It is the use in physics and engineering of model to mean an actual physical model as, for example, in the phrase 'model airplane' and 'model ship'. He believes that one may think that it is impossible to put under one concept the several uses of the word model exhibited in his quotations. However, Suppes very strongly presents his view concerning model and we now wish to quote him directly. He says, "I claim that the concept of model in the sense of Tarski may be used without distortion and is a fundamental concept in all of the disciplines from which the above quotations were drawn. In this sense I would assert that the meaning of the concept of model is the same in mathematics and the empirical sciences. The difference is to be found in the use of the concept." We might proceed further with an examination of the statements of Suppes, but we will terminate our treatment of his very interesting paper with what we consider to be a definitive position for him. He says, "Sufficient examples do now exist to make the point that there is no systematic difference between the axiomatic formulations of theories in well-developed branches of empirical science and in branches of pure mathematics." To emphasize this latter position he says further, "By remarks made from a number of different directions I have tried to argue that the concept of model used by mathematical logicians is the basis and fundamental concept of model needed for an exact statement of any branch of empirical science." Further on, however, he does say, "I am myself prepared to admit the significance and practical importance of the notion of physical model current in much discussion in physics and engineering. What I have tried to claim is that in the exact statement of the theory or in the exact analysis of data the notion of model in the sense of the logicians provides the appropriate intellectual tool for making the analysis precise and clear." We have now said enough so that one may get a clear idea of the thinking of Suppes concerning the concept of model and wish to state wherein we agree with him and also where we think we differ from him.

Suppes represents a fairly large class of serious thinkers on the concept and use of model. Their position has been suggested in our references to the thinking of Suppes. Now we wish to outline briefly our understanding of their position and, also, our consideration of what we think is a much larger and possibly more important view of the model. With some recollection one can readily detect the thought structure of what we might call the Suppesians. First of all we admit that they have a partially justifiable interpretation of model. However, in the last

analysis, it is limited to those disciplines that are totally axiomatizable. We saw that after a very long history mathematics was reduced to a totally sufficient base of axioms. The period from Euclid to Hilbert clearly demonstrates the situation. At this point one can reasonably ask what was the nature of the development of geometry from the time of Euclid and his axioms to the time of Hilbert, in the twentieth century, and his complete set of axioms. It is only sensible to note that geometrical development was in progress a very long time before the advent of Hilbert. We accordingly think that the difference between the view of the Suppesians and our own lies in the real meaning of the evolution of knowledge. We are looking at the entire process of the long period of development of mathematics and Suppes is concentrating on the end product, at a time when the subject is completely axiomatized. Logically, Suppes may be in a satisfactory position for his own immediate purposes, but such a position is not really useful for gaining the most insight for the total development of knowledge. He sees and uses a science in its deductive aspects, after complete development, and either underestimates or ignores the importance of the inductive and experimental phases for the development of radically new knowledge. Always of great importance to mankind are invention, discovery, and new design. These do not come from the science of mathematics as a completed body of axiomatic structure. No branch of knowledge comes into being instantaneously. In its history there is a more or less long period of gestation. For mathematics this period extended over thousands of years. However, once man learns thoroughly from one developed discipline, such as mathematics, he can gain immensely by applying the knowledge in other related disciplines. by applying the knowledge in other related disciplines. It can be seen that such has happened and is continuing to happen, for example, in mechanics which is really a branch of physics. Strong axiomatic foundations are now being laid down for mechanics. One may argue that the history of mechanics is as old as that of mathematics. We do not deny that fact but we do see that it was in mathematics that axiomatization was first developed and applied. It was in learning from the mathematicians of modern times that the scholars in mechanics really began their work on the foundations of that subject. Another fact should also be noted with respect to mechanics. It truly did develop in close proximity to mathematics from its early beginning and required the tools of mathematics for its development. Some people have even referred to mechanics, on occasion as a branch of mathematics. Such of course is not true, however, and one needs only to review such examples as the Rutherford-Bohr model of the atom to appreciate the fact.

In contrast to the treatment by Suppes we can see that Leo Apostel has a more varied interpretation of the concept of model. After first reviewing Apostel's definition of model we will compare it with our [A, B] definition.

We agree that Apostel covers the axiomatic treatment which was stressed by Suppes, however, he extends his notion of model far beyond. First, let us state his observations on the function of models in the

empirical sciences. He says, "Still, it is true to say that the aims mentioned -- theory formation, simplification, reduction, extension, adequation, explanation, concretization, globalization, action or experimentation -- constitute a kind of system." This is truly a large view of the concept of model. His examples, which we will not repeat here, truly illuminate the bare schematization of the various aims of modeling. For the empirical sciences, Apostel treats at length the various aspects under the following suggestive titles: Models and the progress of research, Models and experience, Models and experimentation, Models and explanation, and finally, Simplification and model-building. With respect to these, Apostel says, "The mind needs in one act to have an overview of the essential characteristics of a domain; therefore the domain is represented either as a set of equations or by a picture or by a diagram." We wish to pause here and examine a bit the thrust of this last quotation. It is truly reminiscent of Polanyi's attitude on machine, to which we referred in an earlier chapter. We suggest to the reader a comparison between the intent of "The mind needs in one act an overview" and Polanyi's thought that the principle of a machine must be grasped in one act of the mind. We also focus attention on the above statement that "therefore the domain is represented either as a set of equations or by a picture or by a diagram." With it we contrast our own suggestion that picture plays a basic role in the concept of model. We refer to our modelistic dyad of picture-theory and also our triad of picture-theory-experiment. A classical example from physics is the Rutherford-Bohr model and its corresponding theory of the atom. Finally, we think it useful to here deviate from the empirical sciences and recall the role of picture (metaphor) in the book of Genesis in the bible, which we consider to be an ancient and classical non-mathematical use of model. Finally, we emphasize the use in the above statement by Apostel of the expression 'set of equations'. The instant this expression enters our considerations of model it must be confessed that we are engaged in mathematical modeling.

In the latter half of his paper, Apostel turns to a development of the subject which is more in consonance with that of Suppes. Here he treats what he calls classical models. He deals with algebraic models, semantic models, and syntactical models. In these matters he is distinctly dealing with axiomatics, logic, and pure mathematics.

We cannot drop the version by Apostel until we quote his definition of model given at the end of a lengthy analysis of the whole subject. He says, "This will be our final and most general limit towards the definition of model: any subject using a system A that is neither directly nor indirectly interacting with a system B to obtain information about the system B, is using A as a model of B. The definition of 'using', 'purpose', and 'information about' are problems formal pragmatics is already beginning to tackle. While we do not think that this type of definition of the model concept is very fruitful (the syntactical, algebraic and semantic study of the various special model concepts seem to us immensely more fruitful), we are convinced at least that a general

definition along these lines is possible, adequate and formal." We again contrast this definition of model with our statement that A models B, where A and B are any two different things in the universe. It is quite clear that Apostel does not wish to come to this most general statement about model, especially in the light of his parenthetical remark above. We are of the opinion that he does not wish to do so primarily because he is concerned with mathematico-logical type models. We wish to go further into the matter in our later treatment of the general subject of models.

We may now return to a consideration of the contents of the Venice conference to which we referred above and see how diverse a treatment of the subject of modeling is possible. The emphasis is now on such subjects as the Institute for the Future, planning, simulation, game theory, computer modeling, and human interaction. In one of the articles N. Teodorescu deals with cybernetics and mathematical model. He claims that cybernetics derives from similarities between the working of different complex systems: mechanisms, living organisms, social, economic or administrative institutions. He admits that these are very different organizations, but they can be simulated by models representing their common characteristics, among which the more specific are the presence of feedback and dynamic equilibrium. A particularly telling remark of Teodorescu illuminates his viewpoint as well as that of many others. Because of its significant role in any general consideration of models we will now quote him directly. He says, "Such models, called 'cybernetic', can be considered from a unitary point of view which can only be that of mathematical modeling, because mathematical models by their generality and abstraction, are independent of the particular nature of phenomena and modeling objects."

The various articles are peppered with things mathematical. There are equations, theorems, algorithms, computer programs, diagrams, tables, and maps. It is quite clear that the subject is highly mathematical. We grant such a character of modeling but we wish now to stress our insistence that models and modeling have other facets.

We can do no better than to go back to our fundamental statement that A models B, where A and B are any two different things in the universe. Such a definition of model is no doubt the broadest that can be devised. What some may call its glaring defect is obvious to us. We do not deny that if we choose an A and a B at random they may have very little in common. The important thing, however, is that they must have something in common, if only their existence. Whatever it may be it can serve as a starting point for finding an ultimately effective model. As we have already shown, in modeling a sequence of models $\{m_i\}$ is usually considered. It is hoped that as we proceed in the sequence we converge closer and closer to the prototype. We plan to exhibit such processes in the chapters yet to come.

Since we are viewing mathematical modeling in the present chapter it is well to focus on mathematical equations, systems, or branches as either the A or the B of our broad definition. The assumption then is that there is a significant isomorphism between the mathematics and the physical, societal, military, industrial, political, or general scientific system to which it is considered to correspond.

We well know that some investigators do indeed attempt to use very crude mathematical modeling in a simulation process. If one thinks our A, B definition is uselessly broad he may examine some of these specific attempts to apply mathematics to obtain needed information in various disciplines. We have no fault to find with those who must start out with very crude mathematical models, but do insist that a very critical attitude be maintained at all times and that painstaking efforts be made to develop and improve the model.

It is obvious that in mathematical modeling one should expect to find mathematics playing an essential role. We wish now to devote considerable attention to all aspects of model and modeling. Accordingly, we will attempt to do this by studiously examining many important disciplines and their use of models. In our final chapter we will attempt to provide a clear overall view of the broad aspects of models and modeling.

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CHAPTER 17

MODEL AND ITS MODIFIERS

As we have implied in our treatment of modeling so far, we consider the presentation to be prolegomenal in nature and certainly not a treatise on the foundations of the subject. We hope that others more competent than ourselves will provide, in the not distant future, a definitive work. Notwithstanding, however, our present position concerning the matter, the previous chapter, which treates mathematical modeling, signals the immediate need for an examination of the problem of adjectival modifiers applied to model. As we have previously stated, others have attempted to develop definite classifications of models and to use specific modifiers. In order to help clarify our meaning, we shall now examine in some detail papers on the subject by two different authors.

The first is G. A. Mihram who presented a paper entitled The Modeling Process [1]. We have already referred to this work but wish to do so again in order to direct specific attention to that author's use of adjectival modifiers and to attempt to clarify what we consider to be an essential problem with respect to the concept of modeling. It may be recalled that Mihram recommended a classification which comprised twenty-four categories. These groupings were obtained by the division of models into those which are material and those which are symbolic. Each of these were further divided into sets of three each. The division was further extended by the introduction of the designations static and dynamic. Each of these latter concepts were further modified by the adjectives deterministic and stochastic. The prime division into material and symbolic significations was credited to Rosenblueth and Wiener. As the reader should know, these two scholars have contributed substantially to the advancement of our knowledge of models and the associated mathematics. We still consider that such rigid classification is premature and that it may do a disservice to the potentially broad and necessary treatment of the subject which now seems possible. Before leaving the method of Mihram, we wish to recall a few of his categories and their designations. Amongst others, he uses such adjectival modifiers as the following: deterministic dynamic model, stochastic dynamic analogue model, and dynamic stochastic descriptive model. Because no unified viewpoint on modeling has so far been developed, such clumsy multi-adjectival designations seem necessary. We agree with some need for the modifiers which have been introduced, but wish to consider them from a somewhat different viewpoint in the following exposition. We hope to develop an insight which provides greater unity and flexibility of what we consider to be the modelistic nature of universal reasoning. Before doing this, however, we wish to introduce the study of the second author mentioned above.

In a paper dated about a decade before that by Mihram, G. Frey presented a paper on symbolic and iconic models at a colloquium held at

Utrecht in 1960 [2]. This paper serves excellently as the point of departure for our treatment of the subject. Frey's short paper gives a detailed contrast of the iconic (which, as the reader may recall is an image or physical thing) with the symbolic. Using just these two principal terms, he becomes as involved with multiple modifiers as did Mihram. In fact he resorts to the use of primary and secondary as modifiers for iconic model. He, also, refers to iconic-symbolic model and non-iconic-symbolic model. We do not wish to pursue further the matter of multiple modifiers, but rather go to the root difficulty. A great deal of the problem seems to arise from the nature and role of mathematics.

As we have done in the past, so we wish now to take an historical view and see if the long evolution of human thought does not account for the apparent difficulty. As it turns out we do not have to look long for our point of departure. We think that classical Greek philosophy supplies the clues. As the reader knows, the Greeks of the Classical Period provided the first really systematic basis for philosophical thought and everyday reasoning. In particular we may begin and end with Plato. To make our point clear we will refer to the history of the subject and see that Plato associates mathematics very importantly with all thinking. In fact, at times, he appears to be almost mystical about the subject. That great Greek philospher was born as long ago as the fifth century B.C. Since his time much of philosophy has been under the spell of his thinking but more importantly for us now, it was his thoughts on mathematics that are important. He seems to represent, in a special way, the whole of mankind with respect to the formative impact of mathematics on thought in general. In his history of philosophy, Windelband says, "The importance which mathematics had possessed from the outset in the development of Plato's thought ---. The mathematical structures are the intermediate link, by means of which empty space which is not, is able to imitate in phenomena the pure forms of the world of ideas. Hence mathematical knowledge, as well as purely philosophical knowledge, has to do with an abiding essence, and is therefore comprised together with this, as rational knowledge, and set over against knowledge of phenomena" [3]. If such is the character of Plato's thinking and if it indicates the real nature of the thought process, we see clearly the role of mathematics with respect to modeling. We will attempt to expand on our thinking along these lines, but first we must refer to the ideas of another historian about the position of Plato. In his history of philosophy, Copleston says, "According to Aristotle, Plato declares that:

1. The Forms are Numbers

2. Things exist by participation in Numbers

3. Numbers are composed of the One and the great-and-small or 'indeterminate duality'" [4].

Copleston refers to Plato's thinking as pan-mathematization. He seems to take exception to such a position but does say that pan-mathematization and idealism might even support one another. Copleston also says, "The more Reality is mathematized, the more, in a sense, it is transferred on to an ideal plane. While, conversely, the thinker who desires to find the true reality and being of Nature in an ideal world might easily grasp the proffered hand of mathematics as an aid in the task." The influence of such thinking of Plato has persisted to the present time. To us it seems to underline the relationship of mathematics to the total thought life of man. In the light of history, we can perceive how the present day formal treatment of the concept of model is controlled by mathematics. We do not need to pursue further this phase of our subject. It is surely a keystone for anyone attempting a philosophy of modeling as others have done for mathematics itself.

It is quite easy to appreciate that if every A in the universe models a B, as we have suggested for a starting point in any modelistic thinking, then the A or the B can readily be conceived of as a set of mathematical equations or a mathematical system. We now know that business men conceive of mathematical equations as models of certain aspects of business processes. We stress the fact, however, that even though mathematics provides models for real world situations it also enjoys a unique position in the field of universal ratiocination. It may be recalled that in our original enumeration of model types such as the iconic, the analogic, the similitudinous, the Newtonian, and the extended Newtonian, mathematics played a definitve role. In most of these cases, however, if not in all, we were interested in one material or real system modeling another. Mathematics served the purpose of a bridge connecting the two systems. For example, the analogic model involved two or more physical systems whose behavior was precisely described by the same mathematical equations. Also, for similitudinous models the mathematics was used to relate the performance factors of the scaled physical model with that of the prototype. In other words, the performance of the prototype, say a ship, was obtained from the measured performance of the small scale model by means of a mathematical analysis. We will now temporarily leave this mathematical aspect of the modeling problem and consider more in detail some other important adjectival modifiers: static, dynamic, deterministic, and stochastic. We shall treat these terms in pairs which seem to be antinomies.

The static contrasts with the dynamic and the deterministic with the stochastic. Apparently the paired terms are essential to each other for comparison purposes and, also, for basic comprehension. Notwithstanding these relationships, however, there is sometimes an attempt by engineers, scientists, and even theologians to consider as inherently identical the two modifiers in each antinomic pair. Again the use of a brief historical commentary may demonstrate the growth from static to dynamic and from deterministic to stochastic.

We refer again to the paper by Mihran. There he makes a revealing statement about the four modifiers. He says, "Clearly the dynamic model, whose attributes alter with time, is a generalization of the static variety, and, in any case, deterministic representations are merely special cases of stochastic models." In a more specific vein he says, " A model is said to be dynamic or static depending on whether its features or symbols do or do not respectively alter perceptibly with time." Perceptibly is a key word because it is obvious that in the final analysis there is nothing that does not change with time. We think it useful to again view such problems in an historical sense and see that such distinctions as static and dynamic plagued man as his knowledge of the world developed. The pre-Socratics in Greece pondered a world in flux, but a total flux leads to confusion. A counteractive stance to flux is a static, fixed, absolute, ideal world. Man seemingly cannot tolerate the conception of a wholly static or a wholly moving world scene. The apparent dichotomy of the static and the dynamic must be adapted to the mentality of man. As far back as Heraclitus [c. 536 - 470 B.C.] the essential difficulties were apparent. Windleband in his history, on page 36, says, "His (Heraclitus) doctrine ---. Not only individual things, but also the universe as a whole are involved in perpetual, ceaseless revolution: All flows and nothing abides. We can say of things that they are; they become only, and pass away in the ever changing play of the movement of the universe." Windleband further says, "The conception, however, which Heraclitus has grasped with complete clearness, and carried through with all the strength of his austere personality, is that of order, a conception nevertheless, whose validity for him is as much a matter of conviction as of knowledge." And finally, on page 38, he says, "--- neither the world-stuff or cosmic matter of the Milesians, nor the 'fire-becoming' of Heraclitus, nor the Being of Parmenides were available for explaining Nature. Now the imperfection of the first had become clear through the contrast which separated the two latter as a gulf, and with the recognition of this, occasion was given for the more independent investigations of the next period to separate in their conceptions the two motifs, being and becoming, and by setting them over against one another to think out new forms of relation, out of which permamently valuable categories for the knowledge of Nature resulted." We have mentioned this matter in some detail to emphasize the fact that the problem of change, which is change in time, has been a matter of central concern since the beginning. We may even go back to pre-historic days to obtain indications of the confusion. While the history of rational development is very long and beyond our capabilities to properly portray, it is indeed clear to us that anyone with a little reading and reflection will discern the outline of the evolution of the ideas with which we are now concerned. The practical value of the long development is patent in the twentieth century. Mark Reiner, the grand old man of what has come to be known as the science of rheology, referred poignantly to the role of Heraclitus in foreshadowing the recent thoughts about change in the Cosmos. As we may know, rheology is the study of the flow and deformation of matter. It implies change with time. Reiner was almost biblical in his contemplation of the flow of

even such things as mountains. We have had occasion to speak not only about the practicality of rheology and its relationship to modeling but also its interdisciplinary thrust in its relation to biology, spawning the new science of biorhelogy [5].

We conclude our remarks on static and dynamic by observing that undue clinging to the static model may lead to dangerous preoccupation with fixity. Structural engineering, for example, has suffered in the past by undue reliance on the static model and denying the dynamic. Religion, also, provides an example of some who cling to a static theology in the face of need for a process or dynamic theology. In religion the static model led to Vatican II and in engineering it led to the disastrous failure of the Tacoma Narrows bridge. The reactions to the growing awareness of man's difficulty in clinging too strongly to fixity lead to a very salutary re-examination of our state of knowledge about everything. We must now leave consideration of the static and dynamic models in order to examine the two apparent antinomies which we call deterministic and stochastic.

As in the case of the historical development of static and dynamic, we find much the same situation in the case of the deterministic and the stochastic. One seems to come before the other in the evolution of thought, but the other always seems to be hovering in the background. With regard to the strictly scientific development of these ideas the problem is clear. To truly create dynamic analysis and stochastic analysis, the required mathematical theory had to be available. Much of the required technique and development in mathematics was not available until very recent times. So we can see that there should be no mystery as to why stochastical and dynamic analyses lagged those in the fields of the static and the deterministic.

There are many philosophical and historical treatments of the subject such as the little monograph by G. Spencer Brown [6]. Instead of referring explicitly to these, however, we shall now take a specific example to illustrate the various points that arise in any substantial treatment of these topics. In order to do this we shall refer to an experience of the senior author which occurred back in the early part of the sixties. At that time he happened to be a member of the Research Committee on Random Vibration in the American Society of Mechanical Engineers. The committee had the pleasure of monitoring a research project by S. H. Crandall and W. D. Mark of the Massachusetts Institute of Technology. The project resulted in the publication of a small monograph on Random Vibrations in Mechanical Systems [7]. While the two authors of that little work claim no originality for subject matter or treatment we can recommend their effort as a clear presentation of principles and applications to definite problems. For our present purpose we wish to examine in some detail one of the two specific problems of random vibration which they investigated. It may enable some of our readers to see more clearly what is involved in the analysis of deterministic and stochastic processes. It also gives an opportunity to view briefly the static and the dynamic as they occur in engineering.

The case for present study is the damped linear oscillator as it is called in physics or the single-degree-of-freedom vibratory system as it is usually designated in engineering. Its defining differential equation is as follows:

$$c_1\ddot{x} + c_2\dot{x} + c_3x = \Phi\ (t)$$

where the dot above the x means derivative with respect to time as used by Newton in his study of fluxions. The x may be considered as a response or a displacement which is assumed to be a function of time. The c_i are the system parameters and the ϕ function represents a driving or forcing factor which stands for the imput to the system. In order to make the use of these quantities clearer we refer them to a definite physical system which we consider to be a mass particle suspended from a fixed base by a spring. A damping dashpot is attached to the mass particle which is driven by some external force ϕ. It may be recalled that in our earlier discussion of model types the present differential equation was used to characterize several different analogic models. Also it can be seen to constitute a simple example of what we called a Newtonian model.

Now we wish to examine various possibilities which arise with respect to the differential equation and its related model. First assume that the c_i are fixed with respect to time (i.e. constants) and that the ϕ is a definite known function of time. If initial conditions are specified, that is the position and velocity of the mass particle are known at some time, a solution exists and we can definitely determine the location and velocity of the mass particle at any subsequent time. Obviously we would call such a system dynamic and dterministic. If the c_1 and c_2 are zero and the function Φ a constant, that is independent of time, we would have a very simple equation which gives the statical deflection of a spring supported mass under the influence of an external force. This very simple problem which we are now considering demonstrates rather clearly why dynamical analysis in engineering lagged in development behind the static. The inertia and elasticity of this simple problem can be imagined replaced by a more complicated elastic solid which is loaded by external forces. Again the analysis for the statical cases, even though it may sometimes be very difficult, is very much simpler than for the case in which the applied forces vary with time. For the case of the vibrating body suspended by a spring, the c_1 is actually the mass of the body, the c_2 is the damping coefficient, and the c_3 is the spring constant which specifies the elasticity of the system. With this simple physical model we have exemplified the static, the dynamic, and the deterministic. It remains only to see how the problem can become random, probabilistic, or stochastic. For the present we are implying that these three terms suggest the same thing.

Now if the c_i coefficients are known constants but the ϕ function is given only in terms of some statistical data, the process described by the differential equation is no longer deterministic but is said to be probabilistic or stochastic. It means that we do not have complete knowledge of the input to the system as we did when ϕ was a well-defined given function. However, we may still have interest in the output of the system if we can define in some statistical manner the function ϕ. Engineers and scientists of many sorts are now rapidly coming to realize that many of the important systems with which we must deal, if not all of them, are of such a nature.

We would like to emphasize that for many engineers, and probably many non-engineers, the little monograph by Crandall and Mark may serve as an excellent introductory approach to the study of random processes and stochastic variables. The exposition is clear and precise. Such matters as random process, probability distribution, ensemble averages, temporal averages, stationary and ergodic assumptions, and autocorrelation are simply presented and applied in connection with concrete examples. They say, and we concur, that they show in principle how it is possible to give complete probabilistic information about a random process. Also, when such information is available, it is a simple task to calculate statistical averages for the process. The autocorrelation function is simply explained and related to the mathematical expectation, which concept is so important in practical decision making.

The importance of the foregoing in the technical literature is illustrated by an article on the subject by Rex and Roberts [8]. In their brief paper we find the use of correlation as a measure of the similarity between two wave forms. The authors stress its importance in every kind of research and engineering. The need for such studies occur in connection with electrical, mechanical, acoustical, medical, nuclear, and many other disciplines. They outline some uses such as detection of signals hidden in noise and they stress that autocorrelation is uniquely successful in the detection of unknown periodic signals. Whereas the autocorrelation function of a waveform is a graph of the similarity between waveform and a time-shifted version of itself, as a function of time, cross-correlation measures the similarity between two non-identical waveforms $x(t)$ and $y(t)$. The cross-correlation is used in many ways. For example, an approximation to the impulse response of a linear system can be determined by applying a suitable noise signal to the system input, then cross-correlating the noise signal with the system output signal.

Because of the great importance of the stochastic model we wish now to devote considerable attention to it and its mathematical foundations. We are convinced that this is the roadway to progress, not only for engineering, but for all systems analysis of the future. Mankind is finally coming of age in a probabilistic world and is increasingly

conscious of the fact. Since probability and statistics are basic in the consideration of stochastic processes, which we now consider are coextensive to some degree with all processess, we wish to give consideration to their discovery and development. First we will present a brief historical review.

There is no doubt that some notion of probability and statistics was available to man since his earliest days. The chance that one might be killed or the size of harvests over the years must have been of concern from ancient times, crude though those ideas might have been. Notwithstanding this probabistical and statistical type of sensitivity of man it was not until the 17th century that both probability and statistics came into being in any manner that resembled scientific method. The fact is that sufficient mathematical knowledge was not available until then.

It seems that statistics was introduced as a formal subject by John Graunt, a haberdasher who was born in London in 1620. He published his work in a thin book entitled Natural and Political Observations made upon the Bills of Mortality. It was the first attempt to interpret mass biological phenomena and social behavior from numerical data. Available to Graunt were the fairly crude figures of births and deaths in London from 1604 to 1661. His tract appeared in 1662. Thirty years later the Royal Society published a paper by the astronomer Edmund Halley on mortality rates. Halley was a distinguished scientist and accorded great acclaim in his time. However, the lowly tradesman, Graunt, was not overlooked. He was elected member of the newly incorporated Royal Society through the good graces of Charles II.

The problem which initiated the vast theory of parobability was proposed to Blaise Pascual by the Chevalier de Méré, a gambler. It was a problem to determine the chance which each player has at a given stage of winning a game. The nascent theory of probability grew out of correspondence between Pascal and P. Fermat concerning matters which related to such problems. It is pointed out by E. T. Bell that it is appropriate to consider Pascal, who was born in 1623, and Fermat, who was born in 1601, as the cofounders of the mathematical theory of probability [9].

It was Jacob Bernoulli I [1654 - 1705] who produced in 1713 the first great treatise on probability, entitled Ars Conjectandi, which might be roughly translated as the art of guessing. The work was really a definite start towards applications to insurance, statistics, and the theory of heredity. Following this Bernoulli there were other great mathematicians who contributed to the early development of the subject. Even the great Joseph Louis Lagrange [1736 - 1813] applied the differential calculus to the theory of probability.

The Marquis Pierre - Simon de Laplace [1749 - 1827] of Méchanique Céleste fame, again demonstrated his mathematical prowess with a treatise

on probability in 1820. It may be recalled that the great man was himself somewhat of a gambler. Bell quoted Laplace concerning his opinion of the subject. Laplace said, "We see ... that the theory of probabilities is at bottom only common sense reduced to calculations; it makes us appreciate with exactitude what reasonable minds feel by a sort of instinct, often without being able to account for it .... It is remarkable that [this] science, which originated in the consideration of games of chance, should become the most important object of human knowledge." The statement is obviously a very strong one, but one cannot consider seriously the matters of everyday life without giving some degree of assent to it.

At this point we wish to consider three points of view in connection with probability. One brief description of these is contained in volume 2 of the World of Mathematics by James R. Newman [10]. There it is stated, "There are three main interpretations of probability. The classic view, formulated by Laplace and De Morgan, holds that the notion refers to a state of mind. None of our knowledge is certain; the degree of strength of our belief as to any proposition is its probability. The mathematical theory of probability tells us how a measure can be assigned to each propositon, and how such measures can be combined in a calculus. Another view defines probability as an essentially unanalyzable but intuitively understandable, logical relation between propositions. According to John Keynes, a principal exponent of this interpretation, we must have a logical intuition of the probable relations between propositions. Once we apprehend the existence of this relation between evidence and conclusion, the latter becomes a subject of rational belief. The third view of probability rests on the statistical concept of relative frequency. This interpretation was developed during the last century by the Austrian philosopher and mathematician Bernhard Bolzano (1781 - 1841), the English logician John Venn (1834 - 1923), the French economist Cournot, and by Charles Sanders Pierce; and in our time by R. A. Fisher and Richard von Mises among others. Statistical probability stems from the idea of the relative frequency of an event in a class of events. Thus for example when it is said the probability of surviving an attack of pneumonia if sulfa drugs are proptly administered is 11/12, what is meant is that records show that 11 out of 12 persons who have had this disease and received this treatment have recovered. Most scientists today think of probability in this sense".

Before leaving the historical aspects of our subject we wish to refer to some strongly influentail happenings at the turn of the present century. These particular events greatly influenced the use of probabilistic and statistical methods in the physical sciences and furthermore set the mood for the vast growth of the applied side of the subject. We first refer to the pioneering work of Josiah Willard Gibbs, the great American scientist, on statistical mechanics [11]. In his last work, Elementary Principles of Statistical Mechanics,

Gibbs became a member of that small group of scientists who at the beginning of the 20th century introduced, developed, and applied the statistical method to problems in science. We may do well to let Gibbs speak for himself and so accordingly we quote from his preface. He says, "The usual point of view in the study of mechanics is that where the attention is mainly directed to the changes which take place in the course of time in a given system ......

For some purposes, however, it is desirable to take a broader view of the subject. We may imagine a great number of systems of the same nature, but differing in the configuration and velocities. And here we may set the problem, not to follow a particular system through its succession of configurations, but to determine how the whole number of systems will be distributed among the various conceivable configurations and velocities at any required time, when the distribution has been given for some one time." Gibbs further says that Clerk Maxwell called such studies statistical. Here we have one of the two first modern thrusts of statistics and probability into physics. The other one concerns the study of Brownian motion.

We may recall that a certain scientist Brown (1877) observed that small particles immersed in a liquid exhibit ceaseless irregular motions. This motion, now called Brownian motion, was first explained by Albert Einstein in 1905, the same year in which he published his famous paper on the theory of Special Relativity. Einstein postulated that the particles under observation are subject to perpetual collisions with the molecules of the surrounding medium. The analytical results, of a statistical nature, were later verified experimentally by various physicists. Although we have only mentioned two scientific giants who used statistical methods at the very beginning of the century, maybe we should add a third. He was the great Boltzmann who increased our understanding of gas behavior at the microscopic level.

Even though the biological sciences have begun to use mathematical models on a large scale only in very recent years, it can be shown that some very important fundamental work of a statistical nature was done as far back as the eighteenth century. We have already mentiond the publication by Graunt who, although he was actually a tradesman, treated statistically biological data. We might almost have mentioned the work by Thomas Robert Malthus [1766 - 1834] which resulted in the famous Essay on the Principle of Population as It Affects the Future Improvement of Society. We know that Malthus studied at Cambridge and distinguished himself in mathematics.

An epoch making investigation in statistical biology was conducted by the then little known Augustinian monk Gregor Mendel. For seven years (1856 - 1863) in a little monastery garden he performed thousands of crossing experiments on pea plants. Of course he had to employ the methodology of statistics and combinatorial analysis in studying the vast quantity of information that he compiled. As a consequence he

introduced the mathematical studies of heredity and foreshadowed the great work of the 20th century in genetics. His work, although in the beginning was ignored, is of such a calibre that some have even considered Mendel the father of applied statistics. In more recent times the mathematical theory of natural selection, where inheritance is Mendelian, has been developed by R. A. Fisher, S. Wright, and J. B. S. Haldane. Haldane has said, "The permeation of biology by mathematics is only beginning, but unless the history of science is an inadequate guide, it will continue, and the investigations here summarised represent the beginning of a new branch of applied mathematics." In order to stress the value of interdisciplinary studies we close our present observations on biology by reference to some of the work by the physicist Erwin Schrödinger. That 20th century giant of the physical sciences made a study of the relation of heredity tc quantum mechanics. In an article on the subject, he says, "Much more important for us here is the bearing on the statistical concept of order and disorder, a connection that was revealed by the investigation of Boltzmann and Gibbs in statistical physics. This too is an exact quantitative connection and is expressed by:

$$\text{entropy} = K \log D$$

where K is the so-called Boltzmann constant (3.2983×10^{-24} cal/°c) and D a quantitative measure of the atomistic disorder of the body in question."

To demonstrate that biologists themselves have contributed significantly to the mathematics which is applied to statististics one need go no further than Sir Ronald Fisher. He has held a professorship of genetics at Cambridge University and was a pioneer in the theory of design of experiments. We shall trace out the model aspects of these subjects in later chapters.

Along with the great developments in biology and statistical mechanics which have finally surfaced in the present century we must emphasize certain important activities of a different nature which markedly influence the development of modeling theory and modeling awareness. These are game theory and operational research.

During and since WWII, resemblances between games and socio-economic models seemed to justify the belief that the study of games might be a fruitful approach to an understanding of rational behavior in social and economic processes. Much serious though was devoted to the subject and this eventually resulted in the great treatise entitled Theory of Games and Economic Behavior by Von Neumann and Morgenstern [12]. Its aim is not merely to show analogy between the competitive aspects of games and those of economics. It was destined to convince students that economic behavior and games of strategy model each other and that, as usual for models in general, the defining mathematics is identical in nature. The authors never claimed that their treatise provided a

complete mathematical theory of society, as well it could not. They limited their analysis to just a few economic problems and even there the theory and application are only at the beginning of possible future developments. However, their effort is very promising and it is only just to give the authors credit for laying the foundations for penetrating mathematical studies and suitable model analysis in social sciences.

Finally, we wish to include a very potent modeling science that came out of WWII. It has universally been called Operations Research. No one questions that it materially contributed to the development of every major weapon from guns to atom bombs. It may appear ironic to some and cruel to others, but the fact is that along these line of scientific analysis war has contributed to civilization since the earliest days of man on earth. A vast host of scientists since before Achimedes have contributed. The peculiar activity which has been formalized as Operations Research consists of an amazing methodology. It uses a conglomerate of methods. Some have defined it as "A scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control". One can see that we are here involved with the science of decision making. As always in our histories someone is usually considered the father of a major subject. In this case it is Philip M. Morse who in his earlier years was known as a competent acoustician. An examination of his biography in American Men of Science will sketchily reveal the evolution of an Operations Research Father. A perusal of a small paper that he wrote, along with George E. Kimball the chemist, will illustrate something of the nature of the new science. It is entitled How to Hunt a Submarine.

It is not appropriate to pursue all of these matters further at this time but we will mention two scientists who have provided brief and incisive studies of some of the things we have been examining in connection with stochastic models, Specifically we have in mind the mathematical aspects of the subject. A relatively recent treatment of probability and statistics, which uses such old devices as a bag model (reminiscent of our early school days when games of chance were exemplified by dice, cards, and picking colored balls from a bag or urn), is provided by Hans Freudenthal [13]. The reader may find this book interesting but we must admit it is very sketchy in nature. Along with a reading of Freudenthal we recommend for the uninitiated a quick review of combinatorial analysis. There have been a number of excellent treatises written on this subject alone in recent years but for present purposes we refer to an article by A. A. Bennett, the author of Tables on Internal Ballistics, in the Encyclopedia Britannica [14].

The general subject of the mathematical foundations of probability and statististics with practical applications is so important to our subject of modeling that we will devote the next chapter in its entirety to it.

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CHAPTER 18

MODERN MATHEMATICAL METHODS USED WITH STOCHASTIC MODELS

Our objective, from the outset, has been the examination of the concept and uses of models. We have indicated some of the possibilities for the definition of the concept and have provided many examples of the uses. From what one sees there certainly appears to be a great need for a philosophy of model just as there has finally come to be a philosophy of mathematics. When that is fully accomplished at some future date, we see the chance then of obtaining a definitve treatment of mathematics as related to model. One thing is clear from the variegated models now in existence. The two disciplines, mathematics and models, are independent. In fact, on occasion, we have said that any A in the universe may be considered a model of any B, however imperfectly. A set of mathematical equations may be considered a model of physical phenomenon or of an economic system, for example. In the last chapter we dealt with four important adjectival modifiers of model — static, dynamic, deterministic, and stochastic. In the present chapter we wish to clarify and extend our notion of the important concept of stochasticity. Also, we wish to discuss the relationship of mathematics to stochastic models. Some definite knowledge of that relationship is essential for the comprehension and use of stochastic models. Before proceeding further with such an investigation, however, we consider it necessary to examine in some detail the concepts of system and process, which are so often related to the treatment of stochastic models. It is true that the words system and process are used with great frequency. Also, any systems analyst is surely conscious of what he is analyzing. However, in books written on the subject of systems or in books using the word process, there is seldom a specific definition or description provided at the beginning. It is usually assumed that one knows what is being studied. Despite that fact we will now examine some definitions given in the American Heritage Dictionary and in the Oxford Dictionary for system and process.

First we quote the Heritage Dictionary. It says:

"System - 1. A group of interacting, interrelated, or interdependent elements forming or regarded as forming a collective entity.
2. A functionally related group of elements, as:

a. The human body regarded as a functional physiological unit.

b. A group of physiologically complementary organs or parts.

c. A group of interacting mechanical or electrical components.

d. A network of structures and channels, as for communications, travel, or distribution."

Also, they define Process as follows:

"1. A system of operations in the production of something.

2. A series of actions, changes, or functions that bring about an end or result."

Similarly, the Oxford Dictionary says that a System is:

"An organized or connected group of objects

1. A set or assemblage of things connected, associated, or interdependent, so as to form a complex entity.

In Physics - A group of bodies moving about one another in space under some particular dynamical law, as the law of gravity."

and for Process:

"A particular method of operation in any maufacture; something that goes on or is carried on; a continous and regular action or succession of actions, taking place or carried on in a definite manner."

Then as an example of system we have solar system and of process we have Bessemer process. The former is simply the sun and the set of planets which move around it. The latter is defined as a method for making steel by blasting compressed air through molten iron, burning out excess carbon and other impurities. We can also speak of such things as the Armed Forces, the Church, the Government, and the Stock Market as systems. Besides Bessemer, autofrettage, spinning, weaving, and computing by machine may be spoken of as processes.

We can also speak of Process in a more profound sense. An outstanding philosopher, Alfred North Whitehead, has significantly introduced the concept into twentieth century philosophy. His book entitled Process and Reality is a trail blazer [1]. In it he speaks extensively of "the process of time" and "the process of the temporal world". The reader might do well to read an interpretation of Whitehead in an article by an old friend of our days spent at the Hopkins University, Professor Victor Lowe [2]. Lowe says in his introduction, "Whitehead's amazing philosophical achievement is the construction of a system of the world according to which the basic fact of existence is everywhere some process ---."

Process models are discussed by E. H. Cousins in connection with contemporary theology [3]. He says, "Modern man senses the dynamism of nature, the reality of time, and the possibility of novelty. Out of this experience the process vision has emerged. It was nurtured by the revolution in the scientific view: through Darwin's theory of evolution and Einstein's theory of relativity. It was formulated by philosophers in the nineteenth and twentieth centuries. Through Alfred North Whithead and

Pierre Teilhard de Chardin it is having an increasing influence ---."

Stochasticity as related to models has recently been treated in a book entitled Simulation: Statistical Foundations and Methodology by G. Arthur Mihram [4]. We mention this monograph for several reasons. It is a very recent publication and it is concerned with the subject of models. We mentioned earlier that Mihram attempted a categorization or taxonomy of models in an I.E.E.E. paper. That effort demonstrated that he is seriously interested in the more general aspects of models. Now however, he uses the word simulation, which is a modeling process, in his new book title. Also he claims to be the author of a statistical foundations. We would take this occasion to emphasize our opinion that statistical foundations and probabilistic foundations for the study of models are to be found completely and competently treated elsewhere. Furthermore, it is apparent that authors, such as Mihram, are becoming involved exclusively with the mathematical aspects of modeling. In his recent book, on page 207, Mihram refers to a dynamic, stochastic simulation model. We consider that such expressions as simulation model contain a redundancy. We have mentioned such inappropriateness of some modifiers in the last chapter. Notwithstanding our critical comments, however, we recommend the book to our readers. Mihram is seriously concerned with the use of models and recognizes that statistics and probability can sometimes play essential roles in their treatment. Our reference to his work provides us with a starting point from which to approach the mathematical aspects of the subject. We propose to examine literature which truly provides us with foundations. In what follows we encounter such expressions as stochastic model, stochastic system, and stochastic process. The use of the term system is illustrated in such books as that by Mickle and Sze on Problems in Systems Engineering [5]. On their page 252 they define model of a system as follows. They say that "A model of a system can be defined as a mathematical representation of the system relationships." They, also, define stochastic system negatively as follows. They say, "A system in which the state resulting from a decision is uniquely determined by the decision is termed deterministic. A system which is not deterministic is termed stochastic."

Our aim now is to examine the subject of stochastic model in the light of suitable foundations provided by modern authors who have considerable mathematical competence. First, however, we wish to recall some elementary notions concerning probability and statistics. Even beginners in mathematics deal with some of the ideas with which we are concerned but usually not from the standpoint of models. The history of the subjects quickly supplies the rudimentary concepts. In the beginning it was games of chance which led Pascal and Fermat, as we have seen, to initiate a theory of probability. Laplace, one of its important developers, also was interested in the gambling possibilities. So for our purposes we can approach the theory of stochastic modeling by reexamining the simplest cases of probability. As every reader knows there are games with dice, cards, and lottery drawings. Permit us to review the

well known. In all of these examples there is a chance determination of a number, either by rolling a die, selecting a card, or spinning a roulette. We know, for example, that each of the six faces of a die contains a different number, from one to six. If we roll an unloaded die, a random single number appears. The probability for any one number turning up is one-sixth. It is important to stress the fact that for such a process one never knows for certain what number will appear. Such a lack of knowledge is characteristic of all probabilistic or stochastic processes. If we wish we can reasonably call the games of dice, cards, or roulette, models. We may even think of dice as the model of cards, or vice versa. If we have six cards each of which has a different symbol and we draw one from a well shuffled pack we encounter the same probability of obtaining a given card as we do for obtaining a given number when we roll a die. So we see that the mathematics for two physically different games is the same. We might even use the term analogue in such a situation. Furthermore, the statistics which will be developed by rolling or selecting a large number of times are similar. We can readily generate many variations from the elementary games. For example, we may roll sets of dice, play with cards of several suits, or select colored markers from multiple urns. As we have mentioned before, much of the theory associated with such processes arises in a serious mathematical subject called combinatorial analysis. A clear, concise, and useful presentation of the subject appeared during the last decade in a small monograph by C. Berge [6]. As we now know, the bases for many important human activities are to be found amongst these probabilistic games. We agree with the dictum of Laplace that probability is one of the most important objects of human knowledge. The subject, as a mathematical discipline, is just a little over three hundred years old. However, in recent times there has been a considerable development of the subject and an ever proliferating application.

During the last few decades satisfactory textbooks, which may serve as foundations for model analysis, have appeared. We wish now to cite an excellent example. In 1968 the third edition of An Introduction to Probability Theory and Applications by W. Fellers appeared [7]. It seems appropriate for us to consider somewhat the constitution of that text.

In its preface Feller says, "When this book was first conceived, more than 25 years ago, few mathematicians outside the Soviet Union recognized probability as a legitmate branch of mathematics. Applications were limited in scope, and the treatment of individual problems often led to incredible complications." This is a significant statement but somewhat misleading. From what we have said previously many mathematicians have dealt effectively with the subject during the last several centuries. Feller would certainly not deny that Pascal, Fermat, Jacob Bernoulli, K.F. Gauss, Laplace, Lagrange, and Poincaré are mathematicians and contributors to the theory of probability. The last named of these wrote a book in 1912 about the foundations of science [8]. He discussed probability and his chapter IV is an incisive analysis of the concept of chance. The reader can readily see that that mathematician must have had a profound

understanding of probability and its applications. What we think Feller does mean can be illustrated by an experience that dates from the days of the Great Depression. At that time a long line of mathematical statisticians came to Washington. Some were the authors of impressive mathematical treatments of statistics, which essentially use the theory of probability. Despite this fact one may reasonably question whether this particular group of investigators, with their great mathematical prowess, did much to get us out of the Depression. They were, however, part of a growing tradition which is now becoming effective in solving stochastic or probabilistic problems of an improtant kind. After WWII there arose great interest in another important technical problem of the probabilistic type. It was the problem of turbulence in rapidly moving fluids. The phenomenon is essentially stochastic and will really only admit of probabilistic analysis. However, at that time, the engineers and physicists who were mostly concerned were only beginning to become learned in the application of the necessary mathematics.

We agree with Feller on a very important point in his implications. Certain mathematical devices and modelistic concepts are essential in order to apply the probabilistic thinking effectively. In his book one can find a cogent treatment of the necessary developments. There is only needed a perusal of the table of contents of his text to verify the coexistence of the new concepts and old techniques which are required for applications. These include sample space, combinatorial analysis, random walk, combination of events, conditional probability, stochastic independence, probability distributions, unlimited sequences, random variables, expectation, law of large numbers, generating functions, branching processes, recurrent events, ruin problems, Markov chains, and time-dependent stochastic processes. Feller poignantly observes, on his page 2 that, "The philosophy of the foundation of probability must be divorced from mathematics and statistics, exactly as the discussion of our intuitive space concept is now divorced from geometry ---." He is dealing here with a thorny problem. His rather ineffective dealing with the difficulty emphasized our conviction that there is a great need for a philosophy of models. We have seen already in our perusal of the model in mathematics and the relation of logic to mathematics that the 20th century has witnessed the arrival of an appropriate philosophical treatment of mathematics, which for us includes the mathematical theory of probability. We now know that these subjects are ultimately axiomatizable. Feller certainly senses this situation when on his page 3 he says, "Historically, the original purpose of the theory of probability was to describe the exceedingly narrow domain of experience connected with games of chance, and the main effort was directed to the calculation of certain probabilities. In the opening chapters we too shall calculate a few typical probabilities, but it should be borne in mind that numerical probabilities are not the principal object of the theory. Its aim is to discover general laws and to construct satisfactory theoretical models". The underlining is ours. It is clear that Feller, as well as the other analysts writing in the same vein, are concerned with important applications of stochastic theory to present day problems.

As an example of how computations may be made on systems, only one of which may ever be actually constructed, he cited the automatic telephone exchange, which is a multibillion dollar investment. The exchange is designed on the basis of probability models in which various possible systems are compared. The theoretically optimum system is constructed and the others are discarded. A commonplace application of probability and mathematical statistics is the business of insurance which needs to know something of the probability of ruins. It is shown how probability theory is used to avoid undesirable situations. We know that probability analysis is useful even in situations for which numerical data are not available.

On his page 419, Feller observes that the term "stochastic process" and "random process" are synonymous and cover practically all the theory of probability from coin tossing to harmonic analysis. It seems that stochastic process is used mostly when a time parameter is introduced.

As we previously implied, we refer to the text by Feller as a valuable contribution to the probabilistic foundations of stochastic models. The reader could well use it as a guide to the modern treatment of the subject. Before leaving our comments on one of the very useful modern writers, we wish to specifically agree with him that intuition develops with the theory and that modern probability is at base statistical. Furthermore, it can be reduced to an axiomatic system. Feller acknowledges that a fully axiomatic treatment was developed by A. Kolmogorov in 1933 [9]. Along the lines of such modern developments we should mention a book on the theory of random processes by Gikhman and Skorokhod [10]. Those authors say that in their first five chapters they treat measure theory and axiomatization of probability theory. They stress the fact that the theory of random processes has recently developed into a separate branch of parobability theory. We agree with their opinion that the construction of a mathematical model allows a rigorous and formal definition of random process. Their classification of processes includes those with independent increments, Markov processes, Gaussian processes, and stationary processes. The mathematics used to calculate the probabilistic characteristics of random processes include differential and integrodifferential equations for the Markov processes, integral equations with symmetric kernel for the Gaussian processes, and Fourier transform theory together with the theory of complex variables for processes that are stationary or have independent increments. The stationary processes are processes whose probabilistic characteristics do not change with displacement of time. Gikhman and Skorokhod stress the fact that the set-theoretic axiomatization of probability theory that is currently accepted was proposed by Kolmogorov in 1929 and expounded in the monograph which we mentioned above.

For the reader who may wish a useful modern reference on mathematical statistics we recommend the textbook by Hogg and Craig [11]. Those authors cover the ground thoroughly and use the most effective type of

terminology and exposition. The model analyst can readily utilize their treatment of the subject. We wish to cite here a few examples which, we think, substantiate our point. The authors define random experiment and sample space as follows. On their page one they say, "Suppose we have such an experiment, the outcome of which cannot be predicted with certainty, but the experiment is of such a nature that the collection of every possible outcome can be described prior to its performance. If this kind of experiment can be repeated under the same conditions, it is called a random experiment; and the collection of every possible outcome is called the experimental space or the sample space." They consider, and we concur, that the primary purpose of having a mathematical theory of statistics is to provide mathematical models for random experiments. Statisticians may use the models to make inferences about random experiments. A logical theory of probability is based on the concepts of set and function of set. Frequent examples of sets are sets of numbers. The alternative to set of numbers is set of points, which may be more useful in application. We take the liberty of reproducing for the convenience of the reader and to fix attention on essential features of the subject several of the definitions and descriptions which the authors provide. For the purpose we cite the important concepts of:

(a) random variable

(b) probability density function

(c) distribution function

and (d) mathematical expectation

Definition - Given a random experiment with a sample space C. A function X, which assigns to each element of $c \varepsilon C$ one and only one real number $X(c) = x$, is called a random variable. The space of X is the set of real numbers $A = \{x; x = X(c)\}, c \varepsilon C$.

The notion of probability density function can be grasped as follows. Suppose X is a random variable in space A and A' is a subset. If $P(A') = P_r[X \varepsilon A']$; that is we know how the probability is distributed over the subsets of A. We speak of the distribution of X (distribution of probability).

Furthermore, if X is the probability set function P(A') where A is a one-demensional set and x is a real number, A is the unbounded set from $-\infty$ to x then:

$$P(A) = P_r\ (X \varepsilon A) = P_r(X \leq x)$$

The probability depends on x and is denoted by:

$$F(x) = P_r(X \leq x).$$

The function F is called the cumulative distribution function. Then if f(x) is the probability distribution function we have:

$$F(x) = \sum_{w \leq x} f(w)$$

for discrete type of random variable and

$$F(x) = \int_{-\alpha}^{x} f(w)\,dw$$

for the continous type.

One of the most useful concepts in probability involving distribution of random variables is that of mathematical expectation.

If X = random variable

f(x) = p.d.f.

and let u(X) be such that

$$\int_{-\infty}^{\infty} u(x)f(x)\,dx$$

exists, if X is continous

or

$$\sum_{x} u(x)f(x)$$

exists, if X is discrete the integral or sum is called the mathematical expectation. This concept is essential in the process of decision making.

We cannot finish our reference to the mathematical theory of statistics without calling attention to the superb treatise on the subject by Professor S. S. Wilks of Princeton University [12]. Here we only note that he too stresses that the subject showed a spectacular rate of growth from about the middle of the present century. He wisely notes that the growth results in a flow of new material with which no single individual can keep pace. We agree with his opinion that both aspects of statistics, that is, the mathematical theory and the statistical methodology based on the theory, are most effectively combined in research paper, monographs, and books restricted to specific topics.

Brief but useful papers on statistics abound in the literature. An example provided by A. W. Flux is to be found in the 14th edition of the Encyclopedia Britannica [13]. A related subject to which we shall refer later is also given in the Encyclopedia [14]. It is the extremely important field of biometry. The article is by the foremost authority R. A. Fisher. It is in this paper that Fisher states that Mendel, to whom we have previously referred, is now recognized as a pioneer in the introduction of statistical methods in biology.

Biology is a subject that should impress upon us how the role of large amounts of data characterize the essence of certain important disciplines. We all know that biology is only one such area of experimentation and analysis. An instructive exercise for anyone interested in probabilistic models is to study the data provided in the Statistical Abstract of the United States [15]. Here we find important data on population, housing, health, education, employment, income, prices, business, banking, science, defense, trade, government finance, foreign country comparison, and many other divisions of current importance. Brief data studies can readily be found in the World Almanac [16]. One can see from such tabulations the vast number of important topics which relate to the stochastic model and why they must do so.

Now that we have discussed probability and statistics at some length we wish to shift our attention to the theory of stochastic process. There is no better author than J. L. Doob to follow in such a project and, for the purpose, we will take the liberty of using his terminology and mode of expression. We do this despite the fact that his textbook entitled Stochastic Processes is more than two decades old [17]. In his preface Doob defines stochastic process. He says, "A stochastic process is the mathematical abstraction of an empirical process whose development is governed by probabilistic laws. The theory of stochastic processes had developed so much in the last twenty years that the need of a systematic account has been strongly felt by students of probability and the present book is an attempt to fill this need." He considers that probability is simply a branch of measure theory, with its own special emphasis and field of application. He too says that the basic paper on probability as measure theory is that by A. Kolmogorov. Markov processes were called stochastically definite processes by Kolmogorov in the thirties.

To illustrate variation in terminology it is observed that processes with independent increments were called differential processes in Doob, homogeneous processes in Cramer, integrals with random elements in Lévy. The systematic study of such processes was introduced by Finetti in 1929.

The special time concept referred to as ergodic theory, which was centrally established by George Birkhoff in his classical paper in the 1931 Proceedings of the National Academy of Sciences, is effectively introduced in the study of stochastic processes by Doob.

He also stresses that the law of large numbers for stationary processes in the wide sense, also called L_2 ergodic theorem, is due to John von Neumann who gave its proof in his National Academy of Science paper in 1932.

We would now like to cite the technical definition of stochastic process given by Doob. On his page 46 he says, "From the non-mathematician's point of view a stochastic process is any probability process, that is any process running along in time and controlled by probabilistic laws. Numerical observations made as the process continues indicate its evolution. With this background to guide us we define a stochastic process as any family of variables $\{x, t\varepsilon T\}$. Here x is in practice the observation at any time t, and T is the time range involved." Several stochastic processes are treated at length by Doob. One of these is said to be Gaussian if the joint distribution of every finite set of the x_t is Gaussian. In passing it is said that the Gaussian processes are important because the Gaussian hypothesis simplifies the theory of least squares.

His technical definition of Markov process is given on his page 80 where he says, "A (strict sense) Markov process is a process $\{x_t, t\varepsilon T\}$ satisfying the following condition:

for any integer $n \geq 1$, if $t_1 < \ldots\ldots < t_n$ are parameter values, the conditional x_{t_n} probabilities relative to $x_t, \ldots\ldots, x_{t_{n-1}}$ are the same as those relative to $x_{t_{n-1}}$ in the sense that for each λ

$$P\{x_{t_n}(\omega) \leq \lambda \mid x_{t_n}, \ldots, x_{t_{n-1}}\}$$

$$= P\{x_{t_n}(\omega) \leq \lambda \mid x_{t_{n-1}}\}$$

with probability one."

Another stochastic process of considerable interest and use is the Martingale, a term introduced by Jean Ville in 1939. The definition given by Doob is:

A stochastic process $\{x_t, t\varepsilon T\}$ is called a martingale if

$E\{|x_t|\} < \infty$ for all t and if, whenever $n \geq 1$ and $t_1 < \ldots < t_{n-1}$,

$E\{x_{t_{n+1}} \mid x_{t_1}, \ldots\ldots, x_{t_n}\} = x_{t_n}$ with probability one.

Doob devotes an entire chapter, VII, to the treatment of martingales.

We can recommend a thorough reading of the entire book by Doob because of the meticulous care he uses in treating the subject. However, we must admit that there are other treatises of more recent vintage that should receive consideration. One of these, by Samuel Karlin, is entitled A First Course in Stochastic Processes [18]. It is systematically developed and provides many applications. He stresses the importance of Markov chains which relate to a large number of physical, biological, and economic phenomena. He also treats one-dimensional random walks, discrete queueing, inventory model, and branching processes. In the area of genetics he discusses a model which was introduced by S. Wright for the purpose of investigating the fluctuation of gene frequency under the influence of mutation and selection. Karlin omits any treatment of martingales and of stationary processes.

For one interested in a special treatment of problems associated with the Brownian motion there is a monograph entitled Stochastic Integrals by H. P. McKean Jr. [19]. It is a considerable study of Brownian motion, which was ultimately put on a solid mathematical foundation by N. Wiener and K. Itô. Properties of stochastic integrals are studied in some detail.

Finally, we wish to cite a most recent treatment of stochastic processes. It came out in 1973 and the authors are Melsa and Sage [20]. As might be expected they say stochastic processes are playing an ever more popular role in all fields of engineering. The reason is that many phenomena can be satisfactorily described only by probabilistic means. Also, increased interest in the study of complex systems leads naturally to probabilistic models. Examples are systems analysis, control, and communication.

Numerous conferences, colloquia, and seminars have been held for the study of random processes and stochastic models during the last decade. A typical one held at Prague in 1965 had 45 papers presented [21]. These were on Markov processes, information theory, random systems, optimization, stochastic processes, computers, diffusion processes, ruin problems, control theory, Feller processes, and Monte Carlo simulation. The organizing committee had asked David G. Kendall of Cambridge to present an expository paper on recent developments in the theory of Markov processes. His opinion is that the theory of such processes has grown to such an extent that it threathens to engulf the whole of probability theory. Writing on random systems, G. Adomian of Athens says, "although considerable literature exists on random equations, most of it applies to first order differential equations with constant coefficents with the element of randomness arising from either a random forcing function or random boundary conditions." An interesting paper on the Monte Carlo simulation of Bush-Mosteller stochastic models for learning was presented by R. Theodorescu of Bucharest. He refers to the systematic treatment of models which occurs in the mathematical learning theory

treated by R. R. Bush and F. Mosteller in a book entitled Stochastic Models for Learning in 1955. Theodorescu extends the work in his monograph. Finally as an illustration of the papers presented at the conference we wish to mention an especially interesting one on adaptive control systems by Milan Ullrich of Prague. He stresses the fact that in recent years probabilistic methods, especially statistical decision functions, are often used for solving automatic control problems. The model of statistical decision corresponds better to practical decisions in the optimal control than the classical model. Ullrich says that when applying statistical decision theory, all the probabilistic properties of signals and disturbances and also the probabilistic properties of signals and disturbances and also the probabilistic properties of individual elements in the given control system are assumed to be known. Then the determination of the optimum controller leads to the determination of Bayes solutions of corresponding decision theory.

We end the present chapter on modern mathematics with reference to two very recent applications in the field of stochastic models. The first is a 1974 book on random processes applied to nuclear reactors by M. M. R. Williams [22]. The second is a 1974 book on Stochastic Models in Biology by N. S. Goel and N. Richter-Dyn [23]. In order to provide the reader with some idea of the nature of these applications we will very briefly review the books.

Williams says that his book is an attempt to complement those of Thie (1963) and Uhrig (1970), which are the only existing publications on random processes in nuclear reactors of any depth written in English. He is applying the methods of random processes in the relatively new branch of engineering known as Nuclear Engineering. It is interesting to observe that his opinion is that many of the techniques developed for the understanding of older problems, in particular in biology, are of interest to the nuclear engineer who can often borrow them with only a small change in notation and reidentification of the random source. The author includes a highly suggestive list of references to various 20th century problems in diverse fields.

Williams says that he considers the specific application of random processes in neutron diffusion and reactor behavior by means of the Boltzmann equation. It is from this equation that the basic equations of neutron transport are derived and therefore an understanding of the reduction is of importance in locating possible noise sources in reactor problems. His opinion is that the concept of reactor noise is best explained by reference to the steady-state reactor. If one considered the output of a reactor operating at steady power it would simply be a constant with respect to time. This is the power output from an ideal deterministic steady-state reactor. However, he stresses the fact that an examination of the output of a detector which actually measures the power shows superimposed on the steady output an apparently random fluctuation. Williams classifies the reactors as zero energy systems and power reactors. Very different sources of noise are prevalent

in the two different systems. In zero-energy systems effects due to temperature and mechanical characteristics are absent. The noise source is entirely nuclear in origin. In the power reactors, in addition to specifically nuclear effects, there are noise sources arising from mechanical disturbances. These are caused by vibration of mechanical parts, boiling of the coolant, fluctuation of temperature and pressure and other phenomena peculiar to a particular reactor. The mathematical treatment of this type of noise is based on the Langevin technique. It is generally agreed that such important study of noise in reactors, which is fundamentally a stochastic process, is just in its infancy.

A stochastic study of biological problems by Goel and Richter-Dyn is every bit as interesting and important in its discipline as that by Williams for the nuclear reactor. The content of their book may be of interest to the reader so we shall indicate the type of material contained in its ten chapters. There are an introduction and then brief treatments of random processes in general with respect to time and state space conditions, population growth and extinction, population growth of two-species systems, dynamics of a population of interacting species, population genetics, firing of a neuron (discrete and continous models), chemical kinetics, and photosynthesis. The authors emphasize the fact that mathematical modeling of biological phenomena has grown considerably during the last two decades. Their monograph is an attempt to demonstrate the usefulness of the theory of stochastic processes in understanding biological phenomena at various levels of complexity -- from the molecular to the ecological level. They maintain that modeling of biological systems by means of stochastic processes allows the incorporation of secondary factors for which a detailed knowledge is lacking.

The monograph presents two basically different approaches to the probabilistic model. In both it is assumed that the behavior of the system is memoryless or Markovian; i.e. its future depends only on its present state and not on the past. In modeling a complex biological system, the authors pick one or two important components of a system whose evolution in time is describable by one or more known deterministic dynamical equations. The remaining components are assumed to behave like noise and affect the system accordingly. Such a procedure transforms the deterministic variables into random variables whose probability density satisfies the Fokker-Planck equation, which is really a forward Kolmogorov diffusion equation.

We end the present chapter on modern mathematical methods used with stochastic processes by referring at random to a university catalogue which lists courses that are intimately related to the subject which we have just been discussing. From the 1975-1976 catalogue of the Evening School of the Johns Hopkins University we find the following listings. Under a course designated as Stochastic Systems the description is: Studies of the behavior of stochastic systems, ..., compounding and generalization of distributions, ..., discrete and continous time Markov

processes such as linear birth process, contagion process, Poisson process, and time homogeneous immigration-birth-death process. Stochastic systems such as single and multiple channel queues, ... Another course is called Introduction to Probability and statistics. Its description is as follows: Probability models, random variables, distributions, stochastic independence, conditional probability, applications. A third course is Inventory Systems. Its description is as follows: Art of building and analyzing models as applied to inventory systems. Theoretical and quantitative approach to problems of balancing, carrying costs, shortage costs, and replenishing costs. Optimal decision rules ......, deterministic and probabilistic demand, zero and non-zero lead time, price discounts, multi-item systems, equivalence systems, choice of optimal policies. Applications of sensitivity analysis, simulation, mathematical programming, Markov chains, and computers. And finally in this group of courses there is one called Computer Modeling of Social Systems. Its description is as follows: Methodology for computer modeling of social systems, ..., System dynamics models of the city, the world, the U. S. economy, and other social and material systems will be examined, ..., other system modeling methodologies to be covered include cross-impact analysis, KSIM, probabilistic system dynamics, and scenario analysis. We have cited the catalogue material at length in order to emphasize to some of our readers that currently there is an extensive and serious mathematical analysis of stochastic processes underway. It should also suggest to the non-mathematically inclined person that progress is presumably being made in certain areas of modeling which may bode well for the future. We will go more into some of these questions in our final chapters.

In the next chapter we shall examine important aspects of certain models for exploration of technical matters, models for decision making, and models for design. These include questions concerning control theory, stability, and optimization.

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CHAPTER 19

SYSTEM, PROCESS, AND MODEL

Up until now we have seemingly been dealing with the term model to the exclusion of the thing which is modeled. In the last chapter, which we devoted to modern mathematical methods used with stochastic models, we had occasion to define the terms system and process. The principal reason for doing this was that the various authors, treating the mathematics, used such expressions as stochastic system, stochastic process, and stochastic model. Our fear now is that in an attempt to consider the term model on the broadest basis confusion can arise. In an attempt to remove ambiguity and to emphasize the need for proper definition in any generalization of the subject of models we must face up to the obvious requirement that satisfactory definitions are needed. As we have said in the past, our attempt to treat the general subject is prolegomenal and a definitive treatise must be written by some qualified person or persons. However, even for our present purpose some consideration must be given to these matters and we must forwith proceed to do so.

First we may recall that in dealing with the old classical models, such as the iconic, the analogic, the similitudinous, we used the term prototype to contrast with model. Such a usage can in itself now lead to some confusion and we will abandon it. In its place we shall attempt to prescribe terms and terminology which may be suitable, but which in any event we must use in order to further conduct our discourse. The situation was not so serious when we were treating the well-defined classical models, but now greater definiteness is required. We will proceed to remedy the matter, but first we will cite some pertinent definitions from standard dictionaries.

Since the word prototype is so loosely used we will examine its definition and then proceed to replace it in our modelistic vocabulary. From the Oxford dictionary we find:

> Prototype - The first or primary type of any thing; a pattern, model, standard, example, archetype.

And from the American Heritage dictionary we have:

> Prototype - An original type, form, or instance that serves as a model on whcih later stages are based or judged.

An examination of both of these definitions reveals that the word prototype is identified with some kind of model albeit it is a primal consideration which leads to later stages in a process of development.

We would consider that it is almost the first element in our concept of a sequence $\{M_i\}$ which leads to the final object. We may recall that M_i is the ith stage in a modeling sequence which leads us on to the thing which is being modeled. We certainly have a confusion if we identify M_i wiht the first model, the prototype (by the definition in the dictionary) and the object modeled. We completely avoid such an occasion for ambiguity by relinquishing the term prototype as meaning the thing to be modeled and replacing it by some other term or expression. In order to do this we first wish to examine the definitions of two words: entity and thing.

In the Oxford dictionary we find for the definition of entity the following:

> Entity - 1. Being, existence as opposed to non-existence; the existence as distinct from the qualities or relations of anything. 2. That which makes anything what it is; essence, essential nature.

And for the definition of thing the following:

> Thing - An entity of any kind: That which exists individually in the most general sense, in fact or in idea; that which is or may be in any way an object of perception, knowledge, or thought; a being, an entity.

In the American Heritage dictionary we find for the definition of entity the following:

> Entity - Something that exists independently, not relative to other things; a particular and discrete unit; an entirety.

And for the definition of thing the following:

> Thing - 1. Whatever can be perceived, known, or thought to have a separate existence; an entity. 2. The real substance of that which is indicated as distinguished from its appearance or from the name, word, or symbol denoting it. 3. An entity existing in space or time; an object or fact.

In the light of these definitions we shall now agree to use interchangeably the terms entity and thing in a philosophical manner (we consider our action to be in conformity with most philosophical systems, at least the Aristotelian and the Scholastic). Also instead of using the term prototype to stand for the thing to be modeled, we will agree to speak, simply, of the thing or entity to be modeled. And in contrast with such a thing we will speak of model. Hence we have the thing to be modeled and the model itself. Such a decision is in agreement with our earlier assertion that every A in the universe models every B, where A and B are any distinct things. So if we agree that B is

the thing or entity to be modeled, then A is the model. The very definition implies that there are always two things that are being compared in order to discern similarities and of course, also, dissimilarities.

We can see that just as mathematics may be applied to systems of things in order to make certain kinds of analyses so modeling may be applied to systems of things to make certain kinds of analyses. From our point of view it can easily be seen how a system of mathematical equations is sometimes called a model. However, it can equally well be seen that mathematization and modeling are not identical things. Possible grounds for confusion for some minds can also be understood.

Now we may reintroduce the important terms, systems and process. For certain purposes it may be that they are the things or entities to be modeled. For such a purpose we consider the cited dictionary definitions are satisfactory. The context of any particular use will supply anthing further which may be required for complete comprehension. It can be seen that system and process are very special kinds of things just as mathematics and modeling are special kinds of things. We may make models of systems and processes for the purpose of analysis in contrast to the manner in which mathematics is used. It would seem that a modeler <u>per se</u> is not identical with a mathematician <u>per se</u>. The systems analyst uses both mathematics and models. It is interesting to note that he is quite distinct from the engineer, the physicist, the chemist, the physicaian, the military leader, et cetera, in his profession, although he may very well be classified as one or more of these, also. His principal concern is the production of reliable bases for making important decisions in any of these areas. His tools are mathematics and modeling. His success, however, must obviously depend upon the seriousness of his knowledge in the various fields which require systems analysis.

Having said these things and arrived at some definitions, we will now examine systems and processes with respect to those features which require analysis. We may recall that examples of actual systems are: military organizations, governmental departments, churches, and chemical plants. The corresponsing processes are those of defense, legislation, worship, and production of chemicals. The functions of these processes are to protect, permit satisfactory operation, enhance spiritual value, and provide for the physical needs.

Most systems and processes are dynamic and stochastic, however, certain important cases may be treated as static and deterministic. Whatever may be the case, however, there are certain aspects which require close attention in any analysis. With respect to a system or process, an analyst is interested in one or more of the following: control, stability, expectation, gain, loss, and optimization. We will make a general examination of all of these, especially in the light of the technical literature, but first we shall cite a particular case history in order to concretize our thinking.

The system we wish now to examine briefly is a ship in a seaway. The problem is that of excessive roll. The objective is to minimize the undesirable motion. The means are suitable controls. The production of the controls require methods of analysis and a competent analyst. The case history concerns Nicholas Minorsky who is probably best known for his book entitled Non-Linear Mechanics [1]. At a later date he published another book which is of present interest to us. It is entitled Theory of Nonlinear Control Systems [2]. We will refer to these books later but first we wish to cite a paper by Minorsky, which relates immediately to our story. It is entitled Problems of Anti-Rolling Stabilization of Ships by the Activated Tank Method. It is published in the Journal of the American Society of Naval Engineers [3].

In order to set the stage for our story about Minorsky's research on anti-rolling, with the U. S. Navy in the latter half of the 30's, we will now refer to some of the content of his 1935 paper. In that communication he states that attempts to stablize ships against rolling date from the latter part of the nineteenth century. He points out that Sir Ph. Watts in England invented water ballast stabilization and that it was improved by Dr. Frahm of Germany. Later, O. Schlick of Germany invented the gyroscopic method of stablization and this was improved by E. A. Sperry of the U. S. A. In addition to these methods there were the movable keel or vane and solid moving weight stabilization. The last cited method in his categorization is the controlled water ballast method which is generally designated as activated or active tank method. As the name implies, the activated system differs from the earlier passive tank system in that the flow of ballast between the tanks instead of being governed by the ship's rolling motion, is actuated by a local source of power, such as impeller pumps, controlled by instruments responsive to rolling.

The theory of motion of the ship on which Minorsky based his analysis was suggested by William Froude, whom we mentioned earlier in connection with ship model basins, and is as follows:

$$I\ddot{\theta} + K_1\dot{\theta} + K_2\dot{\theta}^2 + wh\theta = \varepsilon wh\psi \sin \omega t$$

where θ = angle of roll

I - moment of inertia of ship

w = weight of ship

h = metacentric height

ψ = max. angle of effective wave slope

ε = empirical constant

and the dot above θ means differentiation with respect to time.

We reproduce the differential equation here partly because some refer to it as the mathematical model but more particularly because it is the starting point in the extensive analysis made by Minorsky. We also wish to refer to the corresponding physical models used by Minorsky in developing his full scale control devices.

He stresses the fact that it took a long time to try the active tank system because of the difficulty of designing a suitable control system. It finally turned out that controls which had been developed in other connections became available.

Minorsky further says, "It is of interest to note that the theoretical conclusions reached by the analysis have found a complete experimental confirmation during the experiments which have been carried on recently with a model of 2000 pounds; this model was arranged to represent, on a scale reduced by the law of similitude, the performance of a normal ship in a regular as well as an irregular seaway."

The preliminary success in these matters led the U. S. Navy in the late 1930's to conduct an investigation, under the leadership of Minorsky and designed to lead to the introduction of stablization controls on fighting ships. At that time there was a great need for motionless turrets from which to fire projectiles. Of course since that time control of projectile motion has become such an advanced art that the then proposed methods are obsolete. However, the experience demonstrates the effective use of models in solving an important engineering problem.

We shall now very briefly recall some of the story of Minorsky's experience with the Navy. At first, from a desk in the Navy Department in Washington, he attempted analyses of various stablizing devices, including fins. Finally, however, it was decided to concentrate on the activated tanks which would be controlled by variable impeller pumps. After preliminary design attempts were made in Washington, the project was moved to the Material Laboratory in the New York Naval Shipyard. In a shack outside our laboratory, Minorsky set up his small scale model experiments. There is no doubt that he performed many enlightening experiments in the Yard. However, the final result was disappointing. The variable pumps designed and built for use on a particular ship did not perform satisfactorily because of improper blade design. The result was that the ship nearly capsized near Gravesend Bay, south of New York City. The debacle could scarcely be charged to Minorsky, however, because he was an electrical engineer who was charged with the problem of providing too much of the design for the mechanical equipment. Also, another reason for such a situation was undoubtedly the pressure of approaching war and the consequent lack of assignment of competent engineers to a project which at best looked doubtful over the short range.

One of the reasons we have looked at such an engineering project in some detail is that we were familiar with the performance and also to indicate that important lessons can be learned from models even in an apparently losing cause. Minorsky made very good use of the experience and as a consequence, in later years, became one of our leaders in non-linear mechanics and control theory. Before leaving his story we should recall that he did enjoy an earlier success with the Navy in connection with automatic steering. He provided some interesting methods of steering control for the U. S. S. New Mexico which are described in an appendix to his book on Theory of Non-Linear Control Systems, mentioned above.

We hope that from our brief story on ship stabilization the uninitiated reader, at least, can get some of the ideas which are pertinent to control theory. In order to provide a more general definition of control, we now wish to quote from a recent book in the field. It is the text which is entitled Dynamic Optimization and Control [4]. It was written by W. Kipiniak. In his introduction he says, "The control of a given system can be viewed as the process of varying those of its parameters which can be manipulated by means external to it, so as to make its behavior in some sense best. In general, the ideal behavior is unachievable because of dynamic limitations of the element being controlled - henceforth to be called the plant - and because of uncertainty as to its characteristics and those of the external disturbances acting upon it. Thus control is a matter of manipulating system inputs so as to optimize system performance, that is, to maximize the expected value of a preassigned performance criterion." With such a broad definition, one can see that not only does the cited control problem of Minorsky fit it, but also that of many other fields, including even such diverse fields as economics and ecology.

It can be fairly said that the important developments of automation and automatic control occurred exclusively in the present century. All texts on the subject give some introductory remarks on the history. We will now refer to one of these which indicates that there were a few isolated but important contributions before the 20th century [5]. In his book, C. R. Webb says, "Automatic control, although its widespread application began in the 1930's, was probably used several centuries ago. Examples include the use of a fantail for automatically facing the mainsail of a windmill into the wind, simple pressure cookers, and the speed governing of steam engines. ---

The first servo-mechanism appears to have been the application of power steering to ships. The year 1930 saw the beginnings of the widespread application of control in the chemical industry and in oil refineries, a field which tended to develop separately and to become known as process control. Military requirements accelerated the development of fire control systems during the second world war and this was accompanied by a scientific evaluation of the possibilities and limitations of automatic control techniques." Webb also mentioned some of the contributors

to the theory, who presented their work in English. These are Maxwell, Routh, Trinks, Minorsky, Nyquist, Hazen, Ivanoff, Callender, Bode, Ziegler, and Hall.

It may be observed that in some of the books on control theory there is little or no use of the term model. We think that such an omission is interesting but understandable. Authors of such books are motivated by what may be called theory or more accurately mathematical analysis. In this connection we wish to quote from the introduction of a textbook by Sheldon S. L. Chang [6]. Chang says, "The modern theoretical development of feedback control systems started in the 1920's and 1930's and is marked by Minorsky's paper on the steering of ships (1922), Nyquist's paper Regeneration Theory (1932), and Hazen's paper Theory of Servomechanisms (1934). Before that the development of feedback controls was mainly in the hands of inventors. While there were isolated instances of successful applications of the concept of feedback control such as Watt's application of the flyball governor to the steam engine (1788), Whitehead's torpedo control (1866), and Sperry's gyro stabilizer (1915) there were many, many more attempts that were left unrecorded because they failed. The lack of theory prevented consistent success and economical design toward a prespecified objective." The reader may be able to infer something of the motivation of Chang from this single quotation. Apparently it is lost on him what the role of the invention, which always associates with a physical model, really is. It should be clear that the field of control, like so much other physics and technology, did not begin with mathematized analysis but with experimentation on models arising from mechanical inventions. At this point we may do well to recall the failure of Minorsky in his ship stablization experiment at Gravesend Bay. The valid engineering conceptions, inventions, and designs are essential for the sucess of the final objective. Having said this, however, we should not ignore the important use of mathematics in the development of controls.

A useful textbook on the subject of controls, which has recently been published, is that by Katsuhiko Ogata [7]. He too does not use the term model systematically except in a very specialized manner beginning on his page 786. Here he introduces what is called a model-reference control system. The book, however, is literally full of iconic models in the form of block and circuit diagrams. In some portions of his book he also has line drawings of models of both mechanical and electrical control devices. In his introductory chapter he attempts to give definitions of system, process, and control.

The use of the simple model as a teaching device in the study of system control can be seen in the following elementary example. It is a case of biofeedback which avowedly involves very complicated psychology and physiology, but does provide a simple operational picture. Consider the case of a man pointing out to his companion a galloping horse by continuously following the motion of the horse with his index finger. Even the novice in automatic control must get some idea of feedback by

consideration of such a model. After he commences to seriously think about the matter he must ask himself how the individual can keep his finger trained on the moving animal. Obviously the angular position of the finger (and arm) must be corrected continuously to correspond to each advancing position of the horse. A little reflection leads him to see that he is dealing with a complicated optical, muscular, neural, cerebral system that is supplied with feedback control.

With respect to the acknowledge use of the concept of model in sytems analysis, we take heart with a very recent book by Fred C. Schweppe [8]. In his introduction he says that the three main topics of interest in his book are:

1. Modeling of uncertainty

2. Analyzing the effects of uncertainty

3. Designing systems to remove or compensate for uncertainty.

In his chapter 2 he defines system, disturbance, state output, control input, and measurement control. Also on his page 15 he says, "One of many ways to classify the basic problems of system theory is in terms of modeling, analysis, estimation, and control." He then proceeds to discuss each of these aspects. We must concede, however, that he is essentially keeping the mathematical model in the fore. He says that the problem of developing a mathematical model which adequately represents the physical situation is the most critical step. If the model is no good, the subsequent mathematical and computer manipulations are useless. He gives as general rules:

1. The modeling must be done by someone who thoroughly understands the nature and behavior of the actual system.

2. The model must be usable, i.e., capable of being analyzed.

We think that the first rule should always be stressed. It is of course a somewhat sticky question as to whom " the some who throughly understands" phrase applies. However, it is clear that ideally it is an absolute requirement. It also seems to underline the position that that someone is not one who is solely interested in mathematical theorizing. Schweppe's entire chapter 3 is devoted to mathematical models.

An important concept which relates to system or process is that of instability. Before looking closer at the phenomenon we would like to clarify a possible confusion. Referring to the ship stabilization problem which we discussed in relation to the investigation of Nicholas Minorsky we wish to say that the use of stablization in his sense is not the removal of an instability of the type which we now wish to discuss. What he means for the ship stabilization problem is that one wishes a control device to lessen or remove undesirable motions such as those caused by waves. What we now mean by stability or instability is a

different but well-known problem in the theory of differential equations and the corresponding field of physical problems. We can illustrate instability in such a sense by a simple model which we constructed and with which we experimented many years ago. It is the so-called inverted pendulum which consists of a straight vertical rod which is free at the upper end and pinned at the lower. The pin is attached to a moveable horizontal platform which can have imposed upon it a sinusoidally varying amplitude of a given frequency. For certain combinations of amplitude and frequency the rod can be maintained in a stable condition in the neighborhood of the vertical position. However, for certain other combinations the rod will become unstable and fall to the horizontal platform. The differential equation used in the analysis of such a problem is called the Mathieu equation and is discussed by S. Timoshenko in his book on vibrations [9]. The regions of instability are shown in Fig. 107 on page 164 of his book. We happily recall presenting a simple physical model, which exemplified the problem, at a pleasant seminar on vibrations at New York University, conducted by Richard Courant in the 1940's. The present problem is only one of many that involve instability and we may proceed from this simple physical system to many complex systems which manifest the condition of instability.

To provide a modern review of the problem of instability of systems we refer now to a treatise by Hsu and Meyer [10]. On page 9 they define stability as follows: They say, "Stability in a system implies that small changes in the system input, in initial conditions, or in the system parameters do not result in large changes in the system output. ---. For linear time-invariant systems, stability is relatively easy to define and analyze. Powerful tests such as those of Routh and Hurwitz exist, which provide not only the necessary but also the sufficient conditions for stability. When the linear time-invariant system passes the test of stability, it means that (1) in the absence of input, the output tends to zero irrespective of the initial conditions, and (2) when the system is excited by a bounded input, the output is bounded.

For nonlinear systems, because of the possible existence of multiple equilibrium states and other anomalies, the concept stability is difficult even to define. Futhermore, stability with or without input can be two entirely different matters for nonlinear systems." No use of the word model occurs in this book, except on page 352 the so-called adaptive system which actually uses a model is described. While the term model is not explicitly used the iconic model is used throughout the book and simulation of a system is discussed on pages 11, 233, and 236. An extensive bibliography on the subject of systems and controls is provided by the authors.

Having said something about control and stability we should now consider briefly the question of optimization. Like most of the experience of man throughout history, the subject of optimization has arisen and been dealt with in some sense, although possibly without any mathematical precision. Man has intuitively tried for time immemorial

to obtain the most for his efforts. Just as in gambling he has always had the question of chance foremost in mind, so too he has had maximum profit in mind. Since the seventeenth century the theory of maxima and minima, just as the theory of probability, has been the concern of mathematicians. Ever since Euler initiated the subject called calculus of variations by deriving the differential equation of the vibrating rod from variational considerations of energy, we have had a developing theory of that subject. Here again we see a close relationship between mathematics and the actual everyday problems to which it is applied. Here we have the model and the thing to be modeled.

As we have done so frequently in the past we now refer to some of the pertinent literature which deals with optimization as applied to systems. There is an extensive set of publications on the subject, but we choose to begin with a book entitled Foundations of Optimization by Wilde and Beightler [11]. In their preface, they say, "Dealing as it does with achieving the best - maximum gain or minimum loss - in a rational manner, optimization theory naturally holds great interest for the practical professions of engineering, economics, administration, and operations research. Its development over the centuries by architects, physicists, politicians, merchants, astronomers, clerics, and philosophers gives optimization a colorful history and a claim to be considered a branch of mathematics, for most of its contributors are posthumously called mathematicians. Yet no one recognized this body of work as 'optimization theory' until the middle of the twentieth century, when high-speed computers implemented forgotten procedures of the past and stimulated research on new methods." We find this rather breezy description of history rather interesting, however in looking back at University days as students in courses on the calculus of variations we remember very well how the subject was organized and taught by superb mathematicians and teachers. It is true that computers made the subject more applicable to daily programs but they certainly did not provide the basic ideas and concepts of variational calculus and of the theory of stationary values. Apropos of the question of who did what and when and who is doing what we would like to refer to a rather recent book entitled System Theory by Zadeh and Polak [12]. In their introduction, they say that system theory to some is not much more than an assortment of various mathematical techniques for system analysis, such as theories of differential equations, variational calculus, functional analysis, probability, control, circuitry, automata, information, games, et cetera. To others, system theory is a discipline in its own right. System theory may be viewed as a collection of general methods for dealing with problems in system analysis, synethesis, identification, optimization, and similar things. They further say that a system theorist is a generalist whose interest and expertise cut across many established fields. It does not matter whether a system is electrical, mechanical, economic, biological, chemical, or what not in nature. What matters is whether it is linear, discrete-time or continous-time, lumped or distributed, deterministic or stochastic, continuous-state or discrete-state, passive or active. It is the mathematical structure of

a system, and not its physical form or area of application, that is of interest to a system theorist. We are interested both in the statements of Wilde and Beightler on the one hand and by Zadeh and Polak on the other. Their enthusiasm and hard work are commendable, however, we again insist on the contrast of mathematics and model analysis. We consider that these two intellectual pursuits are sovereign in their own rights. Also, we would disagree with Zadeh and Beightler concerning the interest of the system analyst in the specific practical field in which he may be studying at any one moment. He certainly is not concerned only with "mathematical structure". It would clearly appear that his supreme interest is in good decisions in the field in which he may be involved. What we think that Z. and B. are attempting to say is that we should keep separated the subjects of mathematics, modeling, and systems analysis. With that we would certainly agree. Before leaving these two important books we would recommend them to the reader who may not have perused them already. Z. and B. book is not uniform in quality, but the authors present challenging and competent analyses in many areas which are of concern in the study of systems.

Indications of how mathematical developments are progressing in current system analysis and control can be found in the proceedings of a conference held at the University of Bath in 1972 [13]. The papers are divided into five areas as follows: Stability of Non-linear Systems, Optimal Control, Filtering Theory, Control of Systems Governed by Partial Differential Equations, and Algebraic System Theory. We cannot resist quoting at length from one of the articles because we think it emphasizes the confrontation between the so-called purist and the practical operator. In article no. 5, page 69, entitled Optimal Control by J. H. Westcott of the Imperial College of Science and Technology at London, we find the following statement. Westcott says, "The topic of optimal control is today a broad one. It is proposed to restrict these remarks on recent developments to that part of the field most familiar to the author, namely deterministic control. The march of progress here has been conditioned more by the pressing need by engineers for tangible answers to practical control engineering problems than by the desire for elegant mathematical formulations. ----. The days of the grandiose sweep forward as exemplified for example in books by Bellman that rarely needed to descend to the difficult matter of tangible answers, is over." At the end of the paper which Westcott presented, Leigh of the British Steel Corporation said, "It is well known that most industrial control systems are very far from optimal in their performance. However, this does not necessarily represent a deplorable state of affairs, since in many of these cases, very careful studies have ensured that investment in the control system has been sufficient to guarantee adequate though not optimal performance. ----. Putting this another way, optimal control theory identifies the summit of the cost function hill but tells little about the shape of the slopes below the summit where most applications necessarily lie." In reply, Westcott said, "This is a real problem and you have put your finger on the exact issue. If you are a purist and stick rigidly to your optimal control then you must

have your full state vector and do the whole thing properly. You could argue as a purist that anything outside this is not optimal control." We cite these remarks at length because they emphasize the point which we have been trying to make all along. Such polemics would vanish if we carefully distinguish mathematics from modeling. Mathematics by its very nature is precise and rigorous. On the other hand, a model cannot completely represent the thing that is modeled. By its nature there must always be a discrepancy between the model and the thing to be modeled. One cannot speak of true or false models, but only of good or bad, appropriate or inappropriate, useful or useless. The target at which we aim is the thing to be modeled, which is the actual system. However, as Leigh of British Steel implied, this is not a disturbing fact. The systems analyst must ultimately make a judgement both about the mathematics and the model. The final decision maker must take into account the recommendations by the analyst, based on both, when he makes his decision.

We consider that we have looked long enough at the meaning of system and its relation to such matters as control, stability, expectation, optimization, gain, and loss. As the reader must know the libraries contain seemingly endless references to all of these things. The texts to which we have referred contain excellent bibliographies on these matters.

We terminate our reference to the technical literature by citing two textbooks. One, by R. C. Dorf is a recent example of control system analysis which should interest engineers [14]. The other, by White and Tauber, is devoted simply to systems analysis [15]. Both of these books are rather elementary, with many illustrative examples. Our present interest in the first of these references is that although it is limited to mechanical and electrical systems it does demonstrate the level of work on systems now being attained in engineering colleges. The second reference, while a bit older, having been published in 1969, ends with a chapter on the extension of systems theory to areas besides those which are of interest solely to mechanical and electrical engineers. In fact White and Tauber mention biological sciences, bioengineering, bionics, economics, industry, and management. They also work out several illustrative examples to demonstrate the possible diversity of application. These are (1) a military problem which concerns the interruption of the enemy's tank production by the strategic bomber command (2) a production scheduling problem and (3) problem in the area of city planning.

It is hoped that our considerations have led to the notion that system and process are extremely general in relationship to man's view of everything with which he comes in contact. In fact we would suggest that these terms can properly be used to describe every action and every frame of action in which man has any interest.

Also, we consider that our reflections on the subject of modeling have led, at least, to a basis for a unique definition of model. As we said at an earlier stage, it is a dyadic notion consisting of picture and theory, or of a triadic notion consisting of picture, theory, and experience (experiment). A physical example which we suggested for consideration is the Rutherford-Bohr model of the atom. The unique nature of the concept of model is much the same as that of machine as described by Michael Polanyi. As we may recall Polanyi says that machine qua machine cannot be reduced to mathematics, physics, or chemistry. However, all of these disciplines are used in the design of particular machines. Man must grasp the principle of machine as a whole. Somewhat the same situation is true of the concept of model. It cannot be reduced to mathematics or any other science, although all of these may arise in various considerations of model.

Before ending our study of the concept and use of models, we would like to divide the subject into two principal areas of human activity and thought. As we said in our introductory remarks about classification of models, we do not consider that a final and definitive position can be taken at the present time. However, we wish to explore to some extent the possible divisions of modelistic thought and provide some groundwork for the future scholar to take as a point of departure in the great task of ultimately formalizing the subject of universal models. We consider that we have sufficiently examined the iconic, the analogic the similitudinous, the Newtonian, the extended Newtonian, and the disclosive models. Much of this subject matter is now highly standardized and well understood, with the possible exception of the last mentioned. The use of the terminology disclosive model was only tentative and followed the original suggestion of I. T. Ramsey. It was he who said that such models provide at least the basis for dialogue and for the probable development of knowledge. It is hoped that in the next two chapters we may find examples of what Ramsey had in mind. For want of a better plan we shall divide the entire spectrum of conceivable models into mechanistic and humanistic.

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CHAPTER 20

MECHANISTIC THING-TO-BE-MODELED

For convenience, and possibly for philosophical distinction, we divide the universe of thing-to-be-modeled (TTBM) into mechanistic and humanistic. In order to provide some background for these terms it seems appropriate to refer to some standard dictionaries. Accordingly we choose to cite the Oxford Dictionary and the American Heritage Dictionary of the English Language.

In the Oxford Dictionary we find:

1. Mechanistic - Pertaining to mechanics or mechanism. Also pertaining to mechanical theories in biology and philosophy.

2. Humanistic - Pertaining to the humanists.

3. Humanists - A student of human affairs, or of human nature.

4. Humanism - The quality of being human; devotion to human interests.

And in the American Heritage Dictionary we find:

1. Mechanistic - (a) of or pertaining to mechanics as a branch of physics; (b) of or pertaining to the philosophy of mechanism; specifically tending to explain phenomena only in reference to physical or biological causes.

2. Humanistic - of or relating to humanism or the humanities.

3. Humanism - (a) the condition or quality of being human; (b) a philosophy or attitude that is concerned with human beings, their achievements and interests.

Of course there are other variations of these definitions, but we set these particular ones down simply to get some working basis for our use of the terms mechanistic and humanistic. In a sense we could have made a distinction solely on the basis of man and his world. However, we wish to suggest, as we have done in the past, that there is usually a long history of ideas. We may have noticed the contrast of mechanistic and humanistic in our earlier references to history. It should be stressed that we say mechanistic things-to-be-modeled and not mechanistic models. Also the same applies to humanistic. This point of view may seem like a quibble but we do not think it is for the following reason. When we say thing-to-be-modeled we direct our attention to an unlimited world of things. On the other hand when we say model we really refer to a specific thing. Granting the distinction,

however, we can see how one may readily fall into the habit of saying mechanistic model and humanistic model.

Undoubtedly there is a certain amount of arbitrariness which characterizes our stipulation of the disciplines and systems which are to be associated with the mechanistic and the humanistic. We will deal with this matter the best we can and leave the ultimate classification to future students of the subject.

In a sense we consider that the term humanistic relates to the mind and the spirit of man qua man. The term mechanistic more generally refers to the total environment of man. It would seem that anyone would grant that the environment is both the source of nutrition for the brain, on which the mind and spirit so substantially depend, and a source of hostile beings which threaten the existence of man.

In order to concretize our thinking on the matter we wish now to list some of the subjects which we consider to belong to the mechanistic field and some which belong to the humanistic. In terms of our dictionary references it might be agreed that such disciplines as physics, chemistry, anthropology (physical), biology, ecology, cosmology, geology, oceanography, meterology, botany, ichthyology, astronomy, seismology, forestry, engineering, et cetera introduce mechanistic things-to-be-modeled; while anthropology (cultural), psychology, psychiatry, art, literature, language, drama, medicine, science of defense, political science, economics, philosophy, theology et cetera introduce humanistic things-to-be-modeled.

We make a distinction of this type in order to present in a somewhat systematic manner our discussion of the ultimate generalization of the concept of model. We have already indicated in some of the previous chapters the possibility of a science of modeling for the study of any process or system that may suggest itself to the mind of man. There is an extensive literature on the subject of modeling used in very diverse fields but we will now choose as a definite reference a recent book that attempts to treat these fields, but in their specific relation to mathematics. We refer to the small monograph entitled Mathematics, Systems and Society by Richard Bellman [1]. According to its preface this book considers the impact of mathematics on society. In an introduction Göran Borg says that Bellman illustrates "the manner in which mathematical concepts and algorithms are produced as the result of efforts to describe, understand and direct 'reality' as it is presented in physical systems or at it is currently presented in technical, economical and social systems." Bellman himself, in his first chapter, asks "How does one determine whether a few billion dollars should now be spent on high energy physics to track down the elusive fundamental building-blocks of the universe, or in the biomedical field on finding a cure for cancer, or in the study of the oceans, or in any number of other areas?" He answers that, "This is the kind of problem we wish to study restricting ourselves to the decisions affecting society

that are going to arise in connection with mathematics and its applications. Our aim is not to provide answers as much as to provide some systematic ways of thinking about major questions of this nature. These methods themselves will be mathematical in nature and in origin." Bellman further puts in a plug for mathematics in its esthetic and artistic aspects as well as in its power combined with rationality, to respond to the conceptual needs of society. Bellman, who properly is considered as an expert in applied mathematics, writes around the boundaries of another science, which is modeling. If he says that mathematics is essential for the study of societal as well as physical problems no reasonable person will dispute him. But this has been the case since the beginning of the evolution of the mind of man. However, it is only in recent times that man has become fully conscious of the role of modeling as we have demonstrated at length in previous chapters. We have sharply distinguished mathematization from modeling. Mathematics is not a third culture, an heir apparent in the social sciences, an art form, a game, or a religious experience as Bellman so enthusiastically proclaims in his last chaprer. It is a sometime language of science as he also states. Furthermore, on his page 77 he is modeling like mad when he compares the profession of mathematics with organized religion. Unfortunately he does not seem to be aware of the fact. Ironically he does use the term model once in a section entitled "Science: A Very Model of - What?" The thrust of Bellman's remark is to suggest the improper use of science as a model for systems in other intellectual areas. In the ensuing sections he wages what we must consider to be an idle battle against the possible application of the methods of physical science to social systems. He finally concludes on page 70 that, "In general, for example in economic, military, political systems, we must accept the fact that cause-and-effect of the type encountered in scientific systems does not exist." The exact meaning of such a statement is not at all clear. It is, also, polemical in nature, to no good purpose. If Bellman properly recognized the subject of modeling all of his straw men would cease to exist. However, despite our criticism of his apparently erroneous exposition of the role of mathematics we heartily recommend his book to our readers. He does concisely state many important things about the nature and use of mathematics. He also effectively enthuses about the future of what we have dared to call the humanistic things-to-be-modeled.

To again orient our discussion vis a vis the essay by Bellman, we wish to emphasize further some of our own thinking about models and modeling. The sequence of ideas which we consider at this time to be important in conveying our notions are: picture, word, sentence, theory, experience, experiment, model, language, mathematics, reasoning, and logic. It is our opinion that these words as presented here are in the probable order in which they originated in time. We should consider the location of the word model in the set. It comes just after picture, word, sentence, theory, experience, and experiment. If we group picture, word, and sentence to represent both visual and word pictures, we see that they suggest the next word, model, in terms of our triadic definition. Earlier we concluded that in some sense a model is a triad which

consists of picture, theory, and experiment (experience). An ideal illustration for us is always the Rutherfore-Bohr model of the atom. Subsequently we noted that such a concept of model permeated all of the physical sciences. We hope to show, before we conclude, that the models of the humanistic things-to-be-modeled are no exceptions.

Now we will examine briefly the roles of language, mathematics, and logic. These disciplines represent the expressive, the calculable, and the canonical or normative respectively. We know that the single word and then the sentence coming gradually over a long period of time resulted in a language by which man expresses himself in any intellectual sense. Mathematics gradually developed as a science of counting and spatial exploration. Ultimately it grew into an abstract theory of relation, implication, and measure, It also was the occasion for fully establishing the power and meaning of axiomatization and theorematic reasoning. The relationship of mathematics to fully developed logic, which is canonical or normative in nature, finally became clear in the twentieth century. All of these disciplines we contrast with that of modeling. It is clear that while the concept of model is related to all of them it cannot be completely subsumed under any one or even the aggregate of them. The concept of model is unique. It is operable and can be used effectively for the study of any human or cosmological activity.

For the remainder of the present chapter we wish to consider in some detail the mechanistic things-to-be-modeled. We repeat our tentative list as follows: physics, chemistry, anthropology (physical), biology, ecology, cosmology, geology, oceanography, meteorology, zoology, botany, ichthyology, astronomy, seismology, forestry, engineering, et cetera. These disciplines deal with the physical environment of man, which is often described as nature. We are keenly aware that man qua man is a part of that nature, but we have elected to divide all considerations of the universe into man and then all those things external to man.

Physics treats of the physical laws of the cosmos and chemistry treats of the chemical laws. These facts are universally known, but what is not so well known is the role that modeling plays in each of these sciences. The great divisions of physics we have enumerated in a previous chapter. Also, one of its most famous models we have discussed. No one can deny that the Rutherford-Bohr model of the atom is an excellent illustration of what we have referred to as the triadic nature of model. There are many other examples of a similar nature in physics but we will not treat of these because it serves our purpose to simply present a few examples which clearly illustrate our intent.

Before leaving physics, however, we wish to add one more reference of relatively recent vintage. We think it demonstrates how the model of Rutherford and Bohr can be extended and applied to ever increasing complexities of particle physics. For the purpose we find no reference

better than the monograph on Models of Elementary Particles by Bernard T. Feld [2]. In his part II he treats at length the use of models. All of the previous portion of the text he devotes to the fundamental physics of particles. After preparing the groundwork he says on his page 227 that "it is useful to explore the relationships among the elementary particles in terms of simple models. The main purpose of such models is to seek, through simple approximations that embody the main features of known conservation principles, to explore and discover the relationships among the particles, to establish possible connections and hierarchies among them, and to provide the basis for an eventual and valid field - theoretic (or alternative) approach." He then discusses the early models of elementary particles: The Fermi-Yang model, the Sakata Model, the Goldhaber model of the Hyperons, the Frisch symmetrical model, and the Doublet model of Schwinger. In part III he extensively examines the knowledge of unitary symmetry and Quark models.

The impetus of modeling in physics has carried over into chemistry. In that science the awareness of the fundamental role of model increases every day. An entire treatise could be written on that subject alone. All of us, from our early courses in organic chemistry, recall the paper and wire models of the benzene ring and similar molecules. It is well known that the technique of modeling in chemistry advanced both formal teaching and scientific discovery. The conscious use of the concept of model in inorganic chemistry came somewhat later but is now established as an intrinsic part of that subject. To exemplify this fact we now refer to a highly enlightening text entitled Models in Structural Inorganic Chemistry by A. F. Wells [3]. In his preface Wells syas, "The problem is therefore to design a course in model-building that is to form part of the practical work carried out by each student. This type of exercise should be introduced at the earliest possible stage in teaching chemistry, and many of the simpler models described in this book could well be built at school rather than the university." He further says, "It is possible to build models of many molecular and crystal structures from drilled balls and spokes ----." He notes the analogy between the three-dimensional models using polyhedra and the two-dimenstional model of the organic chemist for rings of six carbon atoms. Wells properly distinguishes the topology from the geometry of the structural problem. An insight into the meaning of models in chemistry can be gained from a single sentence of Wells. He says, "Only inspired guesses could be made about the geometry of molecules and then only about the local arrangement of bonds around certain atoms." A reading of the book by this chemist clearly demonstrates the essential role of the model in chemistry.

Of course at this late date no informed person questions the need for the use of model in both physics and chemistry. We simply refer to a few cases to re-emphasize the fact and to help clear the way for the understanding of the nascent growth of modeling seen at present in many non-physical areas. Furthermore, we have nothing more to say about the

use of models in engineering because we have covered that subject extensively in previous chapters.

A fundamental concern of mankind is the earth itself. Because it is the immediate habitat of man, a knowledge of its nature and the control of it are essential for survival. In approaching the study of the earth by use of models we need to divide it into the lithosphere, the hydrosphere, the biosphere, and the atmosphere. These domains relate to the sciences of geology, oceanography, biology (including zoology, botany, and ichthyology) and meteorology.

Modern geology is becoming increasingly important and its reliance on models is obvious. That discipline is an important element in obtaining essential oil and minerals. Currently the part of geology known as tectonics, which is the geology of the structural deformation of the earth, is gaining a surprising amount of attention because of its new plate theory of the surface of the earth. The plate model is highly developable and susceptible of many applications. It ultimately will explain more satisfactorily the ancient processes of structural formation. Also, it will enable man to scientifically control the processes of claiming the natural resources in the earth. Very little consideration is required to convince one that the science of seismology, which is an important study of stress waves in the earth, will be revolutionized with the introduction of these new models of the crust of the earth. The future of geology is very bright and the role of the model is impressive. The lithosphere has been studied abundantly in the past but the new tectonics model approach will accelerate perceptibly the increase of knowledge and the ability to increase our resources.

The hydrosphere is another portion of the surface of the earth, which has already been studied extensively but which is also now on the verge of what almost might properly be called a renascence. The very recent studies, which have been called marine science, signal the beginning of a new era of both knowledge and control. The physical, chemical, biological, and engineering studies which are now appearing in connection with the hydrosphere emphasize an increasing conscious use of models. Modeling has led the way not only in developing important conceptions of the hydrosphere but also in the design of undersea craft and systems used for study and control. To teach, to study, and finally to master oceanology and oceanography are prime current considerations and very little thought leads one to observe the essential use of models. The literature of these subjects abounds and is daily increasing. The breakthrough for the sciences associated with the hydrosphere would have been impossible without a fully developed concept of the model.

The biosphere, as the name implies, is the arena of all living things on the earth. It includes the life in the waters, on the land, and beneath the surface of the earth. Man's needs in all of these regions is a signal cause for the motivation of biological studies. In our division of things-to-be-modeled into the mechanistic and the

humanistic forces us to split the concerns of biology into two parts - one which is related to the being of man proper and the other to his environment, which is the biosphere. The sub-fields of zoology, botany, and ichthyology have already shown the importance of the system and model approach. Studies of the habitats of the animals, the plants, and the fishes, as well as of their modes of living, have demanded the use of models. The bird has been a model for the airplane. The fish as a model for the submarine, especially including the effects of exudates on propulsive efficiency, is beyond question. Drag reduction by tiny amounts of high polymers in solution have led recently to modeling processes which relate fish to man-made ocean vessels [4]. Also, using the dolphin as model, studies have been made of communication systems.

The science of meteorology, which deals with the phenomena of the atmosphere, has for a long time used models explicitly to increase our knowledge of the weather and of the characteristics of air currents in general. Like ocean currents which have been modeled in the laboratory, air currents have also been modeled. For a number of years, to our knowledge, small scale spherical models, using water to mimic air currents around the earth, have been studied at the University of Chicago. It is only natural to conceive of wind tunnel models of all kinds as possibilities for model studies of the wind. Just as man cannot investigate the phenomena of the very small, such as for atoms, without models, so he cannot study the very large, such as galaxies, without models. The earth is embedded in the solar system, which together are embedded in our Galaxy. The galaxies as well as planetary systems cannot be studied without models. A very recent scientific comment on our sun is interesting from the standpoint of models. It is contained in a letter by Demarque, Menzel, and Sweigart [5]. The letter is a discussion of Barbara Levi's account of solar-oblateness which may relate to the problem of solar neutrinos. Levi had observed that low neutrino flux is consistent with models of the sun with a rotating core such as the model of Robert Dicke. The authors of the letter point out certain ambiguities resulting from the use of the Dicke model in which most of the mass rotates rigidly with a period of one day and which yields practically the same high neutrino flux as a nonrotating model. They also assert that only those, including a small rotating core in a state of rapid rotation, can produce the desired reduction in neutrino flux. They further state that this type of model implies an internal structure and rotational history for the sun which is different from that proposed by Dicke. Whether our reader is acquainted with the neutrino flux of the sun or not, he surely can see that we are here dealing with our notion of model. There is a picture of a rotating body, reminiscent of the use of picture in the case of the Rutherford-Bohr atom, a theory of neutrino flux, and a suggested experiment for conformation. Also, the example clearly demonstrates the value of a disclosive model. It permits dialogue and generates knowledge, just as Ramsey suggested in his monograph entitled Mystery and Model.

While cultural anthropology is classified by us under humanistic things-to-be-modeled, we consider that physical anthropology is another example of a science which relates to our mechanistic things-to-be-modeled. The reason for such a division seems sound. Skulls of ancient man and pre-man, while they are important objects for study of our organic evolution, are not of immediate concern to man and his livelihood in the modern world. Norwithstanding this fact, however, it is necessary to observe that the model is essential in physical anthropology. Usually only portions of skulls and other skeletal parts of ancient man are available to the scholar for study and this leads to the speculative modeling of the whole. We consider that such a process is valid and essential for the development of the knowledge of physical man and his ancestors. In the sense of Ramsey it is the only way to permit dialogue and develop knowledge.

We certainly have touched the main divisions of what we call the mechanistic things-to-be-modeled in only a very superficial manner. Of course entire libraries about these subjects are in existence. Our sole purpose, however, is only to indicate in a small but very definite way that the concept of model is essential to all of the sub-fields which we have indicated as examples of the vast field of mechanistic things-to-be-modeled. Before terminating our present review of these various topics, we wish to spend some additional effort on one of the divisions which has been in the fore for several years past. It is the decisively important subject of ecology.

The term ecology has really come into being in connection with recent biological studies. Its definition is simply stated as the science of the relationships between organisms and their environments. Ecology then is obviously a most important field for study. An interesting recent phenomenon growing out of the use of the words ecology and ecological is a political one. The terms are bruited in the secular press and have become the cause for much rhetoric in such political arenas as the United States Congress. There is good in all of this interest but there is also bad. The latter comes out when we have prophets blowing trumpets of doom and cartoonists ridiculously exhibiting pictures such as that of an Indian on the banks of a rubbish choked stream with a single tear dropping from his eye. Discounting the usual fanfare that grows out of the introduction of a new phase of science, we critically observe some of the more serious consequences of the scientific introduction of what we call a one-sided model. The concept of model can be exceedingly important to the survival of man, as well as to a balance in that thing that is sometimes mysteriously referred to as Nature. Man needs energy to survive as well as a suitable environment in which to survive. Proponents of the one-sided model always admonish mankind not to disturb the very delicately balanced Nature apparently unmindful that man is also a part of it. The only satisfactory response to such an approach is to demand what we may call a two-sided model. Such a model should optimize the meeting of the needs of all non-human organisms on the one hand and that of man himself on the other. Uses of

large amounts of energy are essential for the sustenance of man and the growth of his civilization. Having said this, however, we are well aware that the total problem is of great complexity. It requires the greatest talents of man to optimize and produce the system which is best suited for the survival and progress of the human race. It is our opinion that the conscious use of the concept of model will go far to answering the needs. Fortunately, a highly useful literature on the subject of ecology is growing. We wish now to make some reference to it.

It seems that sciences which begin to mature turn definitely to mathematics. Ecology is no exception. We see this in the most recent literature. An illustration of what we mean is the 1969 book entitled An Introduction to Mathematical Ecology by E. C. Pielou [6]. In his preface he says, "The fact that ecology is essentially a mathematical subject is becoming ever more widely accepted. Ecologists everywhere are attempting to formulate and solve their problems by mathematical reasoning, using whatever mathematical knowledge they have acquired, usually in undergraduate courses or private study." The latter remark seems to indicate a premature compulsion of the newly rich than a considered judgment of the sound scientist. Such may be the necessary course of history, however, and we may have to await a philosophy of modeling before these matters can be placed in a proper perspective. The author, however, is well aware of the tentative nature of his subject at present. He thinks he has made a suitable compromise in his choice of subjects for investigation. His opening position is that he wishes "to deal in detail with those aspects that seem likely to furnish good starting points in further research." His main emphasis is on population studies and the principal subdivisions are:

I Population Dynamics

II Spatial Patterns in One-Species Populations

III Spatial Relations of Two or More Species

IV Many-Species Populations

The author is alive to the overall nature of his field as can be seen from a comment in his introduction. He says, "Most ecological communities are made up of a vast profusion of living things. In an acre of forest, for instance, an enormous number of species is present, from trees to soil microorganisms."

Despite the fact that the book shows something of a preoccupation with mathematics, there is a growing awareness of the science of modeling. This is well brought out in his introduction. Here Pielou says, "It may be argued that a simple model can never be realistic, hence that realism demands complicated models is that as soon as one forsakes the Occam's razor principle he runs a risk of constructing a model that incorporates more postulates than are strictly necessary; if unrestricted

proliferation of the postulates is permitted, the number of plausible models that will simulate any given sequence of events becomes conceptually infinite and there is no criterion for choosing among them." The reader may perceive that here is demonstrated a great concern with the prime problems of models which we referred to previously. It may be recalled that we spoke of a sequence of models $\{M_i\}$ which may converge on the thing-to-be-modeled M. We frankly admitted that we had no absolute criterion for the study of such convergence problems. However, based on our knowledge of mathematics and the physical sciences, we have an intuition that such a converging process exists. We as well as Pielou will have to await the work of theorists and philosophers of modeling in the future in order to advance our knowledge of such matters. The science of modeling seems to be much the same state now as mathematics was in the time of Gauss, Cauchy, and Weierstrass. In mathematics the required rigor and foundations took many years for establishment.

The very recent Symposium of the British Ecological Society shows some of the same floundering with respect to the meaning of model and the meaning of mathematics. Their Proceedings were published in 1972 and J. N. R. Jeffers was the editor [7]. In his introductory article he unconsciously emphasizes the ambivalence which exists amongst ecologists concerning mathematics and models. At one point he says, "Modern developments in mathematics and in computer technology have made available new techniques and new conceptual models to the working scientists. Ecology, being essentially concerned with the complex interaction of living organisms with their environment has the greatest need for such developments but can ecologists and mathematicians bridge the gap between their disciplines?" If Jeffers considered the matter properly he would see that there is no gap nor can there be a gap. We consider that one of the causes for all of the mystery and difficulty is that mathematics can be a model, by our definition. However, for most of these serious scientists there is hovering in the backgroung a sense of the need for model qua model. This is brought out to some extent by Jeffers when he says, "If it were possible to construct a mathematical model of a complete ecosystem, then it would be possible to test out various ideas about the management or manipulation of that ecosystem in anticipation of the practical application of those ideas. Such a simulator requires considerable knowledge of the basic working of the ecosystem, but the expression of the available knowledge in the form of a model enables it to be used practically at the earliest possible time." The author is obviously sensing the triadic nature of model. He is dabbling with the true notion, because he implicitly treats of a picture, either verbal or graphic, when he says a "simulator requires considerable knowledge of the basic working of the ecosystem." This can only mean a suitable mental picture of the ecosystem. Where we speak of experiment or experience he speaks of "test". And finally he refers to the practical use of a model.

In the same Proceedings, M. B. Usher discusses the Leslie Matrix Model. Usher says, "Leslie's model, which predicts the age structure

of a population of animals after a unit period of time given both the structure at the present time and a matrix whose elements represent age-specific fecundity and mortality." The model is adapted to studies of energy flowing through food chains. The 'fecundity' terms in the matrix represent the input of solar energy, and the 'mortality' terms the loss of energy at successive trophic levels. Aspects of population ecology such as competition can be built into the model. The elements of the matrix are functions of the vector of energy content.

The need for simplicity in models and the recognition that models can never be perfect by their nature is brought out in one of the Symposium articles by David W. Goodall. He discusses model building and testing. Amongst other things he says, "Model building is often facilitated by division of a system into subsystems. Only in simple models can parameters be estimated from direct observations on the ecosystem as a whole." Furthermore, in connection with the essential incompleteness of any model he says, "The testing of an ecosystem model is not comparable to the testing of a hypothesis. The hypothesis that a model is a perfect representation of the ecosystem would never seriously be entertained, so its disproof is without interest."

Finally we wish to make a reference to a very suggestive remark by P. J. Radford, one of the participants in the Symposium of the British Ecological Society. He said, "Within the mind of almost every ecologist is a qualitative model trying to break out and become quantitative." We consider that Radford is definitely thinking of the picture aspect of model. When he proceeds further in his thinking he will come to the aspects which we call theory and experiment. Then he will fully grasp the real nature of modeling.

A study of the papers which constitute the Symposium should greatly interest any student of generalized modeling because they clearly show the growing consciousness concerning models on the part of scientists in a rapidly maturing discipline, ecology. One can readily trace the need for a science and a philosophy of modeling.

Further evidence of the increasing maturity of thinking in ecology in terms of models can be seen in a very recent monograph by R. M. May [8]. It is concerned with the problems of stability and complexity in ecosystems. The author says, "This book surveys a variety of theoretical models, all bearing on aspects of population stability in biological communities of interacting species." We have here another example of the vacillation between mathematics and modeling.

The book by Ramón Margalef entitled Perspectives in Ecological Theory [9] which was published in 1968 is especially interesting to us because it practically never uses the term model. He does use system and process extensively. A number of ecologists use the expressions ecomodel, but Margalef suggests to us a man who has ridden a horse all of his life but does not know the name of the species. His book however is penetrating in many ways as can be seen from the following two excerpts. He says,

"It is a pity that the tropical rain forest, the most complete and complex model of an ecosystem, is not a very suitable place for the breeding of ecologists." At another place he says that, "Evolution cannot be understood except in the frame of ecosystems."

We complete our brief survey of modern ecology with a reference to a very recent two volume work which was edited by Bernard C. Patten. In the work a whole host of model studies is presented. The concept of model is used extensively throughout both volumes. In his preface, Patten says, "This is a book of ecology in transition from a 'soft' science, synecology, to a 'hard' science, systems ecology, in which the lens of H. T. Odum's 'macroscope' on the world of big patterns is the machinery of mathematical modeling, simulation and systems analysis. The book is substantially the creation of young people at a time when youth in America is experimenting with, if not revising and reorganizing, the ethical and moral basis of contemporary civilized life. The systems theme is central in this exploration in its two salient aspects, change and relationship, and its current pervasiveness in science as well as in society seems no accident as the world presses closer together in the last third of the Twentieth Century." The work treats many of the aspects of what we have referred to as the two-sided model in ecology - the natural system outside and man's impact on it.

We can do no better than terminate our treatment of the science of ecology than with a reference to the final article in the publication which is edited by Patten. It is a suggestive article by A. Ben Clymer entitled Next-Generation Models in Ecology. It has a wide selection of subjects which are assumed to come within the disciplines of ecology. This fact can readily be appreciated from a perusal of the topics which are treated. They are as follows:

1. Current Status of Ecomodeling
2. Trends in Ecomodel Evolution
3. Hierarchical Modeling
4. A Fish Population Ecomodel
5. Mammalian Sociodemographic Ecomodels
6. A Model of Human Personality and Interactions
7. A hierarchy of Health Care Systems Models
8. A General Model for a Community Total Health Care System

At the beginning, Clymer states his objectives. He says, "Ecology is adopted as a perspective on all public systems, and at the same time public problems are regarded as a major emphasis for ecology, one that

will provide a prime motivation for future work in ecological systems analysis and simulation. Man as a semirational, semidisciplined exploiter, manager, and manipulator of the ecosystems he occupies, and yet also as an utterly dependent species of animal with characteristic reaching into the top trophic level, cannot afford to exclude himself from the 'ecomodels' that he constructs to represent nature."

We see that Clymer is approaching, at many points, our thesis for the next chapter. We will now take up the subject which we have presumed to call humanistic thing-to-be-modeled.

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CHAPTER 21

HUMANISTIC THING-TO-BE-MODELED

As we stated in the last chapter, we divide the universe of thing-to-be-modeled into mechanistic and humanistic. We gave some definitions of these terms from standard dictionaries and then examined fairly extensively the mechanistic portion. Now we shall examine briefly the humanistic. We consider that in general, and for our purpose of model studies in particular, the term humanistic relates strongly to the mind and spirit of man qua man. Associated with the idea of humanistic thing-to-be-modeled, there is a particular group of arts and sciences. For our present purpose we list the names of the principal ones. They are: anthropology (cultural), psychology, psychiatry, art, literature, language, drama, health science, medicine, science of defense, political science, economics, philosophy, theology, et cetera. We are of the opinion that this group contrasts sharply with the mechanistic one. In the following we will do our best to emphasize the distinction.

A subject that provides a very interesting example of the clear difference between the mechanistic and the humanistic is mathematics. One might wonder in which group we would place it - in the mechanistic, with the physical sciences, or in the humanistic. It may appear odd at first sight, but we definitely place it with the humanistic. Now we shall give our reasons for doing so. As we stressed in our remarks on the model in mathematics and then on mathematical modeling, there is a clear distinction between mathematics and the applications of mathematics.

In order to put into perspective our thinking about the nature of mathematics we make use of an experience during our stay at the Johns Hopkins University a long time ago. It seems that Professor Aurel Wintner, then a member of the department of mathematics, strongly felt that the subject should be considered to be a part of the humanities rather than of the physical sciences. At first this seemed very odd to us but now, at long last, we think we have an understanding of the position which Wintner took. First we must recall that he was no foreigner in the area of applied mathematics. His treatise on spectral analysis from his days at Leipzig is proof enough of that. No, he simply saw the difference between mathematics per se and its applications. It is our opinion that if one sees the real difference once and for all he will no longer have any difficulty understanding the nature of the subject.

We associate mathematics rather closely with certain areas of knowledge which are contained within the humanistic group. These are language, logic, and philosophy. Also we consider that mathematics is a thing which is closely associated with the mind and spirit of man. It is not something that exists <u>in se</u> in the external world, like the plants and the animals. It is not phenomena like storms, floods, and

earthquakes. Obviously these external things act upon the mind of man and produce reactions. The result is the thought life of man, which includes such things as language, mathematics, and philosophy. Having said this we must reasonably concede that man uses mathematics in the analysis of many phenomena in the physical world. Theoretical models of such a world would no doubt be impossible without mathematics. We hope that our previous discussion of the concept of model, both with respect to mathematics itself and its application, will help clarify our position. For us, the distinction is eminently clear and therefore we heartily agree with Aurel Wintner who classified mathematics as a subject that belongs in the humanities. Since we have said so much about the nature, history, and growth of mathematics in previous chapters, we will now leave that subject and proceed to other phases of the humanistic division.

We do not think that anyone will quarrel with us for classifying language, logic, and philosophy in the humanistic category. However, having done so, one may inquire what it means to say that things-to-be-modeled are linguistic, logical, or philosophical. Our response is that since the beginning of history the meanings of these subjects have ever been before the scholar. The what question is perennial. What is language? What is logic? What is philosophy? Anyone with some knowledge of the history of these subjects knows that change has occurred over a long period of time. Now some of the reasons for this continual change are coming to light. Also, it seems to us that the essential role of model in these areas is ever growing in the consciousness of man. An appreciation of the essential meaning of model and modeling will help us to understand more thoroughly the place of logic, language, and philosophy with respect to the activities of the human mind. When we do, we can see that there is no absolute, final, perfect model possible and hence the mind itself will continuously evolve and progress. Alterations and improvements will always be required in our quest for the perfect.

We may form cognate disciplines into subgroups within the main humanistic division. We could readily take together art, literature, and drama in one subgroup; medicine and health science in another; and political science, economics, and defense in a third. It may also be useful to group psychology, psychiatry, theology, and religion together. A field such as urbanization, which is created by man's everyday needs, we consider to be classifiable as humanistic and to be related to the political, the medical, the economic, and defense subgroups. In contrast to ecology, which gets its raison d'etre from Nature, we think that urbanization, which is a result of the power of man to control, belongs to the humanistic. It is a direct expression of the genius of man, not only to know and study the external world, but also to express his inner spirit and satisfy his personal needs. Ecology is the study of the given environment of man. It is concerned with what many have called Nature, the place and condition in which we naturally find ourselves.

Urbanization relates specifically to the planning and construction of our cities, which are essential to our existence. It would seem that in planning for the urban life one must also allow for the rural life. Clearly the disciplines deeply involve the notion of model, as we shall presently show.

In recent years the problems of aging cities have been well recognized. Professor J. W. Forrester, in his excellent treatise on Urban Dynamics, has extensively examined the manifold problems [1]. It is not our intention to examine the detail of urban planning, but rather to indicate that the concept of model is essential to its performance. We select the work by Forrester for reference, not only because it extensively and competently expounds the subject of urban dynamics, but also because it consistently and consciously uses the concept of model right from the start. On his first page he says that his book is about the growth processes or urban areas and that an urban area is a system of interacting industries, housing, and people. Here at the outset we have a recognition of process and system. On his second page he says that his book is about a particular simulation model. Of course again, we do not understand why an author uses the word simulation as a modifier for model. It is a redundancy which reinforces our argument for a good philosophy of modeling. Forrester further says, "The growth model starts with a nearly empty land area and generates the life cycle of development leading to full land occupancy and equilibrium." At another point he says, "The approach presented in this book is suggested as a method that can be used for evaluating urban policies once the dynamic model or a modification of it has been accepted as adequate. Later on we find a discussion and analysis of an urban model. The author says, "When first modeling a social system it is usually best to model the general class of system rather than a specific system."

On his page 15, Forrester provides a picture which is entitled the urban area in its limitless environment. We think that this is an excellent example of our definition of model. We find the author of a serious book on urban modeling actually drawing a picture of what it is that he is studying. He says that his picture suggests an urban area in a limitless environment. However, he does not add that it is an integral part of the full concept of model. In his sixth chapter, beginning on page 112, he treats rather formally the subject of modeling. The section is even entitled On Modeling. In it he says, "A simulation model is a theory describing the structure and interrelationships of a system." According to us a model is not a theory per se and the reader can see by even a casual observation that the author is really talking about a picture when he defines model. He talks about, "describing the structure and interrelationships of a system." Obviously the description, either verbal or pictorial, is what we call picture. Clearly there is always going to arise a theory of the interrelationships of the system. Again we see here picture, theory, and experiment (experience) as the triadic definition of model.

Forrester concludes that models can be useful or useless. He says, "They can be soundly conceived, inadequate, or wrong." This is another feature of model which we have previously emphasized. Unlike an axiomatic mathematical system, model cannot be called true or false. In its nature, model converges on the thing-to-be-modeled. Forrester even says on page 113, "There is no possibility of absolute proof that a model is appropriate for its objective." He proceeds further to show that he has a very sound understanding of model and the modeling process. Our only quarrel with him is that he does not call a spade a spade. Also he does not proceed from principles of model analysis to specific application, which in his case is urban planning. Of course it has been our position all along that scientists and other scholars do not have the benefit of a science and philosophy of generalized modeling.

We can end our reference to Professor Forrester and his fine treatise only by praising him for what seems to be a superb analysis of a difficult problem. The reader is urged to study the work on urban dynamics even if it is not his field. The gain will be substantial not only for a more solid comprehension of modeling, but also for a clearer view of the methodology which is useful in attacking so many modern problems. In a lengthy appendix the author provides a detailed analysis of urban interactions under the title "The Model - A Theory of Urban Interactions." Because this example from the literature has completely served our purpose, we make no further reference to an ever growing series of books and articles on the subject. Interestingly Forrester makes references only to his own work, with one exception.

We appreciate the face that urban studies relate to the more general field of Sociology. Sociology itself is a study of human social behavior. It is a study of the origin, organization, institutions, and development of human society. We will now examine the use of model in these more general areas.

An interesting study relating to certain aspects of sociology is contained in the Proceedings of a Seminar held in Venice in 1971 [2]. One impressive aspect of the Seminar is that it looks toward the future in a very conscious manner. Of course the forecasting of the future is no easy task, if at all possible. However, it provides an excellent opportunity to use the power of modeling. Olaf Helmer in his paper on societal modeling says, "So I repeat that it is important that we address ourselves today to solving tomorrow's problems." The thrust of the paper by Helmer is not toward the needs of tomorrow, but really toward the methods used in sociology. He specifically treats the Delphic method, which we will briefly describe for the use of the general reader. It is simply a technique whereby the intuitive, or instantaneous, judgments of experts are elicited and combined for the purpose of formulating decisions in whatever fields are under study. Helmer makes what he considers to be a suggestion for future work. He specifically advocates the combining of the Delphic method with what he calls simulation techniques. Here again we witness an ambiguity arising because of

a need for a science of modeling. The Delphic method itself is obviously a part of the modeling to begin with and simulation can really mean no more than modeling. According to our understanding of the teaching of Ramsey, the Delphic method is at least a disclosive model. It permits dialogue and helps to develop knowledge. As everyone should know, an experiment must be used to check Delphic decisions. The paper is terminated with a stressing of the need for a Delphic approach, a Cross-Impact method, and a simulation. The author does admit very close connections among all three.

Included in the papers of the Seminar there are some devoted to models for central planning, system simulation to test environmental policy, a dynamic simulation model of the urban development of Venice, aspects of socio-economic models, models of historical processes, analysis of the structure of behavioral models, and modeling of cybernetic systems. A very poignant remark is made by Dennis L. Meadows in his paper on the Dynamics of Global Equilibrium. He says, "It has become clear that a global model already can provide insights into the general nature of the limiting factors on growth and can provide an overall context for conversations and investigations about specific aspects of global problems." Here again we see the recognition of the need for dialogue and modelistic methods for generating knowledge. Of course a striking aspect of the author's treatment is his use of the concept global. It brings forth again the need for viewing problems in their largest aspects. One should not restrict himself at the beginning of a study to a limited one-sided model when a multi-sided model is essential. We must terminate our reference to the important Venice Seminar for we wish now to draw attention to a recent book on the subject.

In 1971 Peter Abell published a monograph entitled Model Building in Sociology [3]. For various reasons we consider that this book is important for the study of sociology but also because the author seems to have a good grasp of the idea of model. In his preface, inept introductions of the principles of physical science to the study of sociological problems is criticized. Also alongside this criticism is enunciated what we think is a very important principle. Abell says, "Broadly speaking, my belief is that sociology should be concerned to fabricate a 'rational' social existence - societies in which people understand the influences under which they labour and the consequences of what they do in complex situations." Since he claims not to be a philosopher or a statistician, he is rather apologetic about dealing with important substance from these fields. His considered opinion is that sociology needs the availability of sophisticated techniques grounded in philosophy. These he maintains his book attempts to introduce. A considerable amount of attention is given to systematic model building. Types of sociological models are discussed along with what are called causal models. Model Building in Sociology is another recent book of broad design which we can recommend to the student of the general study of models.

We do not wish to rule out from the humanistic category professions that deal with the science of human quantification if they are associated principally with man or groups of men. We have chosen two works for our purpose. The first is on stochastic models of demographic processes by G. Feichtinger [4]. The second concerns opinion sampling in a given population. We chose the monograph by Feichtinger as an example of an important recent study of demographic problems, but even more so because it is an incisive treatment by use of models. The author obviously has an important insight into the meaning of model. As we know, demography is the study of the characteristics of human populations. Specifically the concern is with size, growth, density, distribution, and vital statistics. All of these matters are of great interest to anyone concerned with the human condition as influenced by the now rapidly growing population. The demographic phenomenon treated is extensive and is related to a wide ranging bibliography. The substantive matter of a demographic nature includes population analysis, models for analyzing changes in attitudes, stochastic models for social processes, population waves, models for learning, life table and its applications, biostatistics, measures of natural fecundity, fecundity and the family, fertility rates, life contingencies, educational progress, disease incidence estimation in populations, contraceptive acceptance and pregnancy, statistical processes of evolution, appraisal of the fertility trends in the United States, evaluation of population policy, and model of the Norwegian educational system. The list includes an amazing number of interests for the demographer and for the modelistic analyst. We present the widespread designations of so many subgroups of demographic interest not only to underline the extent of professional interest, but also to indicate the range of use of the concept of model by Feichtinger. We consider that he has a masterful treatment of his subject in terms of a consciously applied model methodology. Again we find the case of a scholar who uses modeling and who makes an incisive examination of its nature as related to his field. However, we have another intensive study of model as applied to a specific subject without any appeal to its general nature. Of course one must expect this because, as we have so often emphasized, a particular scholar is completely attached to the subject matter he is studying and modeling seems to be a tool which he has incidentally attained. Notwithstanding, the entire book reveals an enormous preoccupation with the concept of model. We wish now to examine certain specific aspects of his treatment which relate pertinently to our general thesis.

The author writes about his subject in terms of what he calls mathematical models. We have already commented sufficiently on this peculiarity of analysts in identifying model with mathematics, so we will proceed to the more important aspects of the author's understanding of the concept of model. On his page 5 he has an important division of the aspects of models and we will repeat them here. He has a short paragraph on each of the divisions, which are as follows: Model construction, model analysis, model parameters, and model testing. It is interesting to compare this list with the parts of our triadic concept of model. The model construction relates closely to what we call picture, verbal or

graphic; the model analysis is closely related to what we call theory; and parameters, along with testing, correspond to our use of experiment (experience).

There is a classification of models given by Feichtinger which we think is interesting but limited and not entirely consonant with our position on classification. He gives the divisions as follows: micro- or macro- models, discrete or continuous models, deterministic or stochastic models. These may serve his purpose, but they certainly do not satisfy any reasonable criteria for classification. Our position is that the problem of classification is still unsolved and awaits the development of a science and a philosophy of models. Specifically the micro and macro modifiers imply part and whole. The micro is a side or part of a macro, which is referred to as a global model. The author also refers later to a "mehrtypenmodelle" which we think relates to what we described previously as a many-sided model. If may be recalled that we were critical of some ecologists for limiting themselves to a one-sided model in contrast to a two-sided model which we thought was essential to their approach to environmental problems. The words discrete and continuous are only modifiers which apply to any generalized model and do not imply substantive content of a given field. Also, the deterministic and stochastic aspects of models we designated as general modifiers regardless of field. Notwithstanding our critical comments we still think that Feichtinger has made an impressive attempt to properly apply the principles of modeling in his chosen field. His work corroborates our opinion that scholars in many different disciplines are working toward what will ultimately be the generally accepted science of modeling.

We return now to the second reference which we mentioned above, the short monograph on a sociologically important problem by Professor Coleman of the Johns Hopkins University [5]. His prime interest concerns the nature of the opinion polling process. Those of us who have been interested in polls, particularly political polls, will certainly have an interest in the analysis of the subject which is given by Coleman. In his preface he says, "A few years ago, I became interested in the development of some means of studying the distributions of opinions in a population in a better way than has been done." His attack on the problem if properly critical and his systematic use of the concept of model is highly encouraging for a field that to some must have seemed whimsical at times. With respect to this subject, the author pointedly says, "The response uncertainty confounds simple treatment of reliability." He says further, "This book grew out of the attempt to separate these two elements by a model that explicitly incorporated both of them." Here again we sense a feeling for a two-sided model. The problem of multi-sided or multi-elements must be developed substantially by the scientist of the future. Our opinion is that an important aspect of the one-sided model is that it may lead to unresolvable bias. In justification of early effort in the field we may say that the nascent science of polling had to start someplace, just as ecology did. The original models are models, albeit very crude ones, and maybe ones that should now be rejected. Coleman gives

Lee Wiggins credit for the initial development of attitude change. An outcome of the monograph is a plan for future study. Already an improved use of the present model is foreseen.

The science of biology contains much that relates to man himself. Accordingly, we consider that certain parts of biology may properly be classified in the humanistic division. We wish now to examine this subject from the standpoint of model. Of course as any informed person knows biology during the last few decades has become quite analytical whereas in the past it was scarecely more than a descriptive type of subject. The obvious reason for the change is the great and increasing influence of physics, chemistry, and mathematics. As a consequence we now have subjects such as biophysics, biochemistry, and bioengineering. It is no mystery then as to why biology has now come alive and makes use of systematic modeling. We wish to examine briefly some of the references to the new research. An excellent recent study of some aspects of physiology, which is the biological science of essential life processes, was published under the auspices of the Academy of Sciences of the U.S.S.R. An English translation was published in 1971 by the M.I.T. Press [6]. The editors of this Russian work were I. M. Gelfand, V. S. Gurfinkel, S. W. Fomin, and M. L. Tsetlin. Peter H. Greene of the University of Chicago furnished a useful review of the Russian work but also relates it to investigations by himself and his colleagues. For our purpose, the study along with comments by Greene demonstrate the value of models for the investigation of biological systems. In a brief preface to the English translation it is said that, "At the present time it is becoming increasingly clear that the complexity of physiological systems as objects of contemporary experimental research is so great that even a complete description of the elements and their interrelationships is in itself insufficient, for an understanding of a system's principles of operation. This dilemma leads to the necessity of using new methods for the study of complex systems. One of these methods can be, it seems to us, a model description of a system's functioning. In this case the essential demands of the structure and properties of the model are in the first place that it should provide a correct description of the phenomenon of the functioning system; and in the second place that the postulates used in the construction of the model should correspond to the real properties of the elements of the modeled system and to the interrelationships of the modeled system and to the interrelationships of the elements. Furthermore, inasmuch as one and the same phenomenology usually allows several model descriptions, it is desirable to be concerned about finishing up the model to the level where it becomes possible to make conclusions which allow experimental verification." We were greatly encouraged to see that the author is explicitly using all of the important aspects of models which we have previously formulated. He undoubtedly conceives of the need for a picture, whether verbal or pictorial; of a theory based on postulates; and of experimentation for the purpose of verification. Here we have again a clear statement of the triadic nature of model. Furthermore, the author stresses the fact that the modeling process is never complete. He implicitly involves the notion that a model is never said to be true or

exact. Such a viewpoint coincides with our suggestion that a thing-to-be modeled is a limit of a sequence of improving models. According to Greene the book represents an effort to understand the nervous system and other excitable systems at the levels of cells, tissues, and organs. We are particularlary impressed with the fact that Chapters 3 and 7 deal with wavelike propagation of excitation in excitable media. The analogical relation of these waves to the electrical and substantive waves of classical physics is impressive. Here we have a most potent example of the meaning of analogue and model as related to human reason. The wave phenomena in the biological area relate importantly to the cardiovascular, the respiratory, and the neurological functions. We recommend the book as a fine example of the current use of modeling in biology.

A somewhat earlier publication shows the interesting phenomenon of an electrical engineer in the U.S.A. trying to model the nervous system [7]. In his monograph he impressively tries to model physiological phenomena. He even carries his investigation into the difficult but highly important subfields of learning, memory, and pattern recognition. The author specifically exhibits models of feedback loops, of the somatic sensory cortex, of the motor cortex, and of thought processes. We consider that an important aspect of this paper by Deutsch is that it demonstrates how one who is competent in the physical sciences and in the general science of modeling can begin to make progress in important related fields such as electricity and biology.

In order to draw attention to the relatedness of many biological subjects to our humanistic grouping we can do no better than make reference to a very recent monograph entitled The Lives of a Cell by Lewis Thomas [8]. While Dr. Thomas never uses the technical concept of model consciously in his little book he does exhibit the spirit of modeling extensively. Also he relates in a peculiarly effective manner the great biological environment of man to man himself. His whole treatment is truly humanistic. We strongly recommend the book to anyone who is interested in the most general aspects of modeling. There are many interesting chapters and a typical one is entitled Societies as Organisms. Of course here is our old friend the analogy. Also, the writer does use the term model specifically in this chapter. We will presently refer to this again but first it is necessary to stress the fact that Thomas is aware that scientists in the past have been very wary of comparing colonies of animals with social man. In fact on his page 11 he says, "The writers of books on insect behavior generally take pains, in their prefaces, to caution that insects are like creatures from another planet, that their behavior is absolutely foreign, totally inhuman, unearthly, almost unbiological. They are more like perfectly tooled but crazy little machines and we violate science when we try to read human meanings in their arrangements." We are sure that Thomas is not carried away by such a medieval-like dogma. In fact it is clear that he has found the behavior of ants, termites, and bees very suggestive. From the ever present analogy he is inspired to say, "Although we are by all odds, the most social of all social animals - more interdependent, more attached to each

other, more inseparable in our behavior than bees - we do not often feel our conjoined intelligence." He is thus led to make an important observation on science publications and general human knowledge. He says, "The system of communications in science should provide a neat, workable model for studying mechanisms of information-building in human society." Many other embryonic visions of a modelistic nature are provided in the book. His chapter entitled Your Very Good Health is a gem. It is a very brief but incisive review of health-care delivery and the medical world.

We will terminate our references to the strictly biological with a look at the ever growing science of genetics. We recommend to the future developer of the science and philosophy of models a good look at the relatively recent science of genetics. For our present purpose, that science is eminently important both from the standpoint of form and content. An extensive examination of the subject would be highly rewarding, but lack of time and talent prevents us from making it. However, with the help of the work of C. D. Darlington we will give a brief view of the field and its relation to the science of modeling. We shall see the enormously important connection between it and the humanistic as we have attempted to describe it. Man as man is the center of the division which we chose to call humanistic things-to-be-modeled. It may be useful for some readers to have us say a few words about the makeup of the two books which constitute our references. The first is a far ranging treatment of genetics in its historic and philosophical dimensions by Darlington alone [9]. The second is more strictly science and it is done by Darlington with the collaboration of K. Mather [10]. Darlington, an Oxford Professor, provides a short introduction to his paperback edition, which is a revision and expansion of a 1953 publication called The Facts of Life. At the end of the introduction which was composed in 1968 he says, "As the reader of this book will see, it has been a fluctuating line of battle. And today progress is uncertain and victory remote. For many generations men's opinions of themselves; of their purposes and their policies, will move back and forth. Perilous generations they will be. For some men will accommodate their understanding of biology and their view of mankind to one another while others will fail to do so.

The steps to be taken in this attempted accomodation are now, however, becoming clear. The first step will be to use genetics in understanding history, our history. The second will be to reverse the argument, to use history in understanding genetics, our genetics." The word model is not used in this volume, but analogy, the iconic model, and the disclosive model are really used throughout without saying so. We recommend the book to the reader as eminently important and useful for understanding the science of generalized modeling. Now we wish to comment on the volume which was done jointly by Darlington and Mather, and which is the more strictly scientific of the two.

The introduction to the Elements of Genetics was written by Darlington himself. It claims to be the introduction to the 1969 edition

and also claims to be The Elements of Genetics: 1949 - 1969. Darlington says, "This book was first offered to the reader by its authors as an account, the simplest coherent account possible at the time, of heredity and variation. We tried to show the connections between the materials and the processes at work in heredity and variation. And we tried to show them at work on the three levels of extent and duration at which life is conveniently studied, the level of cells, individuals and populations." In this introduction there is a strong reference to model. Just after describing his work on ribose nucleic acid (RNA) and its desoxy-analogue (DNA) he comes to the celebrated work of Watson and Crick (1953). He says that the connections between the various important properties - at once physical, chemical and genetical - were explained by a universal model of the structure of DNA which was proposed by Watson and Cricks. This model assumes that DNA is a double helix. Here we have an example of the use of model in biology, which is now every bit as famous as the model known as the Rutherford-Bohr atom. According to the model of DNA it became the only and sufficient vehicle of heredity. It is the source of its own reproduction, the source also of the production of proteins, and through them, of everything else. Tests have demonstrated subordinate models, which have explained with increasing detail how the materials of genes and chromosomes are put together and how they do their work. Darlington refers to other models, but we shall leave these to the reader for further study. Suffice it to say that the subject matter is crucially important to man and utterly dependent on the concept of model. As a consequence of all the work on genetics, Darlington can now say, "Below the level of mendelism, we can make our way into the physical sciences. Above it, we can climb into the study of evolution, of society and, notably, of man himself." With the two books, for which we have given the references, we can follow the evolution of biology and heredity from Mendel to Darwin and then into the modern genetic era. The student of models can see the clear-cut role of his science in genetics.

We end our comments on genetics and the model by a reference to a very recent analysis by Rollin D. Hotchkiss of the Rockfeller University [11]. Hotchkiss says "Earlier models for genetic recombinations have principally tended to restate the experimental observations of recombination experiments in form convenient for teaching and learning. As formal genetics has been replaced by molecular genetics, the structure, function, and now lastly, the recombinational exchanges of genes are being explained in biochemical terms. As yet, all models contain some arbitrary steps or omit their specifications altogether." Further along, Hotchkiss says, "This is a time however when the earlier formal - or didactic - models of genetic recombination no longer suffice, and many of us now strive to construct models which take into account besides the end results, the properties of DNA and in particular the properties of enzyme systems which act upon it." Finally, we have the author saying that, "There now exist so many models or proposals for genetic recombination that it will be manifestly impossible even to mention them all here, much less to discuss them adequately." We can only say that we are here dealing with a

sophisticated application of the concept of model. The amount of progress that is apparently being made with the use of the science of modeling is encouraging indeed.

Currently there is a very important sector of our humanistic world that should be considered. It is the medical field. There are many facets of it that should be considered in depth, but we must limit ourselves to but a few remarks. The antiquity of medicine and its importance to mankind need no reviewing here, but what is necessary is an attempt to stress the urgent need for a global model. It is our opinion that a veritable revolution in medicine will soon be upon us. Many signs point that way. Some of the appropriate answers to our urgent questions will undoubtedly be obtained in terms of suitable models.

Some of the first considerations should be for new models in medical education and practice. Related to these are functional models of the human body and its needs. What we mean by this is that important physical and chemical knowledge must be brought to the fore and considered parts of the model. Various questions of a serious nature are now coming to light. These have to do with the makeup and functioning of the human body itself. Of course many people tacitly assume that in a medical college a student masters all of the knowledge that should be required for good practice. We must admit that there are serious grounds to doubt this and we will now indicate why. Let us consider some of the important functions of the body. Immediately we recognize the essential importance of systems such as the cardiovascular. These require intimate knowledge of fluid flow for proper comprehension of performance. Now it is well known to some observers that our medical colleges require only a modicum of knowledge of chemistry and physics in their curricula, but even a casual look at the crudest model of man demonstrates that this is far from sufficient to do intelligent work. Passing time can only result in a final acknowledgement of the fact and a vigorous move to improve the situation. Another part of a global model of the human body involves initimately the nutritional aspect. Present day bickerings about vitamins and minerals which are essential to the functioning of the body are bringing into focus our serious, and at times fatal, lack of knowledge and good practice. One thing that is urgently needed in the area of health is a knowledge of the nutritional requirements of each individual instead of the perpetual muddling along with the aid of vague averages. New fields of measuremens must be opened. Suitable dynamometers must become important parts of daily living. Each man's working capacity must be accurately determined. His nutritional needs must be satisfied. Now it is clear that in the past, and for that matter even in the present, the rule of thumb is all that has been, and is, available. It is essential that the vector of vitality for each person be determined if he is to live most effectively.

It is clear that the demands we make are not easy to meet, but we submit that they are essential for effective living. They will not be met fully in the near future, but at least a suitable model should be

carefully constructed to clearly show the needs. The least we can gain will be the removal of many foolish, and sometimes fatal, medical prescriptions. We will not attempt surgery where it serves no good purpose. We will know how people die of malnutrition even when help is really at hand. We will better understand the effects of inhaling smoke in any form, questionable liquids, and deleterious drugs. Obviously no small group of men is going to accomplish all of these things. Many areas of knowledge are involved. In recent times we have seen the rise of biophysics and biochemistry. More recently we have observed the arrival of bioengineering. The resulting complexity of our system of knowledge demands the intelligent use of the concept of model to handle and apply it.

The ultra-global medical model has many submodels but none is more important than the health care delivery component. The administration and operation of hospitals, trauma centers, and medical institutes are now being subjected to systems analysis and related models are being used. Every newspaper brings to our attention some items of the runaway problems which must be controlled. Proper planning is now being demanded and it cannot be accomplished without satisfactory modeling. Suitable estimations and predictions cannot be made without the serious use of models. All of these views can be extensively documented because the literature pertaining to them is growing apace. Our primary intention is to stress the fact that there is still a great need for models in the entire medical profession. The appropriate pictures - verbal, graphic, and tabular - along with the development of theory and use of experiment are needed. These constitute what we have termed the triadic nature of model.

Cognate with the profession of medicine is that of psychiatry. Closely related to both is the profession of psychology. While the conscious use of model has been a part of physics and engineering for over a century, its use in psychology is very recent indeed. Some may even wonder how it can be used there. We wish to examine the situation for two reasons. One is that we have a possible question about its actual use and the other is that we are dealing with one of mankind's most important professions. With our limited effort we will attempt to provide as extensive a view as possible.

We begin our exposition with a peculiar choice of reference. It is a very recent book by a researcher in medical sociology and a physician. Its title even is very interesting. The authors, Siegler and Osmond, call it Models of Madness, Models of Wisdom [12]. Our interest is dictated by the face that we have here medical wisdom, zest for modeling, and even propagandistic propensity. The latter we will treat first, mainly because we think it is false.

On their pages 16 - 18 they present a table which they call Models of Madness. They claim that the models represented are: Medical, Moral, Impaired, Psychoanalytic, Social, Psychedelic, Conspiratorial, and

Family. These form the columns of a matrix whose rows are a set of aspects of the disease. We think the approach is clever but the fundamental conclusion based on it is highly questionable. The total thrust seems to be propaganda in behalf of the thesis that the physician is the only proper professional to handle mental illness. We cannot fault the authors on their enthusiasm but we can certainly do so on the basis of their faulty logic. Having said this we do wish to recommend the book to our readers as a very recent attempt to use the concept of model in a delicate area.

The foreword to their book is by D. Paul E. Huston who seems to have a good grasp of the meaning of model. In fairness to the authors, whom we have taken to task above, we will let Dr. Huston give his opinion. He says, "The second major contribution of the book is the comparison of the medical model with the seven nonmedical models of madness. In the hands of Siegler and Osmond all of these nonmedical models are inadequate in one or more of their dimensions when compared to the medical model. The message for psychiatrists who have deserted the medical model, or only use a part of it and 'bits and pieces' of other models, it is a clear and urgent demand to re-examine their practices. _Psychiatry is a branch of medicine_." The remark about the use of 'bits and pieces' inspires us to remark that here we may have a case of the pot calling the kettle black. At another point Huston correctly says, "A few words about models. Models, like diseases, are abstractions. They are inventions of the human mind to place facts, events, and theories in an orderly manner. They are not necessarily true or false. Models which are the closest to reality and the most comprehensive seem more satisfying intellectually." We cannot go further at this time in our consideration of a very impressive book, but we will end now with a list of topics treated. They are: Discontinuous Models of Madness, Continuous Models of Madness, Medical Model, Medicine and the Submodels, Community Mental Health: What Model?, Models of Madness Compared, The Future of Psychiatry, Models of Drug Addiction, Models of Alcoholism.

In contrast with the work of the two previous authors, we offer for consideration two books which do not contain the term model but in which the authors unconsciously use the concept. These are entitled _The Organism_ by Kurt Goldstein and _Theories of Personality_ by Hall and Lindsey [13, 14]. We leave to the readers the tracing of the use of the concept model in these works. However, we would like to make a few comments. Goldstein supplies a tour de force on a holistic concept of man. He is really dealing with a global model of man. His work is important in the fields of medicine, biology, neurology, psychiatry, and physiology. The holistic theory is undoubtedly an important one and for us it is especially interesting as an example of a global model applied to man. On their pages 301 - 316, Hall and Lindsey refer appreciatively to the work of Goldstein and relate it to their theories of personality. Their chapter 15 is entitled Personality Theory in Perspective. They begin the chapter by saying, "We have now reached the end of our tour through thirteen major types of personality theory." Our opinion is that the title of the chapter would more appropriately be

Personality Models in Perspective. The student of model in general should have an interesting time tracing the hidden use of model in these books.

In a 1973 book by Le Vine [15] we have a study of personality as related to culture. At this late date we see that authors in this field are beginning to use properly the concept of model. The author's part III is entitled Population Psychology: An Evolutionary Model of Culture and Personality. In his introduction, he says, "The basic questions of culture and personality have long been recognized as worthy of scientific attention, and early theoretical statements on the subject provided a convincing sense of connection between personality, society, and culture, which stimulated a variety of research effort in psychology, psychiatry, education, and the social sciences." He further states that, "I return for guidance to the Darwinian model of organism-environment interaction that contributed the original sense of connection inspiring work in this field." His chapter 8 is entitled Basic Concepts in an Evolutionary Model. Here we have a good example of an author who does not use the word theory for model.

In the field of group psychotherapy by Irvin Yalom we have a good use of model [16]. On his page 91, under the title Model-Setting Participant, he cogently describes the therapist as model setter. The highly important field of group psychotherapy is shown related in an effective manner with the concept of model. At other points the author effectively uses modeling but we will leave further study of Yalom's work to the reader who may be interested.

We end our treatment of the humanistic thing-to-be-modeled with a few references to recent esoteric approaches to psychology. The first is a monograph on Phenomenological Psychology which is edited by Amadeo Giorgi et al [17]. In an introduction by Giorgi, Fisher, and R. von Ekartsberg the objective is stated as follows. It is said that, "Through a utilization of the philosophical tenets of existential phenomenology, we are attempting to found psychology conceived as a human science." The whole slant is conveyed by a preamble entitled Concerning the Paradigm of Human Scientific Psychology and a chapter entitled A Reciprocal Participation Model of Experimentation by Robert J. Sardello. In the chapter Sardello says, "If a phenomenological psychology means more than simply attempting to include heretofore neglected problems within established paradigms, a precise statement of the direction of paradigmatic changes is required." In his monograph, Strasser esoterically says, "The model of the stream of consciousness is a good example to illustrate the point. For how do we know that absolute stream? Particular experiences of the stream of consciousness are directly given to use; others are anticipated through a transcendent expectation." [18] Our final reference concerns a meeting in Rome which was reported by J. T. Lester [19]. The occasion was the Fourth Research Conference on Subjective Probability, Utility, and Decision Making. It turns out that most of those who attended are psychologists. The slogan for the meeting could

well have been a statement by Lester. "Making explicit" really means making a model.

In our next and final chapter we will present some conclusions on our investigation of the concept and use of models.

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CHAPTER 22

CONCLUSIONS REGARDING THE CONCEPT OF MODEL

At long last we can draw some definite conclusions about the concept and uses of models, but first we must briefly summarize the subject matter which we have been examining. We have scanned the rise of the concept of model from the grunt of the hominid to the sophisticated mathematical essay of the Oxonian; from the steam engine of Hero to the super jets of London and Paris; and from the metaphors of Genesis to the theological model of the scholar I. Ramsey.

We have used many references which cover a wide range of the general subject of modeling. It was our intention to obtain a broad coverage, but we do not wish to imply that we necessarily chose the best in every field. If we have done violence with our use or interpretation of the works of experts it was certainly unintentional and we are sincerely sorry. Our main purpose was to convince the reader of the universality of the use of model and its great importance for progress in every area of human enterprise.

We have spanned a very wide spectrum of human interests and we realize that no man or even group of men possesses the talents to go in depth into all of them or even into just a few of them. However we may say that it is our opinion that at present someone must begin the attack on all of the problems associated with the basis and the role of models. We realize that our attempt is the barest kind of start and that any definitive work lies in the future. We also appreciate the fact that the final task may be conceived as encyclopedic. Already we have encyclopedias in the related fields of philosophy, mathematics and science.

We hope that we may have excited interest in the most general approach to modeling as a science and, also, that our work may lead to a philosophy of modeling. Our conviction is that there should be a place in the curricula of the universities for the subject. We know that it vitally touches on such subjects as philosophy, science, engineering, and mathematics, but from the fundamental nature of the concept we consider that it has an autonomy of its own. It seems reasonable to demand that every educated person should be somewhat versed in its use and every professional person expert in its application to the specific area of concern. A similar situation now exists with regard to mathematics. Many are familiar with the rudiments of the subject, some are informed about its advanced phases, and a few are actually professional mathematicians.

We submit that in our treatment of the concept of model it was amply demonstrated that there are certain types of models which are now firmly established. These are the iconic, the analogic, and the similitudinous models. Furthermore we consider that we have made a

serious case for what we called the Newtonian model and the extended Newtonian model. The open-endedness of the science we respond to at the present time by suggesting that consideration be given to the disclosive model which was suggested by I. Ramsey.

The possible ambiguity with respect to mathematics as related to modeling we disposed of, we hope satisfactorily, by a chapter on the use of model in mathematics and then by a chapter on mathematical modeling. We, also, stressed the fact that no amount of virtuosity in pure mathematics is equivalent to the intuition of a process, system, or machine. These latter concepts are associated with model in very special ways. We finally attempted to demonstrate that a science has matured when its participants consciously and systematically make use of models.

In our two previous chapters we made a case for dividing the regimes for modeling into the mechanistic and the humanistic. The mechanistic regime which relates more intimately to the environment of man contains mainly the classical examples of model. The humanistic regime which now consciously uses the concept of model is mainly contemporary. It does have, however, some good examples which are more ancient. Moses modeled a moral code in the Decalogue; Dante (1265 - 1321) modeled Heaven, Hell, and Purgatory in the Divina Commedia; and Michaelangelo (1475 - 1564) modeled the Creation in the Sistine Chapel.

We wish now to present a brief summary of our own formulation of ideas and rules which relate to the concept of model. Basic to our position is the assumption that A models B where A and B are any two things in the universe. This axiom we referred to many times in our text. Before we leave it now, we wish to show that others have somewhat the same idea. Take for example a work of Susanne Langer, the logician [1]. In her small philosophical monograph, which is said to be a study of the human mind in relation to feeling, explored through art, language and symbol, she says, "The processes of Nature, especially, may be seen one in another; and those which are hard to observe are generally understood only through a model. Death is seen as an eternal sleep, youth and age as spring and autumn or winter, life as a flame consuming the candle that provides it. The very framework of experience is only thinkable by means of models: time is most readily imagined as a flowing stream, and is, in fact, so hard to conceive without that metaphorical image that many people believe time literally flows." And a little further on she says, "What we do see, however, is that the most various things repeat a few fundamental forms, by virtue of which we can use familiar events as <u>models</u> to understand new ones and tangible objects as symbols of untangible realities." Langer discusses the idea at greater length but we shall now leave any further perusal to our reader. We consider that her thinking is consonant with our axiom that A models B. Having said this however we admit that if the thing-to-be-modeled, say B, has been chosen it requires considerable skill to choose a satisfactory model A. This fact is really what makes the game hard to play and the decisions difficult to make. On the other hand, the process is clearcut.

The next important assumption is that the complete model is triadic in nature. It includes the picture, verbal or graphic, the theory, and che experiment or experience. We illustrated this principle with many examples in our text. One of the clearest cases, to us, is the Rutherford-Bohr model of the atom.

We then considered to be an axiom the statement that for a thing-to-be-modeled. sav B. there is a possible sequence of models, say $\{B_i\}$, which converges on B as the index i is increased. B is really considered to be a limit for the sequence. We illustrated the principle by examples but one of our favorites is that by Julia Apter, the medical scientist, who proposed approaching the studies of the systems of the human body by a series of increasingly more accurate models. At the present time we would like to strengthen our position with a reference to a philosopher. In one of his important works, The World and the Individual, Josiah Royce seems to be thinking along the same lines as ourselves [2]. He says, "---- in terms of which our theory of Being is to be defined, is a process analogous to that by which modern mathematical speculation has undertaken to deal with its own concepts of the type called by the Germans Grenzbegriffe, or Limiting Concepts, or better, Concepts of Limits. As a fact, one of the first things to be noted about our conception of Being is that, as a matter of Logic, it is the concept of limit, namely of that limit to which the internal meaning or purpose of an idea tends as it grows consciously determinate." Royce further develops his thinking but we leave to our reader any further consideration of his text.

An important deduction that can be made on the basis of our axioms is that a model is never final or complete. Also, it can never be completely true or false. It can only be more or less useful. Its real value is produced on the basis of the ingenuity of its constructor.

In order to avoid possible ambiguity, it may be recalled that we dropped the use of the term prototype as applied in the process of modeling and decided to use instead the more definite terms, which are model and thing-to-be-modeled (TTBM).

In our treatment of model we stressed some essential adjectival modifiers. They are: static, dynamic, deterministic, and stochastic. The static model is that which is invariant with respect to time; the dynamic model is that which changes with time; the deterministic model is that which can be completely and systematically determined by rules or mathematical equations (for example the simple oscillator in physics); and the stochastic model is that which is probabilistic in nature and has some features at least, which are not completely determinate.

The peculiar role played by mathematics in the science of modeling we have treated at considerable length. We are of the opinion that it is very important to keep the relation of mathematics to modeling very

very clear. One possible source of ambiguity is the fact that in our [a, B] concept of model and thing-to-be-modeled, a set of mathematical equations may be the A (model). Our main caution to the reader is that he should never unqualifiedly identify the concept of mathematics and that of model. There is a science of mathematics and an entirely independent science of modeling. One can mathematize and, again, one can model. They are not identically the same process. Mathematics, by its nature, has an exactness and truth that does not belong to the model.

The collaboration of the mathematician with the general scientist can be a very productive enterprise. A brilliant piece of intellectual cooperation which illustrates this fact is shown by the mathematical logician Beth and the psychologist Piaget in their great monograph entitled Mathematical Epistemology and Psychology [3]. In their final paragraph they conclude that , "In all, each of the respective activities of the logician and the psychologist reflects the other, not because they are interdependent, but because, whilst remaining entirely autonomous, they are complimentary. So it is this autonomy and complimentarity together, which make the search for an epistemological synthesis not only possible but also necessary." The same could be said for mathematics and the science of modeling.

A very recent example of the use of mathematics is the treatise by Karl V. Bury [4]. He is very much concerned with what he calls the statistical model. The normal, Gamma, Beta, and Weibull statistical models are investigated in considerable detail. We might consider here a thing-to-be-modeled from the statistical viewpoint. We can recommend this book to the reader as an example of a recent treatment of statistical problems associated with the physical and life sciences as well as the general problem of decision making. It may be recalled that in our treatment of the relationship between statistics and modeling, we attempted to give a view of the historical relation of statistics and probability.

It is our conclusion that at the present time the status of model in both physics and chemistry is well established. In biology considerable progress is being made with the use of model. While it may not be in the forefront of current biological research, the book on dimensional analysis as applied in biology by Stahl is very enlightening as to what may be accomplished in biology by similarity considerations [5]. Our survey of biology indicates that that subject is really coming alive with the use of the model and the future looks very bright. As we have tried to show in our text the concept of model plays an essential role in biology. The Watson-Crick double helix model, for example, is every bit as significant in biology as the Rutherford-Bohr model of the atom is in physics.

Probably the professional field which will most radically benefit from the use of model, in the near future, is psychology. Somewhat like

biology, psychology has had a very long history of slow development in coming of age scientifically. Even in very recent years the status of psychology was dubious. Currently, however, real progress is apparently being made. It is agreed by some that the modern scientific aspect of the subject began to improve with the research of Wundt at the University of Leipzig in the last quarter of the nineteenth century. Of course, there were some other intellectual giants such as Ludwig von Helmholz who conducted important experimental research in Germany. For the U.S.A. the growth of experimental psychology may be traced in the old texts of R. S. Woodworth. His pioneering work culminated in a recent treatise of that name [6]. Nineteen authors contributed. However, in the work there is not much specific use of modeling. There are, however. models of psychophysical threshold and of chromatic vision. In addition there is a paradigmatic analysis of probability learning. A relatively recent article on the use of models in experimental psychology is provided by R. C. Atkinson in the Proceedings of a Colloquium on the concept and role of models [7]. Models have been used extensively in recent years in such studies as those on the human memory. A treatise on the subject in 1970 by many authors is edited by Donald A. Norman [8]. The reader will get a very diversified and modelistic treatment of the subject in this work, which clearly shows the current approach of advanced psychology. There can be no doubt that the scientific future of psychology looks bright and its foundation rests on the science of modeling.

Undoubtedly one of the most active scientific areas in which advanced modeling is used expertly is that of urban development. We have already mentioned the important work on urban dynamics by Forrester, which was published in 1970. Since the advent of that publication, Walter Helly has published an important treatise on urban systems [9]. His work illustrates the current use of models in the studies of urban problems. The indications now are that the future of that professional area is bright and mainly because of the intelligent use of sophisticated modeling techniques.

We have not dwelt much in our book on the advanced use of the model in such areas as production and management. However, the current development of these essential activities is demonstrating again the essential use of models. An example of this point is demonstrated in a book on production and operations management by E. S. Buffa [10]. One can readily see that the industrial dimension of life is essential to the success, maybe even the survival, of the contemporary world. Practically of dominating significance is the industrial-military complex. To obtain a proper perspective in this area would require volumes of difficult analysis. Even serious studies would literally be impossible without the aid of the science of models. In an appendix we attempt to provide some description of the nature of military models. The future need for more realistic modeling in these subjects is beyond question.

We now end our monograph with a reference to a little book entitled Man in the Modern World by the biologist Julian Huxley [11]. We do this because Huxley is a member of a relatively small group of scholars who take a broad view of the world and its many problems. Also he is a competent observer of the human scene who can effectively cross intellectual boundaries because he uses analogy with understanding. In a section of his book he begins a discourse on biological analogy in which he chides the layman for an uncritical use of analogy, while he criticizes scientists for being over-cautious and underating its potential value. Despite this view, however, Huxley did not reach a stage in his own intellectual growth which would have permitted him to appreciate the full power of the growing science of modeling. Albeit, others of competence who have the necessary vision are coming on the scene. These will produce the fully developed science and philosophy of models.

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CHAPTER 23

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APPENDIX

MILITARY COMBAT MODELS

An excellent example of the modern model is that which represents combat of military forces. We have pictures and theories of operations as well as methods for checking results of various kinds of simulation. The different models provide useful dialogue and generate important knowledge. In the present Appendix we consider many of the aspects of such modeling.

A special taxonomy of models applies to warfare because serious interest spans a spectrum of military activity which includes the smallest combat units, on the one hand, and the largest array of divisions on the other. In between are the Battalion Level Activities. Models of these are very useful because they permit studies to include sufficient detail and yet are large enough to portray the major interactions of units with various echelons.

The very small "high resolution" models are used to improve insight into and quantification of detailed trade offs such as better fire accuracy, higher rate of fire, and increased lethality. Attempts to use extensive detail in models of combat between large forces greatly increases cost and time to obtain useful results. One can observe this particularly in the case of practical exercises conducted by means of computer simulation. The required variations and replications cause the consumption of large blocks of computer time.

As we have observed in the main body of our text, a sequence of models $\{M_i\}$ is usually the approach to knowledge of M, the thing-to-be-modeled. In recent years combat analysts have resorted to what they call a hierarchy of models in order to deal with the problems of large scale operations. In various war games the small unit, high resolution model, is used to generate the aggregated performance measures for battalion and division level games. The outputs of the latter are used for studies of the corps and the entire army. An illustration is the use of the output of the so-called Carmonette model for use with analytical models known as Comanex. These models are described in a table at the end of the Appendix. Finally, outputs of the Comanex are used as inputs for what is known as the Division Battle Model (DBM). These higher level models and games are necessary to supply answers to important questions which relate to such things as Force Mix, Structure, and Total Theater Force.

Combat models employ various techniques of war games, simulation types, and analytical formulations. As a consequence various amounts of realism, player involvement, automation, and abstraction are obtained. Some war games are more realistic, involve greater player interaction, are less automated, and are not so abstract as others. The purely

analytical model seems to be less realistic, involves no player interaction, is usually very abstract, and is either highly automated or uses no computer.

In war games, operators who simulate the decision makers of actual warfare, use their judgment to make decisions which depend upon environment, resources, and overall strategy. Either manual or automation methods may be used to evaluate the results of their decisions. For computer methods, which are used with the war games, the process of keeping data on unit location, size, and state of the environment are automated to various degrees in order to accelerate the analysis. Assessment routines are also automated to the maximum practicable extent and the direct use of the judgment of the operator is reserved for operations that fall entirely within his personal abilities.

War games are classified as "free" or "rigid", depending on the degree to which the game controller uses his judgment, which is based on his military experience, or bases his assessment on detailed and comprehensive rules. During the last two decades there has been a strong trend toward rigid games. This is the case for two very important reasons. First, game operations and outcomes have been made more reproducible; and second, the high speed computer, which requires full automation, is used extensively to speed up the games.

The benefits arising from the use of rigid games come at a cost which is not always recognized. Opportunity for effectively using imagination in the development of new modes of combat is severely limited. For example, there have been drastic changes in balance among combat arms since WWII. Despite this face, little or no allowance is made for it in simulating non-nuclear combat. Is is conceded that great effort would be required to develop a complete set of rigid rules for any newly conceived combat model. The ultimate cost for the insistence on rigid models exclusively is that one is unconsciously led to plan for wars which are already in the past.

The TBM and Jiffy games are fully operated manually. Some war games can be computer assisted and their assessment routines automated. Examples of the computer assisted war games are THEATERSPIEL, Corps Battle Model, and Division Battle Model. Another step toward complete automation is the player assisted simulation in which the players periodically obtain readouts of the action and place orders to the computer which continues to run the battle. Examples are models which are called DIVTAG and LEGION. In the latter, all company commander and some higher level decision processes are automated.

Some models are complete simulation with human participation. In this type of model events in the different combat processes are essentially in sequence and decisions are based on predetermined rules which are programmed into the automated evaluation procedure. The automated decision rules and the contingency orders are specified in

an attempt to fully simulate the decision making process of commanders. In some cases these rules are difficult to change while in others certain flexibility and modularity permit changes in rules of engagement.

Models generally contain a certain number of stochastic or probabilistic routines in order to simulate the chance element which is characteristic of actual combat. These models require probability distributions for many of the input variables which generate probability distributions for the outputs or results. Input distributions are samples in the appropriate sequence to produce a single output in any one replication. In order to generate the complete probability distribution for combat results, the sampling procedure is repeated a number of times for each analysis. The process is continued until the results appear to converge to a stable output probability distribution. Stochastic models provide an estimate of the probability that a certain outcome will occur and hence provide a basis for confidence.

It is useful to stress two features of the model. One is the generation of combats and the other is the assessment of outcomes of those combats.

Closed war games usually depend upon the control team to decide when engagements will occur. This fact permits the team to pace the combat more realistically. Computer simulation seems to lead to an excessive amount of intense combat as compared to experience based on war history. Accordingly the completely automated simulation is questionable. The decision to make an engagement is dependent on interactions among environmental factors, on the acquisition process, on an understanding of the capabilities of weapon systems, and on the nature of the mission. These factors are difficult to incorporate into an automated process. For example, in battalion level simulations the engagement is dependent on the existence of line-of-sight, the evaluation of a detection probability, and the assessment of weapon effect at a specific detection range. The validity of the generation process is severely limited by the lack of detection data which are based on experiments and field tests. It is also dependent on an understanding of spatial and temporal aspects of the detection process which is not always available. In aggregated player simulations, the event generating mechanism is usually hidden or merely implied. Models that depend on force ratio assessment do not involve detection specifically.

Assessment routines, for the evaluation of attrition of men or materiel or the degradation of unit capabilities, are similar for any one level of models which are based on computer simulation or on general war game techniques. To provide assessments the computer is highly advantageous in that is provides the means to keep track of a great deal of detail and perform large numbers of calculations in a short period of time.

Analytical models are similar to completely automated simulations in that they have no player involvement. For such models, the military process is decomposed into its basic elements. Mathematical descriptions of the elements are developed and these are incorporated into an overall functional representation of the combat process. An example of such a model is a type known as the Bonder Model. Analytical models are usually difficult to fully comprehend, but when validated against a credible standard are simplest to use and require little time. They are very efficient in conducting sensitivity analyses of the effects of errors in input data and, also, the effects of assumptions about the combat process. They are also efficient in making extrapolations from analyses performed by means of the more complicated techniques of computer simulation and general war game techniques. Analytical models permit easier interpretation of results because the dynamics of the combat process are represented by definite mathematical equations. One can develop an understanding of deterministic cause-effect relationships between input parameters and combat results whereas in stochastic processes used with the other techniques the probabilistic inputs provide only probabilistic outputs or results. Solutions to problems may be calculated with simple mathematical devices or with electronic computers. Obviously, stochastic formulations of problems can be accomplished with analytical models as with any other kinds. One simply represents the values of the variables in probabilistic form. The results will then also be probabilistic and can be used to express degrees of confidence. In chapter 17, starting on page 361 of our text, we discussed such problems in connection with the simple linear oscillator.

It may be useful now to briefly summarize the characteristics of the several types of combat models which we have been discussing. The analytical or mathematical model is readily understood with regard to its usefulness. We consider that we have said enough about it. The other two types we may refer to as the Delphic type and the computer simulated type. We think these terms are in general use and they apply to the models which we have been discussing. The Delphic method which was used by Olaf Helmer in his socialogical studies we described in our chapter 21 on page 301. There we said it is a technique whereby the intuitive judgments of experts are elicited and combined for the purpose of formulating decisions in whatever fields are under study. The computer simulation type of model we mentioned in connection with our remarks about G. Arthur Mihram in chapter 18, page 379, of our text. There we are concerned primarily with stochasticity but, as we mentioned, Mihram entitled his book Simulation and he deals with models which may be described as computer simulators. In any event we are now contrasting the Delphic method of war games and the computer simulator as applied to combat operations.

The Delphic type war games provide the most visibility or picture detail and are best suited for direct involvement of the military sponsor. Although such models require much time and expensive resources, which limit applicability, they must be retained as research tools to

develop new concepts of warfare. They are needed to develop a better understanding of the decision making process; they are necessary for the definition of combat scenarios and distributions of combat situations which seem most likely to occur; and, also, they are useful, maybe essential, for the development of automated processes in computer simulators. They have been, and still are, used extensively. A current example is the war game used by the Royal Armament Research and Development Establishment (RARDE) in the UK for the primary purpose of generating engagement situations in a more realistic environment.

The Delphic type war games provides the opportunity of intelligent play by the decision maker; by it the combat engagement can be planned in advance; it permits the adaptation of maneuver to situation; it provides the possibility of studying many tactical situations (employment, penetration, et cetera); it provides an opportunity to gain insight into the single situations which are analyzed; it permits the controller to determine existence and pace of engagement; it provides the greatest visibility for the user; and it permits the direct involvement of the user. On the negative side, it is slow and expensive. Also only a very limited number of engagement can be examined.

By contrast with the Delphic type model the computer simulation model is much faster in operation and, therefore, many situations can be investigated and the sensitivity of key variables can be tested. Despite these advantages, however, it must use stylized decision routines which are usually fixed. Also, it makes little use of intelligence and is limited in planning horizon. It permits very limited maneuver routines and these are stylized as well as limited with respect to change in formations. The combat always tends to be exceedingly intense. On the positive side, some insight is gained by repeating the analysis for many situations, using different values for key parameters. Characteristically there are predetermined scenarios and engagement rules.

A few general remarks may now be made about some essential requirements for making any combat model sufficiently realistic. Important for the success of modeling is a reliable knowledge of the parameters of combat. Foremost among these are the physical characteristics of materiel and of armed forces units. Unfortunately there is always a lack of data on such matters as the vulnerability of materiel to attack by various weapons and the reaction of troops to combat situations. In fact the entire question of human action and reaction is a soft data area. For example, while we do have some data on man's ability to acquire targets or target intelligence, we do not have nearly enough. Furthermore, we are not sufficiently competent to properly incorporate such factors in a model. Models must involve all sources of intelligence to develop targets. Another soft ware area is our ability to predict the extent to which a man may be suppressed under various combat conditions and volume of incoming fire. The development of a satisfactory data base in the human response area is complicated by the fact that it is not

feasible to conduct human response experiments in the same manner as we do for mechanical systems. A man's response is obviously affected by his emotional state and it is not possible to subject test subjects to the same stresses and hazards as those experienced by a combat soldier. These few comments have been made only for the purpose of emphasizing the importance of the human factors elements in the modeling process.

We consider that we have sufficiently surveyed the military problems in order to properly identify the use of modeling in an attempt to obtain solutions. We invite the reader to re-examine our triadic definition of model in the light of the present example of modeling. He will indeed find that there is a picture, even a moving picture, aspect of the military combat model. Continuously one scene evolves into another towards an end result. The ultimate objective of such a complicated process is is victory or at least containment. Used in conjunction with the pictures are theories of combat and warfare. Finally, there is an essential reliance on experience and experiment in order to attain the desired objective. It may also be apparent, as we have so often suggested, that theories are evolvable.

Finally we wish to add, for the interested reader, a list of descriptions of various kinds of combat models which have been devised for different purposes. The descriptions have been taken from a summary of such material by Braddock, Dunn and McDonald who provided them under a contract with the U.S. Army.

SOME EXAMPLES OF MILITARY COMBAT MODELS

1. AMSAA Duel Model. This is a very low-level, small scale, two-sided stochastic model used to simulate brief fire engagements between two armored vehicles. Its purpose is evaluation of effectiveness of weapon system alternatives for armored vehicles. The model plays a defending vehicle which is stationary, and which always fires first. The attacher vehicle is initially fully exposed to the defender. When the defender opens fire, the attacker vehicle either returns fire or seeks cover and then returns fire. The attacker's ability to return fire is dependent upon acquisition of the defender. The engagement ends when a kill occurs or when a time limit (such as two minutes) has been reached. The model was developed in-house by the Armor Branch, Army Materiel Systems Analysis Agency. Point of contact is Mr. Robert Lake, Autovon 870-3675. Model inputs include probabilities of hit and kill, expected time to fire rounds, and probabilities of detection as a function of number of rounds fired. Outputs include probabilities of win for each side and ammunition expended. This model is programmed in FORTRAN IV for the BRLESC computer.

2. ASARS II (Army Small Arms Requirements Battle Model). The ASARS II Battle Model is a two-sided high resolution, dynamic, Monte Carlo simulation of dismounted combat between less-than-company sized units.

ASARS represents, with a high degree of realistic detail, a substantial protion of the factors involved in or impacting on small infantry unit combat. The model was designed to serve as an operations research tool for evaluating the comparative effectiveness and utility of small arms (pistols, rifles, automatic rifles, machine guns, grenades, and grenade launchers) and various organizations, operational concepts, and tactics of weapon employment in an operational context. Movement paths of the units are generated dynamically within the model to reflect leaders' perception of current battle conditions. The dismounted forces can be supported with artillery and mortar fires represented in detail, and firing from aircraft can be approximated. Antipersonnel minefields are repersented, with options to breach, traverse, or bypass. Intelligence representation focuses on line-of-sight acquisition of opposing personnel and small arms, but also includes unattended ground sensors. The model represents decision processes and events in great detail and affords much flexibility for the user to specify situations and tactical decision rules. Although vehicles and direct fire weapons larger than grenade launchers are not represented, model design permits modifications or expansions into many areas. Terrain elevations are specified at 12.5 meter intervals from map-based digitized tapes of the Topographic Command. Each of up to 100 attacking soldiers is individually represented in up to 20 separate maneuver units. Each exposed man is individually assessed for weapon effects from invididual bullets or flechettes and from fragments from each exploding munition. Hits are recorded by five areas of the body. Probability of incapacitation is computed for each body part hit. These probabilities are translated into inability of the man to observe, move, fire, or fire and move. Suppressive effects of hits and misses are also represented for small arms rounds and grenade fragments, but not for artillery and mortar fire. ASARS II was developed by U.S. Army Combat Developments Command, Systems Analysis Group, and documented in May 1973. The model is maintained by the U.S. Army Infantry School, Ft. Benning, Georgia. Point of contact is MAJ Richard Foss, Autovon 835-2015. The model is programmed in FORTRAN IV for the CDC 6500 and required 84k (decimal) of core, plus tape and disc, for a 31 element scenario, which required on the order of 25 minutes of CPU time. A 60-element scenario has recently (November 1974) required 2 1/2-3 hours of CPU time.

3. <u>ATLAS (A Tactical, Logistical, and Air Simulation)</u>. ATLAS is a fully automated, deterministic theater level model of ground and air combat. Purpose of ATLAS is to assist military force planners to evaluate combat force requirements and capabilities in conventional theater war. A part of the FOREWON automated force planning system, ATLAS has the principal advantage of a rapid rate of game play - approximately 6 days of battle per computer minute. This speed is achieved through use of a highly aggregative and relatively simplistic methodology, which causes the model to have some serious limitations. Combat capability of ground combat units, artillery, and close air support are represented by means of aggregate firepower scores, which are essentially unable to reflect variations in tactical situation, force

mix, weapon systems, and target acquisition. Degradation of unit effectiveness is stylistically derived from casualties, which are calculated for each day of combat, based on an empirically tenuous relationship to firepower-force-ratio and historical casualty data. Daily FEBA movement, a principal output, is similarly derived from firepower force ratio and attacker and defender postures, the latter again derived from firepower ratio. The relationship of logistical constraints, is also tenuous. Intelligence is not played. Straight line sectoring and paths of advance lead to questionable results in logistics and air power application. The rate of combat appears to be excessively intense. Development of the model, by Research Analysis Corporation, essentially began in 1965. It has been used for contingency planning by DCSOPS and JCS, but is no longer being used. A substantially revised version, ATLAS-M, is maintained by U.S. Army Concepts Analysis Agency. Point of contact at CAA is LTC Tom Sanders, Autovon 295-1668. ATLAS-M is operational on the UNIVAC 1108 computer.

4. CARMONETTE VI. CARMONETTE VI is a two-sided high resolution, Monte Carlo simulation of small unit combined arms combat involving ground units ranging in size from platoon to reinforced battalion. Activities simulated include movement, target acquisition, communication, and employment of a variety of weapons, including missiles, by infantrymen, tanks, armored personnel carriers, helicopters, and air defense units. Resolution can be set from platoon level down to the individual vehicle or dismounted soldier. CARMONETTE plays a battle area of 60 x 63 terrain cells, with cell size variable from 10 meters to 250 meters on a side (total battle area from 600 meters x 630 meters to 15 km x 15.8 km) (100 meter cell size is normally used). For each cell the average value is input for terrain height, cover, concealment, height of vegetation and trafficability (road and cross-country). Up to 63 units on each side can be represented, 48 of which can be weapon units and 15 can be command, control, and surveillance units. A predetermined scenario explicitly controls the action of all units, with the exception of certain orders whose execution is dependent on knowledge of and action by enemy or other friendly units. Battles as long as 90 minutes can be simulated. Stable results can often be achieved with 5 to 20 replications. CARMONETTE was essentially the first high resolution computer simulation of this type. It was programmed in 1959 and has since been under modification and use by Research Analysis Corporation (now General Research Corporation). Version III was used in the Small Arms Weapons Study (1967); version IV was used for a study of night vision devices (1969); version V was used in the equal Cost Firepower Study (1971). Version VI has just been employed in the SCAT-II helicopter study. The model can be run by GRC and by U.S. Army Concepts Analysis Agency. Point of contact at CAA is Mrs. Agatha Wolman, Autovon 295-1691. CARMONETTE is programmed in FORTRAN IV and requires 65k (decimal) on the UNIVAC 1108 and 175k (octal) on the CDC 6600 for core storage.

5. CEM III (Concepts (formerly CONAF) Evaluation Model III). CEM III is a two-sided, deterministic, theater-level warfare simulation (fully computerized). It was designed to encompass all combat aspects of theater warfare, in dynamic way, covering an entire campaign and permitting evaluation of a force alternative in about one week, while remaining sensitive to important force characteristics. CEM represents ground combat engagements in a given terrain. CEM resolution is at the level of Blue brigade and Red division. Much input of a judgmental or historical data nature is used. DEM concentrates on representing the sequential decision-making at theater, army, corps, and, especially division levels to determine the allocation of resources and the missions to be undertaken by the various units as the battle progresses. Periodically, estimates of the situation are represented, on the basis of which decisions are reached and implemented, at each of those four echelons. At division level, this process is repeated every 12 hours; at corps every 24 hours; at army every 48 hours; and at theater every 96 hours. Decisions are determined by input alternatives and criteria which are compared with status of units, estimated unit force ratios, missions, postures , and anticipated engagement outcomes down to the brigade front level, by minisector. Unit status reflects losses and replenishments. Losses are a function of engagement type and outcome. Replenishments include personnel and materiel. Estimated force ratios and anticipated outcomes reflect imperfect knowledge. Up to 1000 minisectors can be represented, each designating the front of a resolution unit, which may be opposed by one or more adjacent resolution units whose minisector boundaries need not be coincident with those of the opposer. Minisector traces must be specified as pregame input, conforming to map terrain features. Firepower potential is modified to reflect the circumstances of each engagement, in which only the firepower is counted for which there are targets present. To simplify firepower calculations, ground targets are classed as "hard" (e.g. tank weapons), "medium", or "soft". Similarly, ground missions are classed in three categories: attack, defend, and delay. Four types of terrain are defined: roadway passage only, cross country possible with difficulty, no impedance to movement, and barrier. Decisions made include distribution of replenishments, commitment or retension of reserves, assigment of newly arriving (input scheduled) reingorcing units, allocation of close air support and artillery, and the unit mission and posture to be adopted during the next period. Air resources are similarly allocated to air defense, counterair, armed reconnaissance/interdiction, and close air support. Assessment of employment of air resources includes losses to aircraft inventories, aircraft ground facilities, and air defenses. This assessment also determines whether the air environment is friendly for ground forces, whose delays between allocation and availability are affected accordingly. Fire support is allocated to strong units in attack and to weak units in defense. Engagement outcomes are win, lose or draw, and rate of FEBA movement is based on these outcomes plus input data. Although deterministic, the model may yield substantially different results, from similar forces, because of the complex dependent sequence of threshold-type decisions made during the

course of a lengthy battle, making it difficult to relate cause and effect. CEM outputs WIA and KIA based on input adapted from FM 101-10-1. Other elements of theater personnel replacement are similarly treated. CEM III is an improved version of CEM which was developed in 1971 by Research Analysis Corporation (now General Research Corporation (GRC) for use in the CONAF (Conceptual design of the Army in the Field) methodology and study. CEM can be run by GRC and by U.S. Army Concepts Analysis Agency. Point of contact is Mr. W. A. Bayse, Autovon 295-1693. The model is programmed in FORTRAN IV and requires 100k on the UNIVAC 1108 and CDC 6000 series computer for core storage. Two days of combat require about one minute of CPU time.

6. COMANEX (Combat Analysis Extended). COMANEX is Monte Carlo ground combat model designed to rapidly extrapolate to other force mixes the results of the high resolution CARMONETTE model for a given force mix. Results so extrapolated include losses of dismounted infantry, tanks, APC's, helicopters. The short running time of COMANEX also lends it to use, in division level games/simulations, for assessing small unit engagements. Detailed battle history results from a high resolution model are pre-processed by COMANEX to form a set of Lanchester-type parameters which represent, essentially, the kill rates for each weapon-target combination in the engagement. These parameters are then used to predict battle results when varying input specified numbers of these weapons are involved. COMANEX, running about 100 times faster than CARMONETTE, can provide 30 replications of a 30 minute battle in less than one minute. The model is a revision by General Research Corporation (GRC) of the COMAN model developed by Dr. Gordon Clark of Ohio State University in 1970. COMANEX is programmed in FORTRAN IV for CDC 6400 computer and can be operated by CGR. Point of contact is Mr. Lawrence J. Dondero of GRC, McLean, VA. (703) 893-5900.

7. DBM (Division Battle Model). DBM is a division-level computer-assisted manual war game, designed for a study of the combat impact of varying weapon mixes, organizations, tactics, and support levels. It can address a Blue division opposed by a Red combined arms or tank army, with supporting artillery and airpower. Resolution is generally to company on the Blue side and to battalion on the Red side. Up to 350 units per side can be accommodated by the computer program. The game is played on 1:50,000 or 1:25,000 maps, which are reposted each 15 minutes, in an open, semi-closed, or closed mode. In closed mode, a control team is necessary to process gamer orders, according to game rules, translate to computer inputs, and distribute information to gamers. Gamers perform all battle decision-making functions. The computer performs assessment and bookkeeping. In closed mode, a team of 11 can process 2 to 4 hours of combat in a working day. In open mode, speed can be doubled with a smaller team. Normally, about 4 hours of battle (up to some critical event) is laid out by gamers before the computer is called upon to assess losses and replacements and to update the status of units. Assessment employs the COMANEX model to determine unit losses, based on inputs from the high resolution, small unit

engagement CARMONETTE model. Air-to-air, air-to-ground, and ground-to-air engagements, and air-mobile operations can be played in DBM. Conventional and nuclear munitions can be delivered by air or artillery. Computer portions of DBM are programmed in FORTRAN IV for a CDC 6400 computer. DBM was developed by Research Analysis Corporation (now General Research Corporation (GRC) in 1970-1971. It is operated by GRC. Point of contact is Mr. Lawrence J. Dondero, of GRC, (703) 893-5800.

8. DIVWAG. The DIVWAG model is a predominantly deterministic, two-sided division level, player-assisted computer simulation. It was designed for use in force composition and doctrine studies in mid- and high-intensity environments. It simulates combat between up to one Blue division level force and a Red force composed of up to three divisions. The model addresses all the functions of land combat. It achieves this comprehensiveness of functional coverage only through some sacrifice in the resolution with which specific activities are treated. Therefore, the model should be considered as a medium-resolution model. The user retains general control of the battle by issuing order to idividual units. These orders may be given in a manner that the unit will execute them sequentially, or execution may be made dependent upon the condition of some dynamic element in the battle (e.g., time, the location of a unit, the number of personnel remaining in a unit, etc.). Functions and activities simulated by the DIVWAG model include intelligence, ground combat, area fire, air-ground engagement, mobility, engineer, combat service support, air mobile operations, and the effects of nuclear weapons. The DIVWAG model is capable of simulation up to 14 days of continuous combat. Although designed for force composition and doctrine studies its basic design allows a great deal of flexibility in use. In addition to employment in a War Game of successive periods of play, it can be employed as a pure simulation, without gamer intervention. For example, a single period of engagement, using a scenario from a larger game, can be played to examine the performance of specific systems.

Following extensive testing of DIVWAG in 1972, further refinement and testing has been conducted by the War Games Division, Directorate of Combat Operations Analysis, Combined Arms Combat Developments Activity. The model was used to support the Conceptual Armored Division (CONAD) portion of the Concept for a Family of Army Divisions (CONFAD) study at Fort Leavenworth. In 1973, significant revisions were made to the ground combat submodel, and DIVWAG was exercised in support of the Family of Scatterable Mines Study. The DIVWAG model develops casualties from direct and indirect fires of all weapons except that small arms and other short range weapons of dismounted infantry are not fully represented, principally because of spatial aggregation in the model.

The DIVWAG model is operational on the Control Data 6500 Computer at Fort Leavenworth. The SCOPE 3.4 operating system is being

used. The program, with overlays, resuires approximately 41,000 (decimal) words of central memory storage to execute. One private disk pack (used to store the data files), and two tape drives are also required. Approximately 6000 central processor unit (CPU) seconds are required to load the data base onto disk. The time required to perform the simulation is dependent upon the number of units being played and the complexity of the activities in which they are ordered to engage, but approximately one second of CPU time is required to simulate one second of game time. Approximately 1000 CPU seconds are required each game period to perform postprocessing. Point of contact is Mr. Richard Calkins, CACDA, Autovon 552-4006.

9. DYNTACS X. The DYNTACS X model is a two-sided, small-unit, high resolution, dynamic, Monte Carlo, event-sequenced, highly interactive, land combat simulation. The model is capable of representing battalion or smaller size armor and mechanized units. The basic elements are vehicles and crew-served weapons. Dismounted infantry is not played. Casualties for vehicular movement crews of direct fire crew-served weapons and helicopters are represented and accounted for by the model. Systems which can be represented are vehicles (tracked and wheeled), anti-armor ground weapons (large direct fire ballistic weapons, rapid fire ballistic weapons, and guided missiles), indirect fire (cannon, missile, and mortars), air defense weapons (air defense guns, passive homing missiles, and semi-active homing missiles), terminal homing systems for direct and indirect weapons, mine fields, helicopters (reconnaissance, gun and utility), and artillery fire control system. The principle type of operational variables which can be included are terrain type, roughness, trafficability, obstacles, and day/night conditions. Other variables are engagement type and size, and the type, size, organization, doctrine, and tactics of both Blue and Red forces. The operational area addressed tends to be limited by computer considerations (primarily core storage) to 5 x 10 km with resolution of 100 x 100 meters (a potential exists to reduce grids to 6 x 6). The core storage of the computer being used regulates the number of battle elements represented (along with area and terrain resolution) and time required for a replication. Examples of type computer in relation to core storage, number of battle elements, and CPU time for one replication were CDC 6600 (MERDC)/172k words (OCTAL)/47 elements/7.6 minutes; CDC 6500 (CACDA)/105k (OCTAL)/24 elements/10 minutes; IBM 360-65/670k bytes/47 elements/20 minutes; and IBM 390-91 (Johns Hopkins)/670k bytes/47 elements/4 minutes. A typical run involves 20-30 minutes of battle time. Manpower expenditures to run the model run from approximately 2 man-months for a routine exercise, 2 to 3 man-months to convert from one computer to a similar computer, 6 to 8 man-months for introduction of a new or different system into the model, several months for force structure and number of elements for a newly located scenario, to man-years of effort for an entirely new or different type system not currently represented by the model. Points of contact are Dr. J. J. Hurt, USA ARMCOM, Rock Island, Illinois, 61201, Autovon 793-4202-4143;

Mr. Ernest Petty, USA MICOM, Redstone Arsenal, Alabama, AV 746-4622; Mr. David Farmer, USA CACDA, Ft. Leavenworth, Kansas, 66027, AV 552-5258; Dr. Gordon M. Clark, Systems Research Group, Department of Industrial Engineering, Ohio State University, Columbus, Ohio (614) 442-7862.

10. FAST-VAL. The FAST-VAL Model is a two-sided deterministic computer model which simulates the ground engagement between two Infantry forces with and without varying amounts of Fire Support (air, artillery, and mortar). The model was developed to assist the Air Force in selecting weapons, vehicles, and operational techniques for their close air support role. The simulation can represent an infantry force of up to five companies on the defense in prepared and unprepared positions and up to five companies attacking the defender's forces. Infantry units are identified down to company size and the artillery and mortars are played as batteries. Those weapons which are represented in the infantry force are rifles and machine guns. The supporting fires which are represented are air delivered, artillery, and mortar weapons. Two degrees of protection can be given to the infantry and support personnel in dismounted positions. Protection factors can be assigned on a permanent or a temporary basis to bunkers for defenders, and for the attacking force mounted in APCs moving to the line of departure. The battle area is broken down into 100 meter grid squares with personnel and weapons plaued being identified with each grid. Weapons effects (personnel) losses and material losses) are calculated for artillery/mortar rounds, volleys, and for concentrations and air delivered sticks or patterns. The Full Spray Lethal Area Program is utilized to evaluate the effects of fragmentation of the individual rounds. Round ballistic and volley aim dispersions are used to transform Full Spray damage functions into volley pattern damage functions. These pattern damage functions are used to calculate casualties at targets in the vicinity of as well as at aim point. Rifle and machine gun weapons effects are expressed as expected casualties, as a function of range and posture of targets for both single round fire and burst of rounds fire. Provision for reduced efficiency in delivery of firepower and speed of movement due to suppressive fire has also been incorporated into the model. The input requirements for the model include definition of the attacking unit-characteristics of riflemen and support personnel, posture/time tables, weapons/vehicle characteristics, rifle company characteristics, delivery schedule, range limits and firing rates; definition of defending units - same as those for the attacking unit; definition of the Infantry Action - engagement table; advance characteristics - engagement ranges, troop carrier characteristics, influence of suppression, influence of cumulative fraction of casualties upon advance rates, and small arms characteristics for attacker and defender. A summary of the status of all units, and a summary of the status of the several engagements is printed for each simulation cycle. Additionally, the user may request printouts describing the status of all units at the end of each cycle and the aim points selected for mortars, artillery and air delivered weapons. As can be seen by the description, the model is of high

resolution, prodices detailed output and is free running once started. However, limitations are that only rifles, mortars, and machine guns of the infantry force are represented while, in reality, today's mechanized infantry units have many more supporting weapons (grenade launchers, LAWS, recoilless rifles, tanks, etc.) organic to the organization or attached during battle. Additionally FAST-VAL does not discriminate between KIA and WIA although such a capability could be added, as in most of the other models. FAST-VAL does address the small unit engagement area in considerable depth, and significant efforts have been made to compare its results to those of a series of actual small unit engagements in Vietnam. The fact that these comparisons are surprisingly close, in number of Blue casualties incurred, and the outcome or winner of the fight does not so much prove the rectitude of FAST-VAL as a predictor of small unit infantry casualties and fight outcomes, as it confirms what has been shown elsewhere: given operational inputs that are correct in essentially all respects, a reasonably designed model can accurately recreate historical results. FAST-VAL attempts to predict the outcome (winner and casualties) of a fire fight as a function of weapons and tactics employed rather than predicting, for example, what engagements will occur in a battle or how much of what munition will be employed in a fight. The program is written in FORTRAN IV for an IBM 360/65 computer and requires 190k bytes of core storage memory. The model was developed by RAND for Air Force during 1970-1971 to support Air Force requirements for Close Air Support. The point of contact for this model is Deputy Chief of Staff, Research and Development, Attn: RAND Project Office, Headquarters, U.S. Air Force, Washington, DC. 20330, Autovon 227-3001.

11. IUA (Individual Unit Action). IUA is a two-sided, high resolution, large scale Monte Carlo simulation of mounted ground combat. IUA can represent up to a battalion task force in offense, defense, and delay at engagement ranges up to 3,000 meters. The model was developed in the mid-sixties for evaluating the combat effectiveness of equal-cost mixes of armor and antiarmor weapons. A strength of IUA is its ability to simulate in detail direct fire weapon effectiveness to include weapons such as tanks, armored personnel carriers, recoilless rifles, rocket launchers and guided missiles. IUA has a limited capability to portray minefield, artillery, helicopter-borne weapons, and TACAIR. Dismounted infantry is not played. Movement is on predetermined routes. Defender does not maneuver, but can withdraw to a secondary position. The simulation of mobility and line-of-sight are done deterministically by mobility and terrain preprocessor computer programs. The defenders are always considered in hull defilade. Terrain is represented by up to 999 triangles, with map elevation to the nearest meter specified for each vertex. Generally, a battle area of 5 x 8 km is represented. Input can include five soil types, 13 obstacle types, three concealment heights, six terrain roughness types, and three cover heights. The attacking force has one to three prespecified axes of advance. A total of up to 12 routes are prespecified, with two force sections, and developed by Lockheed in the mid-sixties to support TATAWS; the model was improved by

Booz Allen in 1970. IUA has been used in nine studies, such as TATAWS III, ATMIX, and CONFADS. The model requires approximately 162k (octal) of core storage for execution and approximately 10 minutes of CPU time for 30 replications of one case, on the TRADOC 6500 computer at Fort Leavenworth. Input data preparation time, for a new terrain, scenario, and weapon data set requires on the order of 10-12 man-weeks. One full-time analyst, plus programmer support, is required to operate the model. Point of contact is Mr. Kent Pickett, of CACDA, Autovon 552-5258.

12. BONDER/IUA. Bonder/IUA is a differential model based on the IUA (Individual Unit Action) model. As such, Bonder/IUA is a two-sided, high-resolution, large scale, analytic model of tank-antitank combat which can represent up to a battalion task force in offense, and defense (the delay role cannot be played) at engagement ranges up to 3000 meters. Bonder/IUA uses the same terrain, route, and mobility data as IUA, and therefore depends upon the same deterministic mobility and terrain pre-processor programs as IUA in order to simulate mobility and line of sight. The principal difference between Bonder/IUA and IUA is in the attrition assessment portion of the program. While IUA uses Monte Carlo techniques in this area, Bonder/IUA uses analytic techniques involving modified Lanchester equations. Bonder/IUA requires no replication; therefore, Bonder/IUA uses approximately 5 minutes of CPU time to execute on the TRADOC CDC 6500 computer, as compared to approximately 10 minutes for 30 replications by IUA. The short running time of Bonder/IUA and the relative ease of changing tactics and weapon mixes (provided no changes are required in the prespecified routes) enables users to review a number of weapon mixes and tactics with minimal cost. Bonder/IUA requires about 150K (octal) of core storage to execute. With respect to the assumptions made as to the combat process, and the limitations therein, Bonder/IUA and IUA are identical. Bonder/IUA was developed in 1970 by Vector Research, Incorporated, and has been used by the Studies, Analysis and Gaming Agency of JCS for several studies; by ACSFOR for the DRAGON Cost Effectiveness Analysis; by Rock Island Arsenal and the Weapons System Analysis Directorate, OAVCSA, in the MBT Study and the M60A1 Improvement Study, and by USACDC in the ATMIX Study. Although the model must not be considered validated against test data, the model has been validated against IUA, and validation effort underway for IUA applies also to Bonder/IUA. The model is maintained at CACDA. Point of contact is Mr. M. G. Minnick, Autovon 552-5481.

13. CAC Manual Wargame (Jiffy). The Jiffy Game is a manual methodology for two-sided war gaming. Computer support is not currently used. Players manually manipulate forces, using maps and performance indicators developed in previous non-manual studies, to simulate ground combat. The game can handle from platoon through theater level force. It was developed or evolved as a highly flexible, simple and rapid procedure for preliminary investigation of the relative value or effectiveness of different force designs. Units are identified and placed in their position on the map. Firepower scores are aggregated for each side, force ratios are calculated, modified in accordance with the situation, and rates of advance are determined. Previous non-manual

studies and Field Manuals provide the factors used to quantify the performance of weapons systems and to calculate attrition resulting from combat. Personnel and equipment losses and utilization are determined, and requirements for replacements are derived. Utilization and requirements for artillery and engineer support are determined. Elements whose status is specifically addressed include field artillery; ADA; TACIAR; trains elements; dismounted infantry; antitank weapons; armed helicopters; command posts; tanks, APCs, and ICV's; mortars; and minefields. Resolution is to the level required, but normally addresses the battalion. The battle is assessed periodically for periods during which committed combat power remains constant, termed "critical incidents" (significant events). It requires approximately two to five days to run a critical incident, depending on the evaluation objectives assigned. The output of the exercise is a narrative, photographic and statistical display of the progress of the battle to include the listing of personnel losses, major supplies consumed and equipment lost. The most apparent limitations are that the source references used as inputs to the methodology are supportive of a wide variety of specific purposes, not specifically related to each other, nor necessarily in agreement. The value judgements of the gaming team dictate the relationships of study inputs and their specific adaptation to the gaming process. Jiffy Game was developed by USACDC, ICAS. The point of contact is LTC T. W. Buff, Directorate of Concepts and Force Design, USA CACDA, Fort Leavenworth, Kansas 66027, Autovon 552-4731.

14. LEGAL MIX IV. Legal Mix IV is a one-sided high resolution deterministic simulation developed to evaluate artillery mixes at Field Army and lower levels. Artillery Weapons are employed against a time-phased set of acquired targets. Primary uses of the model are to provide data on artillery support requirements and to provide comparative analyses on the effectiveness of alternative mixes of artillery weapons. The model computer percentage of missions lost, personnel casualties inflicted, armored vehicles damaged, missions accomplished, targets defeated, accrued units of military worth for missions accomplished, and cost and weight of ammunition expended to achieve effects. Military worth is an average value assigned to each target processed in the model and was derived from questionnaires in which military officers assigned priorities to the existing Legal III target list. Weapon system rate of fire, ammunition basic loads and resupply rates, predicted and precision weapon circular probable errors, weapon range capabilities, ammunition lethality data, and ammunition costs are used as inputs. Legal Mix IV is written in FORTRAN and is operational on the TRADOC CDC 6500 computer. Required core space is 110K (octal) for the largest of four basic computer programs. Computer run time can take from 8-25 minutes for the effectiveness program. Preparation time is substantail. Point of contact is Mr. William Milspaugh, U.S. Army Field Artillery School, Combat Training and Developments Activity, Autovon 639-5707.

15. TARTARUS IV. TARTARUS is a player-assisted, two-sided, differential model of theater level combat, with resolution to the brigade or division. The model is designed to study the effects of weapons systems and their mixes and can simulate the attack, defense, delay and counter-attack. Firepower scores modified by the interactions are used in the differential equations to assess movement and casualties. The model represents the effects of tank, infantry and tank/infantry forces supported or not supported by artillery or like forces. Target acquisition, engagement, and movement are played. Four to 300 Brigade/Division size units can be played with 94 items of equipment identified (10 weapons classes and 3 firing classes can be programmed). Every battle hour or as input, opposing units are acquired and the target list is updated. The firing interval may be as small as 1 minute. Weapons are assumed to distribute their fires among available targets within range according to a formula based upon unit mission, range to the targets, surveillance factors, and maximum range-firing fraction (a factor given to weapons based upon its capability to fire at maximum range in a particular type mission - i.e., hasty defense, attack, etc.). The computer developed assessment is highly sensitive to the weapons-class versus weapons-class effectiveness factors which combined with unit "hardness indicators" and "breakpoints" will determine the outcome of any simulated engagement. The outputs of the model are a unit status report which gives the general status of the unit; detailed strength and loss report (strengths and losses of each unit by weapon class); ammunition and fuel expenditure report; summary of losses by weapon class and side; number of weapons lost by unit and weapon type and displays (off-line Calcomp Plotter) showing unit location, frontages, unit movement routes, and terrain data set. The model was developed by the U.S. Army Strategy and Tactics Analysis Group (STAG) and is written in FORTRAN V for the UNIVAC 1108 computer. The effort required to run the model is based on the number of units and size of the area played. Points of contact are Miss Pat McGroddy (301) 295-1645 or Mr. Ben Robbins (301) 295-1695, USA Concepts Analysis Agency, 8120 Woodmont Avenue, Bethesda, Maryland 20014.

16. TBM. The Theater Battle Model (TBM) is a comprehensive manual war game of tactical combat operations involving all types of theater forces (land, sea and air) under a conventional or nuclear environment. The level of resolution for land forces is the division; for air elements, the flight for conventional weapons and the sortie for nuclear weapons; and, for sea forces, the Task Force. Research Analysis Corporation (RAC) was directed in 1968 to develop a family of compatible models adding capabilities to simulate CBR, air mobile operations, and counter guerilla warfare operations to the 1963 version of TBM. The models were Theater War Game, Theater Quick Game, Division Operations, Amphibious Warfare and Counterguerrilla Warfare Model. While elements of this TBM appear in several different war games which carry TBM in their name, the version referred to here is a tactical war game which has been reported to acquire 30 gamers and to proceed at a 1:1 ratio of combat to real time. Point of contact is B235 (MAJ Ed Davis), NMCSSC, the Pentagon, Washington, DC 20301, Autovon 225-3780.

17. THEATER AMMORATES (Theater Nonnuclear Ammunition Combat Rates Model). THEATER AMMORATES is more properly called a methodology than a model, since a group of nine models are employed separately, with the results of one being a partial input to another, in a series of off-line steps culminating in a final processing and aggregating run by the Theater Rates model. THEATER AMMORATES was designed to predict Army expenditures of nonnuclear ammunition in hypothetical theater campaigns of 90 or 180 days in Europe and the Pacific. Such predictions are to serve as the basis of Department of the Army plans and decisions on ammunition stockage and procurement, as a part of the annual DOD budgetary process. The Theater Rates model, which generates the final output, is a two-sided, deterministic model of theater level ground warfare, including artillery and helicopters. Theater Rates uses specially developed scenarios, and input data from the various submodels, to simulate a theater campaign, including intense initial periods of conflict and subsequent sustaining periods. The eight submodels are of various types. The Tank-Antitank submodel and the Helicopter Antiarmor model are both two-sided, small unit, high resolution, deterministic models. The Infantry submodel is a two-sided, small unit, high resolution, Monte Carlo simulation. The Helicopter Antipersonnel submodel is a one-sided, small unit, high resolution, Monte Carlo, simulation. The Artillery Casualty Assessment submodel is a one-sided, high resolution, Monte Carlo, munition delivery and target effects simulation. It is supported by a one-sided, Monte Carlo target acquisition simulation and separate deterministic models, for Red and Blue artillery, representing tactical rules and weapons allocation processes of the fire direction center and the availability of weapons to respond to the time-phased fire missions. The Air Defense submodel is essentially a one-sided manipulator of judgmentally-derived input data. As a whole, THEATER AMMORATES represents most of the major types of weapon system-versus-unit interaction that result in personnel casualties Close air support by fixed wing aircraft, however, is not represented, except by Air Force input data. THEATER AMMORATES is unusual, moreover, in being intended to generate, with limited resources, numbers having a reasonable degree of absolute validity, rather than simply the relative validity which is often sufficient for comparative evaluation of forces or weapon systems. Thus, the makeup and development of this overall model reflects some concern with the matter of "representativeness" of the subnumbers used and generated, and with the matter of creating realistic rates of battle activity as far as ammunition expenditures, and to some degree casualties, are concerned. Typically, in operation of the model, military experience is used to define, based on a deatiled scenario, the small unit engagements likely to occur in each of a series of consecutive 6 hour periods, for a typical division slice. As many as 40-50 such engagements may be identified in one such 6 hour period. Based on those defined engagements may be identified in one such 6 hour period. Based on those defined engagements, about 100 representative engagements are simulated with the relevant high resolution submodels. Results of those simulations are used to fill 80 main cells in a limited-situation matrix, reflecting four operation types or

"postures" (attack, defense light, defense heavy, and delay), five types of engagement (infantry, tank-antitank, helicopter antitank, helicopter anti-personnel, and indirect fire support), and four 6 hour portions of the day and night.

18. TXM (Tank Exchange Model). TXM is a two-sided medium-high resolution stochastic simulation model developed to assess tank lethality and vulnerability. A total of 10 elements can be used as inputs and can be either tanks or anti-tank weapons. Any combination of the 10 elements may be examined, a constraint being that attacking tanks must be all of the same type. Attacking tanks are allowed movement along straight line predetermined paths whereas defending tanks and anti-tank weapons remain stationary. Line of sight is prescribed by the user and scoring is on a one-to-one basis. Into each such cell is loaded data on Blue and Red ammunition expenditures, personnel casualties, armor losses, and helicopter losses. The Theater Rates model then accesses this data and aggregates and extrapolates to the theater (U.S. Army Sector). In performing this process, the model updates the Index of Comparative Firepower (ICF) scores of the opposing forces to account for losses, reinforcements, replacements, and returns to duty, and uses the ICF, together with scenario data, criteria, and doctrine to define frontal activity on a period by period basis. A cumulative total of expenditures, casualties, and losses is recorded for each period and at the end of the campaign, for the theater. THEATER AMMORATES was initially developed in 1967-68 by Eyler Associates, Frederick, Maryland for what is now known as U.S. Army Concepts Analysis Agency (CAA). Modification and improvements have been made by CAA, where the model is maintained with a staff of five analysts and has been exercised annually since 1968. It is programmed in FORTRAN IV for the UNIVAC 1108 computer, on which the various submodels each require from 20 to 50K of core and consume from about 1 minute to 3 hours of CPU time per "case" run. Point of contact is Mr. C. E. Van Albert of CAA, Autovon 295-1696.

Two references are useful in connection with the survey. They are listed below.

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1. Review of Selected Army Models by I. Henry, R. Blum, H.Holland, D. Howes, D. Lester, K. Meyers, and R. Zimmerman, Department of the Army, May 1971.

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